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A

PRACTICAL TREATISE

ON THE

STRENGTH OF MATERIALS

INCLUDING THEIR

ELASTICITY

AND RESISTANCE TO

IMPACT.

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PREFACE.

The Strength of Materials is a subject of the very first importance to Engineers and others engaged in the Industrial Arts, forming as it does the basis of all constructive calculation. The absence of reliable Rules, or the misapplication of even correct ones, imperfectly understood, may lead to serious consequences;—on the one hand to a useless excess of strength involving heavy pecuniary loss: or on the other hand to inadequate strength, which may issue in disastrous failure.

Two special objects have been kept in view throughout this work: 1st, that the Rules and Data shall be correct, and therefore trustworthy, and 2nd, that their application to practice shall be clearly understood; for which purpose, every Rule has been illustrated by examples worked out in detail.

To effect the first object, every Rule has been subjected to the test of experience; almost every available experiment having been examined and compared therewith, the error, or rather the difference per cent. between the Rule and Experiment being given in each case. When the theoretical laws did not bear that test, they were relentlessly modified, or abandoned altogether in favour of Empirical Rules whose accuracy was proved by experiment, although they did not admit of a theoretical demonstration. In that case the great object has been so to modify the Rules that the mean results of calculation should practically agree with the mean results of experiment; and this is all that can be done, for the natural variableness in Materials, will always preclude perfect and uniform coincidence.

The authorities for the experimental Data, &c., are given as they occur, but the wonderfully refined and exhaustive labours of Mr. Hodgkinson should be more particularly mentioned. It is matter for regret that he did not fully analyse his own experiments, nor deduce from them all they were capable of teaching, as for example those on the important subject of the Wrinkling Strain in Plate-iron Beams and Pillars. This omission is supplied to some extent,—however imperfectly, in the present work.

Newton-Abbot, Devonshire,
March, 1883.

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CONTENTS.

CHAPTER IV.—Strength of Chain, Ropes, &c.

CHAPTER V.—On the Shearing Strain.

CHAPTER VI.—On the Crushing Strain.

CHAPTER VII.—On the Strength of Pillars.

[redacted]

CHAPTER VIII.—Connection of Pillars with Transverse Strains.

CHAPTER IX.—On the Wrinkling Strain.

CHAPTER X.—On the Transverse Strain.

CHAPTER XV.—On Extension and Compression.

CHAPTER XVI.—On the Deflection of Beams.

CHAPTER XVII.—On Torsional Elasticity.

CHAPTER XVIII.—On the Modulus of Elasticity.

CHAPTER XIX.—On Permanent Set.

CHAPTER XX.—On Impact.

CHAPTER XXI.—Collapse of Tubes.

CHAPTER XXII.—On the Factor of Safety.

[redacted]

CHAPTER XXIII.—Fatigue of Materials.

APPENDIX.

STRENGTH OF MATERIALS.

CHAPTER I.

ON THE TENSILE STRAIN.

THERE are five principal strains to which materials may be subjected, namely, the Tensile, Shearing, Crushing, Transverse, and Torsional strains: essentially all strains are modifications or combinations of the Tensile and Crushing ones, but it will be convenient in practice to consider each of them as distinct and specific.

(1.) "*Central Strain.*"—When the cross-section of a body is of a regular figure, and the tensile strain is in the centre, it is commonly admitted that the resistance is simply proportional to the area, and that every part of the section is equally strained. This may be practically true in many cases, but where the body is wide or large, the central part is more stretched than the edges, and the strain becomes very unequal. For example, Fig. 1 is a plate of very elastic material whose normal form unloaded is *a*, *b*, *c*, *d*, and when strained by the central load W it becomes *e*, *f*, *g*, *h*. Obviously the central part is more stretched and therefore more strained than the edges, and if the load be increased up to the point of rupture, the plate will break first at the centre.

(2.) "*Strain out of Centre.*"—When the strain coincides with one edge of a plate as in Fig. 2, the primitive form *i*, *k*, *l*, *m*, tends to take the form *n*, *o*, *p*, *r*, and we have this remarkable result, that the maximum extension and corresponding strain at *n*, *o*, is progressively reduced towards *s*, *t*, where it becomes *nil*, and between *s*, *p*, and *t*, *r*, the plate is *compressed*, not

stretched, and thus a crushing strain is created by a nominally tensile one.

Say, that we take a spiral spring whose normal length unloaded = 10 inches, and its elasticity such that it extends 1 inch per lb.; also let 4 lbs. be the breaking weight, the maximum length being then 14 inches. Let B, C, D, E in Fig. 3 be four such springs attached at equal distances to two rigid cross-bars F, G: if now a tensile strain of 16 lbs. be applied at the centre-line H, J, obviously the whole of the springs will be extended to 14 inches, each yielding the 4 lbs. due to it.

In Fig. 4, K, L, M, N, are the centre lines of four springs similar to those in Fig. 3, but here the centre line of the strain coincides with L, Q, or the centre of the spring L. Now, it is essential that the forces on the two sides of the centre line should balance one another: they will arrange themselves as in the figure; thus the strain on K being 4, and its distance from the centre = $1 \cdot 0$, we have $4 \times 1 = 4$, as the effect of the spring K. Then, on the other side, $M = 2 \times 1 = 2 \cdot 0$, and $N = 1 \times 2 = 2 \cdot 0$ also: the sum of the two being 4, or the same as K. Then the weight at W, with which the spring K will break, becomes as in the figure, $4 + 3 + 2 + 1 = 10$ lbs., whereas with a central strain as in Fig. 3 we obtained 16 lbs., or 60 per cent. more than in Fig. 4.

(3.) To show how a compressive strain may be generated by a nominally tensile load, let Fig. 5 be an arrangement similar to the preceding, but one where the tensile strain coincides with the centre line of the spring R, or the extreme edge of the combination. In this case the spring R bears the maximum load of 4 lbs., but S = 2 lbs. only: the spring T is *neither extended nor compressed*, but retains its normal length of 10 inches; it is therefore useless. The spring U is *compressed* to the length of 8 inches, and bears a *crushing* strain of 2 lbs.

The tensile load at X from R = 4 lbs., from $S = 2 \times 1 \div 2 = 1 \cdot 0$; from T = 0, and from $U = 2 \times 1 \div 2 = 1 \cdot 0$; the total being $4 + 1 + 0 + 1 = 6$ lbs., whereas with a central load as in Fig. 3 we had 16 lbs.; hence the ratio = $6 \div 16 = \cdot 375$ to $1 \cdot 0$. Mr. Hodgkinson found by experiment that a cast-iron bar which broke with a central load of $7 \cdot 65$ tons, failed with

2·62 tons only when the force coincided with one side of the bar, the ratio being $2·62 \div 7·65 = ·342$ to 1·0, or nearly as we found it by calculation.

These illustrations will serve to show the importance of arranging for the tensile strain to coincide with the axis of the body, or the centre line of the section, and that where this is impracticable, due allowance should be made for the fact.

(4.) "*Experimental Results.*"—Table 1 gives a general summary of the most important experiments on the tensile strength of materials, from which it appears than the mean breaking weight of:—

Cast Iron	Wrought Iron	Steel Bar	Copper Bolts

may be taken at:—

7·142	25·7	47·84	16·0

tons per square inch, which is equivalent to:—

16,000	57,500	107,160	35,840 lbs.

Table 2 gives the breaking weight of round bars from $\frac{1}{4}$ inch to 3 inches diameter, calculated from these data.

(5.) "*Effect of Re-melting Cast Iron.*"—Ordinary cast iron is usually from the 2nd fusion, pig iron being the 1st: it has been found that with some kinds of iron at least, the tensile strength is very much increased by repeated re-melting; thus one set of experiments gave for iron of the

1	2	3	4th

melting, the tensile strength per square inch =

14,000	20,900	30,300	35,785

lbs. Another series gave

11,020	15,942	35,846	45,970

lbs. The mean of the two series in tons per square inch =

5·6	8·2	14·65	18·26

(6.) But Mr. Fairbairn obtained very different results, as given by Table 3, which shows that the transverse and tensile strengths were *reduced* by re-melting so far as the 3rd, then

LE STRENGTH OF MATERIALS.

Breaking Weight per Square Inch. Max.	Min.	Mean.		Authority.
lbs.	lbs.	lbs.	tons.	
62,886	60,075	61,480	27·4	Napier and Sons.
62,231	56,715	59,473	26·5	"
48,232	47,885	48,043	21·4	"
56,805	49,564	53,185	23·7	"
68,848	44,584	57,555	25·7	Kirkaldy, 188 Exp.
57,342	33,288	47,266	21·1	"
47,846	34,344	41,736	18·6	"
66,848	47,095	58,197	25·97	"
55,277	50,127	52,702	23·52	Fairbairn (in joints).
58,464	53,536	54,073	24·14	E. Clark.
51,296	48,384	49,504	22·10	"
..	..	52,193	23·30	"
..	..	53,760	24·00	"
56,005	52,000	54,002	24·10	Napier and Sons.
50,515	46,221	48,368	21·7	"
62,544	37,474	50,737	22·6	Kirkaldy, 167 Exp.
60,756	32,450	46,171	20·6	" 160 Exp.
61,650	34,962	48,454	21·6	"
61,579	43,805	52,486	23·43	W. Fairbairn.
45,743	37,161	41,590	18·56	"
58,286	52,352	53,635	23·94	"
58,915	49,852	55,029	24·56	"
61,588	53,570	54,993	24·55	"
51,191	46,047	48,619	21·70	"
44,291	41,391	42,841	19·12	"
53,349	48,912	51,130	22·82	"

" " Shropshire, solid, crossway ..	49,651	48,912	49,281	22·00	"
" " Staffordshire, solid, lengthway	45,329	42,314	43,822	19·56	"
" " Staffordshire, solid, crossway	49,100	46,798	47,950	21·40	"
" " mean of the four kinds, &c. ..	51,675	47,224	49,208	21·96	"
Steel bar, rolled or tilted	148,294	65,158	107,160	47·8	Kirkaldy, 66 Exp.
" " hardened in water	..	..	90,049	40·2	"
" " " tempered yellow	..	..	100,983	45·0	"
" " " spring temper	..	..	104,888	46·8	"
" " " tempered blue	..	..	112,119	50·0	"
" " highly heated and cooled in oil	..	..	215,400	96·1	"
" hardened and annealed	..	..	121,716	54·3	"
" welded joints	52,784	38,772	45,778	20·4	"
Steel-plate, solid or unpunched	..	..	81,133	36·22	H. Sharp.
" punched plate, not annealed	59,786	38,073	49,235	22·00	"
" " annealed	79,289	67,157	73,562	32·84	"
" drilled plate, annealed or not	83,485	78,893	81,189	36·24	"
" double-riveted joints, punched holes, annealed plate, steel rivets	96,163	87,606	91,885	41·02	"
" solid plate, lengthway	102,593	72,408	90,000	40·1	Kirkaldy.
" " crossway	97,150	67,686	81,960	36·6	"
" mean of the two	99,870	70,047	85,977	38·4	"
Steel rivets in double-riveted joints	58,128	42,000	53,245	23·77	H. Sharp.
Wrought-iron chain, short-linked	48,272	36,557	42,380	18·92	Hawks and Crawshay.
" " stud-linked, cable	48,765	33,376	38,443	17·43	"
" " " "	56,089	45,002	48,160	21·5	Captain Brown.
" drawn welded press-pipe	..	..	14,270	6·37	E. Clark.
" wire $\frac{1}{10}$ inch diameter	83,000	78,430	80,214	35·8	Telford.
Cast-iron, British	23,468	12,716	16,000	7·142	Hodgkinson and Fairbairn.
" Stirling's toughened	32,077	23,461	27,916	12·46	" Owen, &c.
Aluminium bronze	96,300	..	73,185	32·67	Anderson.
Antimony, cast	..	..	1,273	0·568	Muschenbroek.

TABLE 1.—Of the TENSILE STRENGTH OF MATERIALS—*continued.*

Material.	Breaking Weight per Square Inch.				Authority.
	Max.	Min.	Mean.		
	lbs.	lbs.	lbs.	tons	
Bismuth	..	..	3,692	1·65	Muschenbroek.
Brass, yellow, cast	..	..	17,968	8·01	Rennie.
„ copper 2, zinc 1	35,880	23,920	28,940	12·9	Anderson.
Gun-metal, copper 9, tin 1	41,000	22,500	31,280	14·0	„
Muntz-metal, copper 6, zinc 4	..	..	49,280	22·0	„
Copper, cast, pure	34,474	26,410	31,830	14·2	„
Copper bolts, 2½, 2, 1, and ¾ inch diameter	36,400	35,280	35,930	16·0	„
„ cast	22,570	19,072	20,638	9·21	Rennie.
„ hammered	..	..	33,792	15·1	„
„ wire	..	..	63,230	28·2	„
Copper 12, tin 1, cast	..	..	29,000	12·95	Anderson.
„ 11, „ 1, „	..	..	30,700	13·70	„
„ 10, „ 1, „	..	..	33,000	14·73	„
9, „ 1, „	..	..	38,000	17·00	„
Lead, cast	..	..	1,814	0·81	Rennie.
„ wire	3,146	2,547	2,757	1·23	Morveau, &c.
„ milled sheet	..	..	3,328	1·48	Tredgold.
„ drawn pipe	2,611	2,412	2,510	1·10	Jardine.
Silver, cast	..	..	40,902	18·26	Muschenbroek.
„ wire	..	..	38,256	17·08	Sickingen.
Platinum, wire	56,473	52,987	54,730	24·43	Morveau, &c.
Tin, cast	6,650	4,726	5,566	2·48	Rennie, &c.
„ wire	7,129	6,400	6,765	3·02	Morveau, &c.
Zinc, cast	..	..	2,689	1·2	Muschenbroek.
„ sheet	..	..	16,600	7·41	Tredgold.
„ wire	..	..	22,550	10·07	Morveau.

TABLE 1—*continued*.—TENSILE STRENGTH OF TIMBER.

Kind of Wood.	Max.	Min.	Mean.*		Authority.
	lbs.	lbs.	lbs.	tons.	
Alder	..	..	14,186	6·3	Muschenbroek.
Apple	..	..	19,500	8·7	Bevan.
Ash	17,850	15,784	17,077	7·6	6. Barlow.
,,	19,600	16,700	18,150	8·1	2. Bevan.
,,	..	..	12,000	5·4	Muschenbroek.
Beech	11,626	11,388	11,467	5·1	3. Barlow.
,,	..	..	17,709	7·9	Muschenbroek.
,,	..	..	22,200	9·8	Bevan.
Birch	..	..	15,000	6·7	,,
Box	20,348	19,595	19,890	8·9	3. Barlow.
,,	..	..	15,500	6·9	Bevan.
,,	..	..	20,000	9·0	Muschenbroek.
Cedar	..	..	11,400	5·1	Bevan.
,,	..	..	4,973	2·2	Muschenbroek.
Chestnut, horse	..	..	12,100	5·4	Bevan.
,, sweet	..	..	10,500	4·7	,,
,, ,,	..	..	13,300	6·0	Muschenbroek.
Damson	..	..	14,000	6·3	Bevan.
Deal, Norway	18,100	17,600	17,850	8·0	,,
,, Christiania	14,000	12,300	12,900	5·8	,,
,, English	..	..	7,000	3·1	,,
Elder	..	..	15,000	6·7	,,
,,	..	..	10,000	4·5	Muschenbroek.
Elm	..	..	13,490	6·0	,,
,,	..	..	14,400	6·4	Bevan.
Fir	13,448	11,000	12,203	5·5	12. Barlow.
Hawthorn	10,700	9,200	9,950	4·4	Bevan.
Hazel	..	..	18,000	8·0	,,
Hickory	..	..	20,700	9·2	Hatfield.

Kind of Wood.	Max.	Min.	Mean.		Authority.
	lbs.	lbs.	lbs.	tons.	
Holly	..	..	16,000	7·1	Bevan.
Lance-wood	24,696	23,400	24,048	10·7	,,
Larch	..	..	8,900	4·0	,,
,,	..	..	10,220	4·6	Rondelet.
Lignum-vitæ	..	..	11,800	5·3	Bevan.
Mahogany	8,224	7,950	8,041	3·6	3. Barlow.
,,	21,800	16,500	19,150	8·5	Bevan.
Maple	..	..	17,400	7·8	,,
Oak, English	12,008	8,889	10,389	4·6	6. Barlow.
,, ,,	19,800	15,000	17,400	7·8	2. Bevan.
,, ,,	..	..	17,300	7·7	Muschenbroek.
,, ,,	..	..	13,950	6·2	Rondolet.
Pear-tree	11,537	8,834	9,822	4·4	3. Barlow.
Pine, St. Petersburg	..	..	13,300	5·9	Bevan.
,,	..	..	7,818	3·5	Muschenbroek.
,, Norway	14,300	12,400	13,350	6·0	Bevan.
Plane-tree	..	..	11,700	5·2	,,
Plum-tree	..	..	11,800	5·3	Muschenbroek.
Poplar	6,641	4,596	5,618	2·5	,,
,,	..	..	7,200	3·2	Bevan.
Sycamore	..	..	13,000	5·8	,,
Teak	15,405	14,662	15,090	6·7	3. Barlow.
,, old	..	..	8,200	3·7	Bevan.
Walnut	..	..	7,800	3·5	,,
,,	..	..	8,130	3·7	Muschenbroek.
Willow	..	..	12,500	5·6	,,
,,	..	..	14,000	6·3	Bevan.
Yew, Spanish	..	..	8,000	3·6	,,

STRAIN PERPENDICULAR TO THE GRAIN.

Kind of Wood.	Max.	Min.	Mean.		Authority.	Kind of Wood.	Max.	Min.	Mean.		Authority.
Larch	1,700	970	1,335	·596	Tredgold.	Memel	840	540	690	·310	Bevan.
Oak, English	..	..	2,316	1·034	,,	Fir, Scotch	..	..	562	·250	,,

TABLE 2.—Of the ULTIMATE TENSILE STRENGTH

Diameter.	Wrought-iron Rolled Bar.					
	Plain Bar.		Welded Bar.		Screwed Bar.	
Inches.	Lbs.	Tons.	Lbs.	Tons.	Lbs.	Tons.
$\frac{1}{4}$	2,825	1·26	2,320	1·03	2,049	0·91
$\frac{3}{8}$	6,357	2·84	5,220	2·33	4,610	2·15
$\frac{1}{2}$	11,300	5·04	9,280	4·14	8,195	3·66
$\frac{5}{8}$	17,660	7·88	14,500	6·47	12,800	5·71
$\frac{3}{4}$	25,430	11·35	20,880	9·32	18,440	8·23
$\frac{7}{8}$	34,810	15·45	28,420	12·68	25,090	11·20
1	45,200	20·18	37,120	16·57	32,780	14·63
$1\frac{1}{8}$	57,210	25·54	46,980	20·97	41,490	18·52
$1\frac{1}{4}$	70,630	31·53	58,000	25·89	51,220	22·86
$1\frac{3}{8}$	85,460	38·15	70,180	31·33	61,970	27·66
$1\frac{1}{2}$	101,710	45·40	83,520	37·28	73,760	32·93
$1\frac{5}{8}$	119,360	53·28	98,020	43·76	86,560	38·64
$1\frac{3}{4}$	138,440	61·80	113,680	50·75	100,390	44·82
$1\frac{7}{8}$	158,920	70·94	130,500	58·26	115,240	51·54
2	180,820	80·72	148,480	66·28	131,120	58·53
$2\frac{1}{8}$	204,120	91·12	167,620	74·83	148,030	66·08
$2\frac{1}{4}$	228,840	102·1	187,920	83·89	165,950	74·08
$2\frac{3}{8}$	254,980	113·8	209,380	93·47	184,900	82·54
$2\frac{1}{2}$	282,530	126·1	232,000	103·5	204,880	91·46
$2\frac{5}{8}$	311,480	139·0	255,780	114·2	225,880	100·8
$2\frac{3}{4}$	341,850	152·6	280,720	125·3	247,900	110·7
$2\frac{7}{8}$	373,630	166·8	306,820	136·9	270,950	120·9
3	406,830	181·6	334,080	149·6	295,040	131·7
	(1)	(2)	(3)	(4)	(5)	(6)

NOTE.—The diameters of the screwed bars

of ROUND BARS of IRON, STEEL, and COPPER.

Tilted Steel Bar.				Cast Iron.		Wrought Copper Bolts.		Diameter.
Plain Bar.		Welded Bar.						
Lbs.	Tons.	Lbs.	Tons.	Lbs.	Tons.	Lbs.	Tons.	Inches.
5,246	2·34	2,250	1·00	785	0·35	1,756	0·78	¼
11,820	5·28	5,060	2·25	1,766	0·79	3,952	1·76	⅜
21,020	9·38	8,990	4·01	3,140	1·40	7,024	3·14	½
32,840	14·66	14,050	6·27	4,906	2·19	11,000	4·91	⅝
47,290	21·11	20,220	9·00	7,065	3·16	15,810	7·06	¾
64,380	28·74	27,530	12·28	9,616	4·30	21,550	9·62	⅞
84,090	37·54	35,950	16·05	12,560	5·61	28,100	12·55	1
106,440	47·51	45,500	20·31	15,900	7·11	35,630	15·91	1⅛
131,380	58·65	56,200	25·08	19,620	8·77	44,000	19·64	1¼
158,970	70·97	68,000	30·35	23,750	10·62	53,220	23·76	1⅜
189,190	84·46	80,900	36·10	28,260	12·64	63,230	28·23	1½
222,050	99·13	94,970	42·39	33,180	14·81	74,350	33·19	1⅝
257,600	115·0	110,100	49·12	38,460	17·24	86,180	38·47	1¾
295,680	132·0	126,450	56·55	44,170	19·72	98,980	44·19	1⅞
336,450	150·2	143,810	64·20	50,240	22·46	112,380	50·17	2
379,680	169·5	163,000	72·76	56,740	25·33	127,090	56·74	2⅛
425,600	190·0	182,010	81·24	63,580	28·43	142,520	63·63	2¼
474,430	211·8	202,770	90·52	70,880	31·64	158,820	70·90	2⅜
525,500	234·6	224,780	100·3	78,540	35·10	176,000	78·57	2½
579,490	258·7	247,730	110·6	86,590	38·66	193,970	86·60	2⅝
635,940	283·9	272,010	121·4	94,980	42·47	212,880	95·04	2¾
695,070	310·3	297,200	132·6	103,870	46·33	232,160	103·6	2⅞
757,120	338·0	323,580	144·4	113,040	50·54	252,920	112·9	3
(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	

are measured at the *top* of the thread.

gradually rose to a maximum with the 12th, beyond which to the 18th they were progressively reduced. The crushing strain was reduced to its first minimum with the 4th melting, and attained its maximum with the 14th. Taking the iron of the 1st melting as a standard, the maximum increase in strength due to re-melting was 32 per cent. with the tensile; 41 per cent. with the transverse, and 135 per cent. with the crushing strain. It should be observed that Mr. Fairbairn's experiments did not include the tensile strain: col. 4 in Table 3 has been *calculated* (499) from the transverse and crushing strains in cols. 2 and 3; the results in col. 4 are very regular among themselves, despite the irregularities in col. 3.

TABLE 3.—Of the EFFECT of RE-MELTING on the STRENGTH of CAST IRON.

No. of Melting.	Transverse Strength 4½ Feet Bars 1 Inch Square.	Crushing Strength per Square Inch.	Calculated Tensile Strength per Square Inch.
	tons.	tons.	tons.
1	·2187	44·0	9·502
2	·1973	43·6	8·217
3	·1793*	41·1	7·351*
4	·1846	40·7*	7·697
5	·1927	41·1	8·151
6	·1959	41·1	8·349
7	·2005	40·9	8·655
8	·2192	41·1	9·847
9	·2440	55·1	10·07
10	·2531	57·7	10·40
11	·2910	69·8	11·71
12	·3090*	73·1	12·51*
13	·2834	66·0	11·54
14	·2700	95·9*	9·154
15	·1657	76·7	5·366
16	·1568	70·5	5·116
17	..	..	..
18	·1396	88·0	4·196
(1)	(2)	(3)	(4)

NOTE.—Maximum and minimum results marked *.

(7.) It would appear from all this, that the method of obtaining increased strength by re-melting cast iron is very

uncertain; it will also be very expensive in fuel, labour, and waste of metal. With iron such as that in (5), where the mean tensile strength was increased from 1 to $18 \cdot 26 \div 5 \cdot 6 = 3 \cdot 26$ at the 4th melting, it would no doubt be commercially advantageous: in such a case experiments should be specially made on the iron intended to be used (87).

(8.) By maintaining cast iron in a state of fusion for lengthened periods, the tensile strength is greatly increased: thus with iron twice re-melted and kept in fusion for

0	1	2	3 hours

the tensile strength was =

15,861	20,420	24,383	25,733 lbs.

per square inch. In another set of experiments, the time being =

½	1	1½	2 hours

the tensile strength =

17,843	20,127	24,387	34,496 lbs.

(9.) Cold-blast iron is considerably stronger than hot-blast iron; taking the former = 1·0, that of the latter was found at Lowmoor = ·831; at Dowlais, ·835: at Ystalyfera, ·802. The deterioration in strength appears by the American experiments to be proportional to the temperature of the blast; thus the strength of cold-blast iron being 1·0, it is reduced to ·865 with the blast at 150°, and to ·807 at 250°, &c.

WROUGHT IRON AND STEEL.

(10.) The strength of wrought iron increases, as might be expected, with repeated working in the fire and under the hammer. Mr. Clay found that the strength of a puddled bar being 1·0, it becomes 1·36 when piled three or four times, and 1·41 when piled six times; beyond that point, however, its strength declines, and is reduced to that of a puddled bar when piled twelve times.

The same authority has shown that with steel, the strength of a puddled bar being 1·0, it becomes 1·253 at the fourth piling, after which it declines and is reduced to 0·94 at the seventh piling.

(11.) "*Welded Joints.*"—The strength of wrought-iron welded joints appears by the experiments of Kirkaldy to be very variable, the mean from eighteen experiments on bars from $1\frac{1}{4}$ to $\frac{3}{4}$ inch diameter = ·8066, the strength of a solid bar being 1·0; in extreme cases it is as low as ·562, or little more than half, in others as high as ·974, the great difference being due no doubt to imperfect workmanship.

With steel the loss of strength by welding is still more considerable; the same authority shows that the strength of welded steel joints varies from ·55 to ·404 of that of a solid bar. The strength of steel is also affected considerably by hardening, tempering, annealing, &c., as is shown by Table 1: when heated and quenched in oil, Mr. Kirkaldy obtained the extraordinary strength of 96·1 tons per square inch, which is exceptional and anomalous. The same steel made as hard as possible by being highly heated and quenched in water, gave 40·2 tons only: the mean for ordinary rolled or tilted bars being 47·84 tons per square inch: see cols. 3, 9 of Table 2.

(12.) "*Screwed Bolts.*"—There are two ways of measuring screwed bars, namely by the diameter at the top of the thread, or that of the plain bar before screwing, and by the diameter at the bottom of the thread: the former is the most convenient, and will be followed here. Mr. Kirkaldy obtained some curious results; he found that when the thread was chased in the lathe, or cut by *new* dies, the strength was nearly proportional to the diameter at the bottom of the thread as might be expected, and varied with different sizes, between 67 and 82 per cent. of that of a plain bar, the mean being 72·5 per cent. But when old dies were used, the metal seemed to be *compressed* rather than cut, and the strength was much greater than with new dies, varying from 77 to 89 per cent. of that due to a plain bar, the mean being about 85. It will be the safest course to reckon the strength as due to new dies, or 72·5 per cent. of plain bar; see cols. 5, 6 of Table 2.

(13.) "*Plate-iron and Steel.*"—Rolled plates of iron and steel are rather weaker than the same materials in the form of bars, as shown by Table 1; the ratio happens to be nearly the same for both: thus taking Kirkaldy's results, with wrought iron we

have $21 \cdot 6 \div 25 \cdot 7 = \cdot 84$, or 84 per cent., and with steel $38 \cdot 4 \div 47 \cdot 8 = \cdot 80$, or 80 per cent.

"*Effect of the Grain.*"—Experiments have shown that the tensile strength of both wrought iron and steel, lengthways of the grain, is greater than that crossways, as shown by Table 1: thus with wrought iron we have $22 \cdot 6 \div 20 \cdot 6 = 1 \cdot 097$, or $9 \cdot 7$ per cent.; and with steel $40 \cdot 1 \div 36 \cdot 6 = 1 \cdot 096$, or $9 \cdot 6$ per cent., being practically the same for both.

In arranging the plates for girders, &c., those subjected to tension should be cut so that the strain is in the direction of the grain, and in boilers, where the circumferential strain is double the longitudinal (71), the direction of the grain should be arranged accordingly.

(14.) "*Effect of Annealing.*"—It has been found by experiment that the effect of annealing wrought iron in the bar, plate, and chain form is to reduce the tensile strength: this is the more remarkable, being just the reverse of the effect on steel plate, which is to *increase* the strength as much as 55 per cent. (38).

By hammering *cold* the strength of wrought iron is much reduced, but by annealing it is partially restored: experiments at Woolwich show the effect of both processes on bars of different sizes: thus bars

2 inches	1½ inch	1 inch

diameter, gave after being cold-hammered

17·5	16·4	22·3 tons

per square inch. The mean tensile strength of ordinary bar iron is 25·7 tons per square inch by Table 1, hence the loss by cold-hammering is

32	36	13

per cent.: the mean being 27 per cent. After annealing, the strength became

22·1	24·6	23·5 tons.

per square inch. Hence after both processes, there still remains a loss of

14	4	8·6 per cent.

The tensile strength of plate iron also is reduced by annealing as shown by Mr. Kirkaldy's experiments on six kinds of Yorkshire iron $\frac{3}{8}$, $\frac{1}{2}$, and $\frac{5}{8}$ inch thick, which gave the loss = 5·6 per cent. lengthways, and 5·2 per cent. crossways of the grain; Lowmoor giving 4·8 and 1·8; Bowling, 8·0 and 9·1 per cent. respectively.

The effect of annealing chain is shown by Nos. 19, 20 in Table 21, to be $16{\cdot}34 \div 17{\cdot}54 = {\cdot}93$, or 7 per cent. loss of strength (109).

CHAPTER II.

ON RIVETED JOINTS.

(15.) Riveting two plates of metal together may appea
a very simple matter, but the fact is that there is more phil
involved in it than is commonly supposed. The extr
portance of riveted joints, not only as applied to stear
but also to girders, railway bridges, and other stru
justify the most careful attention to the principles b
strength is governed, and the proper proportions are

The strength of a riveted joint is dependent, f
tensile strength of the plate, measured at its w
namely, through the line of the rivet-holes; se
shearing strength of the rivets; and third, on the
plates against one another due to the pressure of t

(16.) "*Strength of Punched Plates.*"—It has
experiment that when wrought-iron plates are
the usual way, the strength of the plate is redu
the removal of the metal punched out, but also
of the fibres of the metal that remains betweer
Direct evidence of this is given by the experim
plates by Mr. Kirkaldy in Table 4, which sh
loss was 13 per cent. with the grain, and 17·26
grain, this being the result of eighteen experi
of iron by different makers, with plates $\frac{3}{8}$,
The plates were in all cases 8 inches wide,

TABLE 4.—Of the Loss of STRENGTH by PUNCHING RIVET-HOLES in YORKSHIRE BOILER-PLATES.

Kind.	Thickness.	Loss Per Cent.	
		Lengthways.	Crossways.
Lowmoor	·38	17·7	..
"	·63	16·3	..
"	·37	..	21·2
"	·63	..	17·8
Bowling	·54	8·55	..
"	·53	..	13·9
Farnley	·40	16·8	..
"	·63	16·5	..
"	·41	..	19·8
"	·63	..	19·1
Cooper's	·39	11·0	..
"	·63	14·4	..
"	·39	..	13·05
"	·62	..	18·9
Taylor's	·52	7·63	..
"	·53	..	12·8
Monkbridge	·51	6·7	..
"	·50	..	16·7
Mean	..	13·0	17·26
	(1)	(2)	(3)

in a line, ·85 inch diameter; hence the ratio of the solid part of the plate to the metal between rivet-holes was $\left\{8 - (\cdot 85 \times 4\right\} \div 8 = \cdot 575$ to $1 \cdot 0$. This proportion is about the same as that adopted in ordinary riveting, a fact which is important, for obviously, the damage to the fibres will be the greatest close to the rivet-holes, and will diminish with the distance: now when the holes are pretty close together as in ordinary riveting, we may suppose that the whole of the metal between them will be affected, but where the distance is very great, the metal at mid-distance may be wholly unaffected, and in that case the *mean* strength would be much greater than in others where the pitch of the rivets is small. Mr. Kirkaldy's experiments are the more conclusive because the strengths of the punched plates were compared with those of unpunched ones cut out of *the same plate*. Moreover, in order to avoid any possible loss of strength by

shearing, which would probably be analogous to that due to punching, the plates were all *cut* out in a slotting machine.

(17.) These experiments appear to be reliable, nevertheless there are some remarkable differences between them and the results obtained by other authorities. For example, Mr. Fairbairn's experiments in Table 1 give for Lowmoor iron 24·56 tons per square inch both lengthways and crossways, whereas Mr. Kirkaldy gives for plates *not* annealed 21·3 lengthways, and 20·3 tons crossways, and for annealed plates 20·1 and 19·2 tons respectively, these being the means for six kinds of Yorkshire iron, and they show a difference of 15, 22, 20, and 28 per cent. as compared with Fairbairn's results. Messrs. Napier and Son's experiments in Table 1 give 24·1 tons lengthways as the strength of Yorkshire plate iron; agreeing with Mr. Fairbairn. Another remarkable difference is that the loss due to punching, which as we have seen (16) was 13, and 17·26 per cent. by Kirkaldy, was as much as 24 per cent. in single riveted joints by Mr. Fairbairn, as shown by col. 4 of Table 5. Probably the fact that the strain is not central or symmetrical, as shown by the broken centre-line in A, Fig. 6, may account for the difference.

(18.) "*Strength of Drilled Plates.*"—When the rivet-holes are drilled, the loss of strength in the metal between the rivet-holes is practically nothing, the mean result of eighteen experiments on six kinds of Yorkshire iron (16) was 1·13 per cent. lengthways, and 0·9 per cent. crossways. Notwithstanding the advantage which is thus shown to accrue from drilling rivet-holes, it is hardly likely ever to be adopted extensively in practice; the extra cost of drilling would not be compensated by the extra strength obtained.

(19.) "*Shearing Strength of Rivets.*"—It is shown in (123) that the resistance to shearing is equal to the tensile strength of the iron, and Mr. E. Clark's experiments gave 22·1 tons per square inch, Mr. Fairbairn's experiments gave 22·04 tons in a single-riveted joint, where the result might possibly be complicated by friction (20), but with such a joint friction would be eliminated at the point of rupture, the surfaces separating by the unsymmetrical strain (17). We may therefore take 22 tons, or

49,280 lbs. per square inch as the mean shearing strength of wrought-iron rivets.

(20.) "*Friction from Grip of Rivets.*"—Mr. E. Clark made some experiments on the friction in riveted joints, and obtained some remarkable results, for he found that the friction increased considerably with the *length* of the rivet. With rivets $\frac{7}{8}$ inch diameter, riveted hot in the usual manner, and $1\frac{1}{4}$, $1\frac{7}{8}$, and $2\frac{7}{8}$ inches long, the friction was $4\frac{3}{4}$, $5\frac{1}{2}$, and 8 tons respectively. The experiments were made in the following manner: three $\frac{5}{8}$ plates were riveted together with one $\frac{7}{8}$ rivet; the central plate, having an oblong hole, was then drawn between the other two, the frictional resistance to which was $5\frac{1}{2}$ tons. Two $\frac{1}{2}$-inch washers were then added, making the length $2\frac{7}{8}$, when the strain due to friction became 8 tons. Two $\frac{5}{16}$ plates and two $\frac{5}{16}$ washers gave a length of $1\frac{1}{4}$ inch, when friction became $4\frac{3}{4}$ tons. This last experiment approximates nearly to the conditions of ordinary riveted joints.

(21.) "*Principles of Riveting.*"—We may now investigate the phenomena which occur with riveted joints, and to do that satisfactorily it will be well to take an experimental case, the reasoning can then be checked by practice. Fig. 7 is a joint of best Staffordshire plate, experimented upon by Mr. Brunel: the main plates were $\frac{1}{2}$ inch thick, and the joint was formed with a front and back plate each $\frac{3}{8}$ inch thick, and twenty rivets $\frac{11}{16}$ inch diameter. This joint failed with 164 tons, by the $\frac{1}{2}$-inch plate tearing through the outer line of rivet-holes B, B: the rivets were not broken in this case, but evidently they must have been on the point of rupture, for in another and similar experiment the whole of the ten rivets in one half of the joint were sheared with a lower strain, namely 153 tons. We may therefore assume that 164 tons would or should rupture the plate and shear the rivets simultaneously. It was also found that a solid or unpunched plate of the same iron broke with a mean strain of 20·6 tons per square inch.

We have no experimental evidence of the damaging effect of punching on Staffordshire plates (16), but with Lowmoor iron, Table 4 shows a *mean* loss of 19·5 per cent. when strained crossways of the grain, and 17 per cent. lengthways; taking 18 per

cent. as a mean for Staffordshire plates, the metal left between rivet-holes will be reduced to $100 - 18 = 82$ per cent. of the strength of a solid plate; hence in our case, we have $20 \cdot 6 \times \cdot 82 = 16 \cdot 9$ tons per square inch. The area through the line B, B is $\left\{20 - (\frac{11}{16} \times 5\right\} \times \frac{1}{2}'' = 8 \cdot 28$ square inches, hence $8 \cdot 28 \times 16 \cdot 9 = 140$ tons, the breaking weight of the plate.

To this has to be added the friction due to the grip of the five rivets in that row; by Mr. E. Clark's experiments (20) this may be taken at $4\frac{3}{4}$ tons per rivet, or in our case $4\frac{3}{4} \times 5 = 24$ tons, making with that due to the plate $140 + 24 = 164$ tons, which happens to be precisely as per experiment.

(22.) "*Real and Apparent Strength.*"—The difference between real and apparent strength will now be manifest; the apparent strength or that of the whole combination is 164 tons, borne by $8 \cdot 28$ square inches, or $164 \div 8 \cdot 28 = 19 \cdot 8$ tons per square inch, but the real strain on the metal between rivet-holes as we have seen (21) is $16 \cdot 9$ tons; the strain on the solid part of the plate at C, C is only $164 \div 10 = 16 \cdot 4$ tons, whereas the breaking weight $= 20 \cdot 6$ tons per square inch.

Thus the normal strength of the solid plate, or $20 \cdot 6$ tons, is reduced by punching to $16 \cdot 9$ tons per square inch, which again is increased by friction to $19 \cdot 8$ tons, being restored within $20 \cdot 6 - 19 \cdot 8 = 0 \cdot 8$ ton of the normal strength.

"*Rivets.*"—The ten rivets were each $\frac{11}{16}$-inch diameter $= \cdot 3712$ square inch area, and being subjected to a *double shear* give $\cdot 3712 \times 10 \times 2 = 7 \cdot 424$ square inches shearing area; then, their apparent strength is $164 \div 7 \cdot 424 = 22 \cdot 09$ tons per square inch, which is almost exactly the resistance given for double shear by Mr. E. Clark's experiments (123).

With certain proportions of double-riveted joints the apparent strength of the metal between rivet-holes per square inch may *exceed that of the solid plate*, a result that seems anomalous, but may be easily explained. Thus, let Fig. 8 be a joint with seven $\frac{7}{8}$-inch rivets in the outer row B, B; then the area of the central plate on that line will be $\left\{20 - (\frac{7}{8} \times 7\right\} \times \frac{1}{2} = 6 \cdot 94$ square inches, giving $16 \cdot 9 \times 6 \cdot 94 = 117 \cdot 28$ tons.

Then, each rivet giving $4\frac{3}{4}$ tons of friction, we have $4 \cdot 75 \times 7 = 33 \cdot 25$ tons, and the total breaking weight of the joint $= 117 \cdot 28 + 33 \cdot 25 = 150 \cdot 53$ tons, or $150 \cdot 53 \div 6 \cdot 94 = 21 \cdot 69$ tons per square inch of metal between rivet-holes. But the solid plate yields 20·6 tons only, hence we have $21 \cdot 69 \div 20 \cdot 6 = 1 \cdot 053$, or 5·3 per cent. in excess of the solid plate, agreeing with Mr. Fairbairn's result in col. 4 of Table 5, which gives 1·0526, or 5·26 per cent. excess. Here the actual strength of metal between rivet-holes is 18 per cent. less than the normal strength or that of the solid plate, but the *apparent* strength is 5·3 per cent. in excess, the difference being due to friction.

(23.) These calculations are not given as absolutely correct, but as serving to illustrate the principles on which the strength of riveted joints depends, and to explain the differences in the apparent strength of various kinds of joints in Table 5. For instance, by col. 6 the mean strength of a solid plate of average British plate-iron = 48,454 lbs. per square inch; by punching, the loss is 18 per cent. as in (21), and the strength of the metal left between rivet-holes is reduced to $48454 \times \cdot 82 = 39723$ lbs., and will be the same with all the different joints in that column. The *oblique* action of the strain in an ordinary single-riveted joint, as shown by the broken centre line o, p at A, in Fig. 6, reduces the apparent strength to 36,898 lbs., and with one back plate as at B, to 39,248 lbs. per square inch, both being less than that of a punched but unriveted plate with a fair central strain, which, as we have seen, is 39,723 lbs. But with a front and back plate, friction becomes more influential and increases the apparent strength to 46,070 lbs.

(24.) "*Kinds of Riveted Joints.*"—There are six principal kinds of riveted joint, which are shown by A, B, C, D, E, F in Fig. 6, and are described or specified in Table 5, which also gives the apparent strength of the metal between the rivet-holes in each kind of joint, and with three different kinds or qualities of plate iron, namely, Yorkshire, Staffordshire, and the general average of British iron. Staffordshire is the weakest of the three, but is more extensively used than any other, and may be taken as a basis for calculation in ordinary cases. Taking it as a standard, British iron gives $48454 \div 44800 = 1 \cdot 082$, or

TABLE 5.—Of the STRENGTH of RIVETED JOINTS in WROUGHT-IRON PLATES.

Kind of Joint.	Breaking-strain of Metal between Rivet-holes, in Lbs. per Square Inch.						
	Yorkshire Plates.					Mean of British Plates.	Stafford-shire Plates.
	Max.	Min.	Mean.	Ratios.	No. of Experi-ments.		
Solid plate, not punched	61,588	49,852	56,264	1·0000	8	48,454	44,800
Single-riveted, simple lap, Fig. 6 A	49,325	36,588	42,847	·7615	10	36,898	34,110
Single-riveted, with back-plate, Fig. 6 B	53,880	37,262	45,570	·8100	2	39,248	36,290
Single-riveted, with back and front plate, Fig. C	58,461	48,534	53,497	·9508	2	46,070	42,600
Double-riveted, simple lap, Fig. D	58,286	45,590	52,503	·9330	6	45,208	41,800
Double-riveted, with back-plate, Fig. E	54,643	53,116	53,879	·9570	2	46,520	42,870
Double-riveted, with back and front plate, Fig. F	62,279	56,175	59,225	1·0526	3	51,000	47,160
	(1)	(2)	(3)	(4)	(5)	(6)	(7)

8·2 per cent., and Yorkshire 56264 ÷ 44800 = 1·256, or 25·6 per cent. greater tensile strength.

(25.) "*Proportions of Riveted Joints.*"—In fixing the proportions of riveted joints, it is necessary to consider the subject under two different heads; 1st for girder-work, where we have simply to secure equality between the shearing strength of the rivets and the tensile strength of the plate between rivet-holes, so that both may fail simultaneously; and 2nd, for steam-boiler work, where we have not only to consider the question of strength, but also the maximum pitch of rivets consistent with tightness of the steam-joints. This will vary with the pressure of the steam and the thickness of the plate; if with a given thickness and pressure the distance between rivet-holes exceeds a certain amount, it will be difficult, or perhaps impossible, to make the joint permanently steam-tight (45). On the other hand, if the pitch is unduly reduced, the proportions of the metal left between the holes to the

solid part of the plate are reduced also, and a loss of strength will ensue.

With girder-work we have the choice of any of the six kinds of joint in Table 5, but for boilers we are practically confined to two, namely, single and double riveted; these alone being easily made steam-tight by caulking.

(26.) "*Diameter of Rivets.*"—The proper size of rivets in proportion to the thickness of plate is to some extent arbitrary, and within certain limits may be varied considerably, so long as the great principle is observed, namely, so to adjust the pitch and thickness as to secure equality between the tensile strength of the plate and the shearing strength of the rivets.

Nevertheless, practice has dictated as expedient, certain proportions between the diameter of rivet and thickness of plate which should be followed in ordinary cases, and may be expressed by the rule:—

(27.) $$d = (t \times 1\tfrac{1}{4}) + \tfrac{3}{16}$$
$$\text{or } d = (t \times 1{\cdot}25) + {\cdot}1875.$$

In which t = the thickness of plate, and d = diameter of rivet-holes, both in inches: col. 2 of Table 14 has been calculated by this rule. It should be observed that the diameter of the *hole* should be taken rather than that of the cold rivet: the rivet is always made smaller than the hole for facility in inserting it; but when riveted hot in the usual way it fills the hole completely, and the strength is therefore governed by the size of the hole itself.

(28.) It is a practical dictum that the diameter of the rivets shall be proportional to the thickness of the plate irrespective of the pressure of steam and other considerations. This leads to no difficulty with girder-work, because we can always adjust the pitch so as to obtain equality between the strain on the rivets to that on the plate (25). But for boiler-work the pitch is restricted by the pressure of steam (45), and we are conducted to the anomaly, that as the pressure is increased, the diameter of the rivets should be reduced, a result precisely contrary to that expected (53).

RIVETED JOINTS FOR GIRDER-WORK.

(29.) "*Pitch of Rivets in Single-riveted Joints.*"—The main principle in riveting, as we have stated, is so to proportion the space between rivet-holes to the area of the rivet as to obtain equality of strength, that is to say, that, theoretically at least, the rivets shall be sheared and the plate ruptured simultaneously. In a simple single-riveted joint, if the shearing strength of rivets per square inch and the tensile strength of boiler-plate were equal, the area of plate between two rivet-holes should be equal to the area of a rivet-hole, and it is commonly assumed that such is the proper proportion. But by col. 6 of Table 5 the mean strength of the metal between the rivet-holes in single-riveted joints is 36,898 lbs. per square inch, whereas the shearing strain of rivets by (19) is 49,280 lbs.: hence the area of the rivet *in this kind of joint* should be $36898 \div 49280 = \cdot 75$ of the area of metal between rivet-holes.

(30.) Thus, with $\frac{3}{8}$ plate, and $\frac{11}{16}$ rivets, as per Table 6, tl area of $\frac{11}{16} = \cdot 3712$, hence the area of metal between two rive holes should be $\cdot 3712 \div \cdot 75 = \cdot 495$ square inch ; the distar between rivet-holes $= \cdot 495 \div \frac{3}{8} = 1 \cdot 32$ or $1\frac{5}{16}$ inch, and pitch $1\frac{5}{16} + \frac{11}{16} = 2$ inches. The ratio of the metal be holes to the solid plate is $1 \cdot 32 \div 2 = \cdot 66$, hence the str the *solid* plate, when the joint is breaking through the rivet-holes, is $36898 \times \cdot 66 = 24353$ lbs. per square inch. *reduced* strain on the solid plate as thus found, is us the purpose of calculation, as we shall find when we apply these results to girders, boilers, &c.

The general proportions of single-riveted lap-joints on these principles are given by Table 6; with st the pressure of the steam has to be considered i itch of the rivets (44), but for girder-work th ven by the Table require no correction.

(31.) "*Single-riveted Joints with Back and Front* vantage of a single back-plate, as at B, Fig. 6, e, as shown by col. 4 of Table 5, but with a te, as in Fig. 8, it is very great, which is c greater *apparent* strength of the metal,

TABLE 6.—Of the PROPORTIONS and STRENGTH of SINGLE-RIVETED JOINTS in WROUGHT-IRON PLATES: for GIRDER-WORK ONLY.

Thickness of Plate.	Diameter of Rivet-holes.	Suitable Pitch of Rivets.	Space between Rivets.	Lap of Joint.	Ratio of Punched to Solid Plate.	Breaking Strain of Metal between the Holes, in Lbs. per Sq. In.	Strain on the Solid Part of Plate, per Sq. In.	
							Lbs.	Tons.
$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	·667	36,898	24,611	11·00
”	$\frac{5}{16}$*	$1\frac{1}{8}$	$\frac{13}{16}$	1	·722	”	26,640	11·89
”	$\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{1}{8}$	$1\frac{1}{8}$	·750	”	27,673	12·35
$\frac{3}{16}$	$\frac{5}{16}$	$\frac{7}{8}$	$\frac{9}{16}$	$\frac{7}{8}$	·643	”	23,725	10·59
”	$\frac{3}{8}$	$1\frac{3}{16}$	$\frac{13}{16}$	$1\frac{1}{8}$	·684	”	25,238	11·27
”	$\frac{7}{16}$*	$1\frac{1}{2}$	$1\frac{1}{16}$	$1\frac{3}{8}$	·708	”	26,124	11·66
”	$\frac{1}{2}$	$1\frac{7}{8}$	$1\frac{3}{8}$	$1\frac{3}{4}$	·733	”	27,046	12·07
$\frac{1}{4}$	$\frac{3}{8}$	$\frac{15}{16}$	$\frac{9}{16}$	$\frac{7}{8}$	·600	”	22,139	9·88
”	$\frac{7}{16}$	$1\frac{1}{4}$	$\frac{13}{16}$	$1\frac{1}{8}$	·650	”	23,984	10·71
”	$\frac{1}{2}$*	$1\frac{1}{2}$	1	$1\frac{3}{8}$	·667	”	24,611	11·00
”	$\frac{9}{16}$	$1\frac{15}{16}$	$1\frac{3}{8}$	$1\frac{3}{4}$	·709	”	26,161	11·68
”	$\frac{5}{8}$	$2\frac{1}{4}$	$1\frac{5}{8}$	2	·722	”	26,840	12·00
$\frac{5}{16}$	$\frac{7}{16}$	$1\frac{1}{16}$	$\frac{5}{8}$	1	·588	”	21,696	9·68
”	$\frac{1}{2}$	$1\frac{3}{8}$	$\frac{7}{8}$	$1\frac{1}{4}$	·636	”	23,467	10·47
”	$\frac{9}{16}$*	$1\frac{5}{8}$	$1\frac{1}{16}$	$1\frac{1}{2}$	·654	”	24,131	10·77
”	$\frac{5}{8}$	2	$1\frac{3}{8}$	$1\frac{3}{4}$	·687	”	25,349	11·32
$\frac{3}{8}$	$\frac{9}{16}$	$1\frac{7}{16}$	$\frac{7}{8}$	$1\frac{1}{4}$	·609	”	22,471	10·00
”	$\frac{5}{8}$	$1\frac{11}{16}$	$1\frac{1}{16}$	$1\frac{1}{2}$	·630	”	23,246	10·38
”	$\frac{11}{16}$*	2	$1\frac{5}{16}$	$1\frac{3}{4}$	·660	”	24,353	10·87
”	$\frac{3}{4}$	$2\frac{3}{8}$	$1\frac{5}{8}$	$2\frac{1}{8}$	·684	”	25,238	11·27
$\frac{7}{16}$	$\frac{5}{8}$	$1\frac{9}{16}$	$\frac{15}{16}$	$1\frac{3}{8}$	·600	”	22,139	9·88
”	$\frac{11}{16}$	$1\frac{13}{16}$	$1\frac{1}{8}$	$1\frac{5}{8}$	·621	”	22,914	10·23
”	$\frac{3}{4}$*	$2\frac{1}{8}$	$1\frac{3}{8}$	$1\frac{7}{8}$	·649	”	23,946	10·69
”	$\frac{13}{16}$	$2\frac{3}{8}$	$1\frac{9}{16}$	$2\frac{1}{4}$	·695	”	25,644	11·45
$\frac{1}{2}$	$\frac{11}{16}$	$1\frac{11}{16}$	1	$1\frac{1}{2}$	·592	”	21,844	9·75
”	$\frac{3}{4}$	$1\frac{15}{16}$	$1\frac{5}{16}$	$1\frac{3}{4}$	·613	”	22,618	10·10
”	$\frac{13}{16}$*	$2\frac{3}{16}$	$1\frac{3}{8}$	2	·628	”	23,172	10·35
”	$\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{5}{8}$	$2\frac{1}{4}$	·650	”	23,984	10·71
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

NOTE.—The rivet-holes marked * are the most suitable for the given thickness of plate, in ordinary cases.

the rivets are subjected to a *double-shear*, and their area in proportion to the plate may be reduced to half.

The apparent strength of metal between holes being by col. 6 $= 46070$ lbs. per square inch, the area of the rivets should be $46070 \div 49280 = \cdot 93$ of the area of metal between two holes. Taking $\frac{3}{8}$ plate, and $\frac{11}{16}$ rivets as before, the area of the rivet with double shear $= \cdot 3712 \times 2 = \cdot 7424$ square inch; the area of metal between two holes $= \cdot 7424 \div \cdot 93 = \cdot 8$ square inch; the distance between the holes $= \cdot 8 \div \frac{3}{8} = 2\frac{1}{8}$ inches, and the pitch $= 2\frac{1}{8} + \frac{11}{16} = 2\frac{13}{16}$; the ratio of the metal between holes to the solid plate $= 2\frac{1}{8} \div 2\frac{13}{16}$, or $34 \div 45 = \cdot 756$, and the strain on the solid plate $= \cdot 756 \times 46070 = 34829$ lbs. per square inch, which is $34829 \div 24353 = 1 \cdot 43$, or 43 per cent. more than with a simple single-riveted joint (30). Table 7 gives the general proportions and strength of single-riveted joints, with front and back plate calculated in the manner we have illustrated.

(32.) "*Amount of Lap.*"—A riveted joint may give way 1st, by the rivets shearing; 2nd, by the plate breaking across through the line of the rivet-holes; and 3rd, by the rivet-holes tearing out: we have considered the two former, [illegible] have now to consider the latter. Theoretically, the metal [illegible] *c*, *d*, in Fig. 10, should be torn out, but the rivet wo[illegible] flattened and deformed by the plate, and we may as[illegible] starting-point for the line of fracture at *m*, say midway [illegible] *a* and *n*: then the sum of the distances *m*, *o* and *p*, *r* s[illegible] equal to the space *c*, *e* between two rivets. Taking, t[illegible] half the distance *c*, *e*, and setting it from *m* to *o*, we o[illegible] lap for single-riveted joints, as in col. 5 of Table 6.

(33.) With a double-riveted joint, Fig. 11, the pla[illegible] break on the line of rivets A, B; in that case the pla[illegible] [t]he rivets may remain intact; conversely, the plate [illegible] [o]n the line C, D, the plate F and all the rivets re[illegible]

If, as in Fig. 12, we make the space T = h[illegible] [m]ight take place on the zig-zag line J, K as easily [illegible] [A], B, or C, D, in Fig. 11, because the breadth of [illegible] [i]s the same in both cases. This shows that T sh[illegible]

In another case all the rivets might be torn o[illegible]

TABLE 7.—Of the PROPORTIONS and COHESIVE STRENGTH of SINGLE and DOUBLE-RIVETED JOINTS in WROUGHT-IRON PLATES, with FRONT and BACK PLATE as in Figs. 8, 9: for GIRDER-WORK ONLY.

Thickness of Plate.	Diameter of Rivet-holes.	Pitch of Rivets.	Space between Rivet-holes.	Distance between the Rows of Rivets.		Back and Front Plates.		Ratio of Punched to Solid Plate.	Breaking Strain of Metal between the Holes. Lbs. per Sq. Inch.	Strain on the Solid Plate per Square Inch.	
				A in Figs. 8, 9.	B in Fig. 9.	Breadth.	Thickness.			Lbs.	Tons.
				STRENGTH, &c., OF SINGLE-RIVETED JOINTS.							
$\frac{1}{4}$	$\frac{1}{2}$	$2\frac{3}{16}$	$1\frac{11}{16}$	2	. .	4	$\frac{3}{16}$	·772	46,070	35,567	15·88
$\frac{5}{16}$	$\frac{9}{16}$	$2\frac{5}{16}$	$1\frac{3}{4}$	$2\frac{1}{8}$	. .	$4\frac{1}{4}$	$\frac{1}{4}$	·757	"	34,876	15·57
$\frac{3}{8}$	$\frac{11}{16}$	$2\frac{13}{16}$	$2\frac{1}{8}$	$2\frac{1}{2}$	. .	5	$\frac{1}{4}$	·756	"	34,829	15·55
$\frac{7}{16}$	$\frac{3}{4}$	$2\frac{15}{16}$	$2\frac{3}{16}$	$2\frac{3}{4}$	. .	$5\frac{1}{2}$	$\frac{5}{16}$	·745	"	34,323	15·32
$\frac{1}{2}$	$\frac{13}{16}$	$3\frac{1}{16}$	$2\frac{1}{4}$	$2\frac{7}{8}$	. .	$5\frac{3}{4}$	$\frac{5}{16}$	·735	"	33,862	15·12
				STRENGTH, &c., OF DOUBLE-RIVETED JOINTS.							
$\frac{1}{4}$	$\frac{1}{2}$	$3\frac{1}{2}$	3	$3\frac{3}{8}$	1	$8\frac{3}{4}$	$\frac{3}{16}$	·857	51,000	43,707	19·51
$\frac{5}{16}$	$\frac{9}{16}$	$3\frac{3}{4}$	$3\frac{3}{16}$	$3\frac{1}{2}$	$1\frac{1}{8}$	$9\frac{1}{4}$	$\frac{1}{4}$	·850	"	43,350	19·36
$\frac{3}{8}$	$\frac{11}{16}$	$4\frac{1}{2}$	$3\frac{13}{16}$	$4\frac{1}{4}$	$1\frac{3}{8}$	$11\frac{1}{4}$	$\frac{1}{4}$	·848	"	43,248	19·30
$\frac{7}{16}$	$\frac{3}{4}$	$4\frac{11}{16}$	$3\frac{13}{16}$	$4\frac{1}{2}$	$1\frac{3}{8}$	$11\frac{3}{4}$	$\frac{5}{16}$	·840	"	42,840	19·24
$\frac{1}{2}$	$\frac{13}{16}$	$4\frac{13}{16}$	4	$4\frac{5}{8}$	$1\frac{1}{2}$	12	$\frac{5}{16}$	·831	"	42,381	18·92
$\frac{9}{16}$	$\frac{7}{8}$	5	$4\frac{1}{8}$	$4\frac{3}{4}$	$1\frac{5}{8}$	$12\frac{3}{4}$	$\frac{3}{8}$	·825	"	42,075	18·78
$\frac{5}{8}$	$\frac{15}{16}$	$5\frac{3}{16}$	$4\frac{1}{4}$	5	$1\frac{3}{4}$	13	$\frac{3}{8}$	·820	"	41,820	18·67
$\frac{11}{16}$	$1\frac{1}{16}$	$6\frac{1}{16}$	5	$5\frac{5}{8}$	$1\frac{7}{8}$	15	$\frac{7}{16}$	·825	"	42,075	18·78
$\frac{3}{4}$	$1\frac{1}{8}$	$6\frac{1}{4}$	$5\frac{1}{8}$	$5\frac{7}{8}$	2	$15\frac{1}{2}$	$\frac{7}{16}$	·820	"	41,820	18·67
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)

both plates being torn, but if the lap be adjusted, as in Fig. 12, so that the area of metal torn out *in each half* is equal to that on the line A, B in Fig. 11, the breaking weight will be the same in all three cases, Figs. 11, 12, 13.

In yet another case all the rivets might be sheared, and both plates remain intact.

(34.) "*Pitch of Rivets in Double-riveted Joints.*"—Say that we take the case of $\frac{1}{2}$-inch plate, double-riveted with $\frac{7}{8}$-inch rivets: the area of $\frac{7}{8} = \cdot 6013$, or for two rivets $\cdot 6013 \times 2 = 1 \cdot 2026$ square inch. In a common double-riveted joint, the apparent strength of the metal between the rivet-holes, by col. 6 of Table 5, is 45,208 lbs. per square inch, and taking the shearing strength of rivets (19) at 49,280 lbs. per square inch, the ratio is $45208 \div 49280 = \cdot 91$; hence for $1 \cdot 2026$ square inch of rivet we require $1 \cdot 2026 \div \cdot 91 = 1 \cdot 32$ square inch of plate. The distance between two rivet-holes will then be $1 \cdot 32 \div \frac{1}{2} = 2 \cdot 64$, or $2\frac{5}{8}$ inches, and the pitch P in Fig. 11, $= 2\frac{5}{8} + \frac{7}{8} = 3\frac{1}{2}$ inches: the ratio of the metal between rivet-holes to the solid plate is $2\frac{5}{8} \div 3\frac{1}{2}$, or $21 \div 28 = \cdot 751$, hence we have $45208 \times \cdot 751 = 33951$ lbs. per square inch on the solid plate.

Comparing this result with that given for the same plate single-riveted by col. 8 of Table 6, we obtain $33951 \div 23984 = 1 \cdot 42$, or 42 per cent. more with a double than with a single-riveted joint. This great advantage is partly due to the improved conditions of the strain by which the apparent strength is increased from 36,898 to 45,208 lbs. per square inch, as shown by col. 6 of Table 5; and partly to the greatly increased pitch, by which the ratio of metal between holes to the solid plate is increased from $\cdot 650$ to $\cdot 751$.

For girder-work this increased strength may be realised, but for steam-boilers the great pitch precludes the use of such proportions, except for very low pressures of steam, for instance, $\frac{1}{2}$-inch plate, $3\frac{1}{2}$ inches pitch, with $\frac{7}{8}$ rivets, would give $2\frac{5}{8}$ space, with which, by the rule (46), the working pressure of steam $= 7 \cdot 4$ lbs. per square inch only.

Table 8 gives the general proportions of double-riveted joints, calculated on the principles we have illustrated.

TABLE 8.—Of the PROPORTIONS and STRENGTH of DOUBLE-RIVETED JOINTS in WROUGHT-IRON PLATES: for GIRDER-WORK ONLY.

Thickness of Plate.	Diam. of Rivet-holes.	Suitable Pitch of Rivets	Space between Rivets.	Distance between Rows of Rivets.	Lap of Joint.	Ratio of Punched to Solid Plate.	Breaking Strain of Metal between the Holes, Lbs. per Sq. In.	Strain on the Solid Part of Plate, per Sq. In.	
								Lbs.	Tons.
3/8	9/16	2	1 7/16	13/16	2 1/4	·718	45,208	32,459	14·49
"	5/8	2 7/16	1 13/16	15/16	2 1/2	·743	"	33,589	15·00
"	11/16*	2 13/16	2 1/8	1 1/16	2 7/8	·756	"	34,177	15·25
"	3/4	3 5/16	2 9/16	1 3/16	3 1/4	·774	"	34,991	15·62
7/16	11/16	2 11/16	2	1 1/16	2 7/8	·744	"	33,634	15·00
"	3/4*	3	2 1/4	1 1/8	3	·750	"	33,906	15·14
"	13/16	3 3/8	2 9/16	1 1/4	3 3/8	·759	"	34,313	15·32
1/2	11/16	2 5/16	1 5/8	15/16	2 1/2	·684	"	30,922	13·81
"	3/4	2 11/16	1 13/16	1 1/8	2 7/8	·721	"	32,595	14·55
"	13/16*	3 1/16	2 1/4	1 1/4	3 1/4	·734	"	33,183	14·81
"	7/8	3 1/2	2 5/8	1 3/8	3 5/8	·751	"	33,951	15·13
9/16	13/16	2 13/16	2	1 3/16	3	·711	"	32,143	14·35
"	7/8*	3 1/4	2 3/8	1 5/16	3 5/8	·731	"	33,047	14·75
"	15/16	3 5/8	2 11/16	1 7/16	3 3/4	·741	"	33,500	14·96
"	1	4 1/16	3 1/16	1 9/16	4	·754	"	34,086	15·21
5/8	7/8	3	2 1/8	1 1/4	3 1/4	·708	"	32,007	14·29
"	15/16*	3 3/8	2 7/16	1 3/8	3 5/8	·722	"	32,640	14·57
"	1	3 3/4	2 3/4	1 1/2	3 7/8	·733	"	33,137	14·79
"	1 1/16	4 3/16	3 1/8	1 5/8	4 1/4	·746	"	33,725	15·05
11/16	15/16	3 3/16	2 1/4	1 5/16	3 1/2	·706	"	31,917	14·25
"	1	3 1/2	2 1/2	1 7/16	3 3/4	·714	"	32,278	14·41
"	1 1/16*	3 15/16	2 7/8	1 9/16	4 1/8	·730	"	33,002	14·73
"	1 1/8	4 5/16	3 3/16	1 11/16	4 1/2	·739	"	33,408	14·91
3/4	1	3 5/16	2 5/16	1 3/8	3 1/2	·700	"	31,646	14·13
"	1 1/16	3 5/8	2 9/16	1 1/2	3 7/8	·707	"	31,962	14·27
"	1 1/8*	4 1/16	2 15/16	1 5/8	4 1/4	·723	"	32,685	14·59
"	1 3/16	4 7/16	3 1/4	1 3/4	4 5/8	·732	"	33,092	14·77
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

NOTE.—The rivet-holes marked * are the most suitable for the given thickness of plate, in ordinary cases.

(35.) "*Double-riveted Joints with Back and Front Plates.*"—With a plate on both sides, as in Fig 9, a double-riveted joint has an apparent strength of 51,000 lbs. per square inch by col. 6 of Table 5, and taking the shearing strength of rivets at 49,280 lbs. (19) the shearing area of the rivets should be $51000 \div 49280 = 1 \cdot 035$, that of the metal between holes being $1 \cdot 0$. Taking the case of $\frac{1}{2}$-inch plate with $\frac{13}{16}$ rivets, each space being now matched by *two* rivets subjected to *double-shear*, and the area of $\frac{13}{16} = \cdot 5185$ square inch, we have $\cdot 5185 \times 4 = 2 \cdot 074$ square inches of rivets, requiring $2 \cdot 074 \div 1 \cdot 035 = 2$ square inches of plate: hence the distance between the insides of two rivet-holes $= 2 \div \frac{1}{2} = 4$ inches; the pitch $4 + \frac{13}{16} = 4\frac{13}{16}$ inches: the ratio of metal between rivet-holes to the solid plate $= 4 \div 4\frac{13}{16}$, or $64 \div 77 = \cdot 831$, giving on the solid plate $51000 \times \cdot 831 = 42381$ lbs. per square inch, being $42381 \div 23172 = 1 \cdot 83$, or 83 per cent. greater strength than the same $\frac{1}{2}$-inch plate with simple single-riveted joint, as given by col. 8 of Table 6. Table 7 has been calculated throughout in this manner.

(36.) "*Chain-Riveting.*"—The various Tables 6 to 8 show
that with every kind of ordinary joint there is a considerabl
loss of strength, due to the metal being punched out to
the rivets, varying from $\cdot 588$, or a loss of $41 \cdot 2$ per cen
single-riveted joints in Table 6, to $\cdot 857$, or a loss of 1
cent. in double-riveted ones with front and back
Table 7. By what has been termed *Chain-riveti*
from this cause may be entirely avoided, and th
strength of the entire area of the solid plate may

Say we take $\frac{3}{4}$ plates, double-riveted with $1\frac{1}{8}$ ri
pitch, as in Table 8, the space between rivet-ho
then the ratio of the metal between rivet-hole
part of the plate or space $\div$ pitch, becomes
$47 \div 65 = \cdot 723$, hence $1 \cdot 0 - \cdot 723 = \cdot 277$, c
of the strength, is lost. Now if instead
rivets in two rows, as in Fig. 11, we place th
in Fig. 15, we have on the line Q, R, $4\frac{1}{16}$
pitch, and the space $12\frac{3}{16} - 1\frac{1}{8} = 11\frac{1}{16}$, th
$\div$ pitch becomes $11\frac{1}{16} \div 12\frac{3}{16}$, or 177 ÷

91 = 9 per cent. loss of strength only, allowing nothing for friction between the surfaces (20). But friction in such a joint would certainly add 9 per cent. or even more if it could be utilised, and thus we find that the full strength of the solid plate becomes available, there being no loss whatever.

The amount of lap in such a joint carried out in the ordinary manner would be very great, but this may be avoided by the arrangement shown by Fig. 15. The bottom plate of the girder, or rather the plate subjected to tensile strain, instead of being made in one thickness, is divided into two plates of half the thickness, and they are arranged upon one another so as to *break-joint*. Thus when a thickness of 1 inch is required we should use two ½-inch plates: then if the top plate extends from A to B, the lower plate would extend from C to D, the junction at C being in the centre of the solid part of the plate A, B, &c. We thus secure all the advantages of spreading the rivets, without any loss by lap.

The only drawback to this method is, that two thin plates will be more subject to damage from rust than one thick one of equal area, not only because they would expose double surface to the elements, but also that the interstice between them would harbour the rain-water. This method is therefore most useful in large structures where thick plates are used, and even then, care should be taken by painting, &c., to obviate deterioration by rusting.

STEEL RIVETED-JOINTS FOR GIRDER-WORK.

(37.) The introduction of the Bessemer process in the manufacture of steel, and consequent reduction in cost, has led to its extensive use for all purposes as a substitute for wrought iron. The full value of its great tensile strength has not been quite realised with riveted joints, from the fact that steel rivets have comparatively a low shearing strength, which differs very little from that of wrought iron (42). This has led to the necessity for larger rivets than would otherwise have been required, resulting in some loss of strength. The best experiments we have are those of Mr. H. Sharp, from which we shall obtain the

strength of annealed and unannealed steel plates, solid, punched, and drilled; also the strength of the metal in riveted joints, and of the rivets in those joints.

(38.) "*Solid or Unpunched Plates.*"—Table 9 gives the strength of solid steel plates, and shows the remarkable effect of annealing or heating to a dull red heat and cooling slowly in sand or ashes, the result being an increase in strength of 51 per cent. crossways of the grain, and 60 per cent. lengthways, the mean of the two = $55\frac{1}{2}$ per cent.

Remarkable as this result is, it is confirmed by the experiments of Mr. Barnaby at H.M. Dockyard, Chatham, on steel plates $\frac{1}{2}$ inch thick, *punched* with holes about $\frac{5}{8}$ inch diameter, &c., as in Fig. 14: the average of eight annealed plates was 32·839, and of eight unannealed plates 21·097 tons per square inch, showing a difference of 38·839 ÷ 21·097 = 1·5566, or 55·66 per cent., being almost exactly the same as with solid plates.

TABLE 9.—Of Experiments on the TENSILE STRENGTH of SOLID STEEL PLATES.

Direction of the Grain.	Breaking Weight. Tons per Square Inch.		Ratio.
	Annealed.	Not Annealed.	
Crossways	33·010	19·390	..
"	29·981	26·369	..
"	30·724	18·885	..
"	31·278	18·005	..
" Mean =	31·250	20·662	= 1
Lengthways	36·929	22·460	
"	35·397	22·781	
"	33·875	16·997	
"	33·522	23·891	
" Mean =	34·43	21·53	
	(1)	(2)	

(39.) This is the more remarkable because annealing with wrought-iron plates was just shown by the direct experiments of Kirkaldy of Yorkshire iron; the mean result of eigh

plates $\frac{3}{8}$, $\frac{1}{2}$, and $\frac{5}{8}$ inch thick being a *loss* by annealing of 5·6 per cent. lengthways, and 5·2 per cent. crossways. With Lowmoor iron, the loss was 4·8 and 1·8 per cent., and with Bowling, 8·0 and 9·1 per cent. respectively.

It would appear from this that steel plates, even in the solid or unpunched form should always be annealed With annealed plates, those strained lengthways of the grain are 10 per cent. stronger than those strained crossways, and with those not annealed, 4·2 per cent.

(40.) "*Effect of Punching and Drilling.*"—Mr. Sharp made experiments on steel plates by punching and drilling rivet-holes of the diameter and pitch commonly used for riveted joints, the results of which are given by Table 10, which shows that by punching cold in the usual way, the metal left between the holes is damaged from 24·1 to 38 per cent., the mean being 33 per cent., which is very great as compared with wrought iron. Mr. Kirkaldy's experiments on Yorkshire iron in Table 4 gives the loss due to punching from 6·7 to 21·2 per cent., the mean of the whole being 15 per cent. only; Mr. Fairbairn's experiments on Lowmoor iron in single-riveted *joints* gave 1·0–·76 = ·24, or 24 per cent. loss, by col. 4 of Table 5.

After the punched plates were annealed, the tensile strength was restored to 35·86 tons per square inch, or nearly to that of the solid plate, which in this case was 36·22 tons, and this again is the strength of the metal in a drilled plate which by Table 10 = 36·3 tons per square inch. From this we find when the holes are drilled the metal left between holes is uninjured, its strength per square inch being equal to that in a solid plate.

It is not very clear how these experiments were made, but it would appear that the plates in Table 10 were all annealed to begin with; then after the holes were punched the strength was reduced from 36·22 to 24·333 tons per square inch, which by annealing *a second time* was restored nearly to its normal value, or to 35·86 tons. The drilled and annealed plates gave 36·3 tons, or practically the same strength as the annealed solid plate, which was 36·22 tons.

(41.) "*Riveted Joints in Steel Plates.*"—Steel plates $\frac{5}{16}$ inch

TABLE 10.—Of the EFFECT of PUNCHING and DRILLING RIVET-HOLES in STEEL PLATES.

Breaking Weight. Tons per Square Inch.		Ratio.	Loss Per Cent.
Holes Punched.	Holes Drilled.		
26·690 ÷	35·22 =	·759	24·1
23·735 ÷	37·27 =	·637	36·3
22·570 ÷	36·40 =	·620	38·0
Mean Results of the whole.			
24·333 ÷	36·30 =	·67	33·0

thick, were riveted together with $\frac{9}{16}$ rivets, $1\frac{3}{8}$ pitch; the joints being in the six different forms given in Table 5. Unfortunately the results were vitiated by the weakness of the rivets. Of course when a joint fails by the rivets shearing it is no test of the strength of the plate: the only fair way is to take those cases where the rivets failed as giving the strength of the rivets, and *vice-versâ*.

Taking from Table 11 the *plates* which failed, we have three with *punched* holes giving 40·98, 43·63, and 39·11 tons, the mean = 41·24 tons per square inch. Then two plates with *drilled* holes failed with 39·25 and 42·93 tons respectively, the mean = 41·09 tons per square inch of metal between holes, which shows that with riveted joints, as with unriveted plates (40), the strength of *annealed* steel plates is the same whether the holes are punched or drilled. The mean of the five experiments on punched or drilled plates is 41·2, say 41 tons or 91,840 lbs. per square inch, which may be taken as the breaking weight of metal between rivet-holes in ordinary double-riveted steel joints. It will be observed that in some of the experiments from which that datum was derived, the joint had a back and front plate, the rivets being therefore subjected to a double-shear, but that fact seems to have made no difference to the results.

(42.) "*Strength of Steel Rivets.*"—The resistance of mild steel rivets to a shearing strain may be obtained from Table 11:

TABLE 11.—Of Experiments on the STRENGTH of RIVETED JOINTS, in STEEL PLATES.

Kind of Joint.	Breaking Strain. Tons per Square Inch.			Mode of Failing.
	On the Plate. Metal between Holes.		On the Rivets.	
	Drilled.	Punched.		
Solid Plate (36·22 tons) Fig. 6	..	..	..	..
Single-riveted „ A	24·928	..	25·53	Rivets sheared.
„ „ A	..	26·254	25·87	„
„ with back plate „ B	23·68	..	24·26	„
„ „ „ B	..	24·53	24·17	„
„ with back and front plate .. Fig. C	36·62	..	18·75	„
„ „ „ C	..	40·98	20·20	Plate broke.
Double-riveted „ D	42·33	..	25·95	Rivets sheared.
„ „ D	..	37·00	21·88	„
„ with back plate „ E	39·25	..	17·18	Plate broke.
„ „ „ E	..	43·63	18·42	„
„ with back and front plate .. Fig. F	42·93	..	9·15	„
„ „ „ F	..	39·11	7·25	„
	(1)	(2)	(3)	

in seven cases the rivets were sheared with strains varying from 25·95 to 18·75 tons, the mean of the whole being 23·77 tons or 53,245 lbs. per square inch, which is remarkably low. It is generally admitted that the shearing and tensile strains are equal to one another, and (123) shows that this is correct so far as wrought iron is concerned. But the mean tensile strength of bar steel is 47·84 tons per square inch, this being the mean of sixty-six experiments in Table 1, so that it would appear that the fibres of steel are very seriously damaged in the act of riveting, the shearing strength being reduced to *half* the normal tensile strength.

This is the more remarkable because it is really lower than the shearing strength of *wrought iron:* the direct experiments of Mr. E. Clark in (123) give 24·14 tons per square inch as the mean of four experiments with single-shear, which is $1\frac{1}{2}$

per cent. greater than 23·77 tons the shearing strength of steel.

Moreover it should be observed that the strength of steel rivets was obtained from *joints* in steel plates, where the friction due to the grip of the rivets (20) must have contributed to the apparent strength, so that the resistance of the rivets alone must have been considerably less than 23·77 tons per square inch, which however must be accepted as the *apparent* shearing strength of steel rivets in double-riveted joints of steel plates.

(43.) "*Proportions of Steel Joints.*"—The pitch and other proportions of riveted joints with steel plates may be determined on the same principles as those of wrought-iron plates, but the relative weakness of steel rivets will affect the pitch very considerably.

We shall take the tensile strength of the metal between rivet-holes in steel joints of all kinds at 41 tons, or 91,840 lbs. (41), and the apparent shearing strain of steel rivets in joints at 53,245 lbs. per square inch (42): hence the rivets are 58 per cent. only of the strength of the plates.

Say we take for illustration $\frac{3}{8}$-inch plates, double-riveted: by col. 3 of Table 14 the rivets should be $\frac{11}{16}$ inch diameter; th
the area of the rivets must be 91840 ÷ 53245 = 1·725, t
the plate between holes being 1·0, hence for two $\frac{11}{16}$ r
each space as with double-riveted joints, whose area =
× 2 = ·7424 square inch, we require ·7424 ÷ 1·72
square inch of plate. The distance between holes w
fore be ·43 ÷ $\frac{3}{8}$ = 1·146, or $1\frac{1}{8}$ inch; the pitch $1\frac{1}{8}$ +
the ratio of metal left between holes, to the solid p
$1\frac{13}{16}$, or 18 ÷ 29 = ·621; and the strain on the
the plate when the metal between holes is bre
91840 × ·621 = 57033 lbs. per square inch.
been calculated in this way throughout.

Comparing steel joints with double-riveted w
in Table 8, col. 9 gives for $\frac{3}{8}$-inch plate 34,17
inch: hence we obtain 57033 ÷ 34177 = 1·
in favour of steel for Girder-work: for Boil

TABLE 12.—Of DOUBLE-RIVETED JOINTS in STEEL PLATES: for GIRDER-WORK ONLY.

Thickness of Plate.	Diam. of Rivet-holes.	Suitable Pitch.	Space between Rivets.	Distance between Rows of Rivets.	Lap.	Ratio of Punched to Solid Plate.	Breaking Strain of Metal between Holes. Lbs. per Square Inch.	Strain on Solid Part of Plate. Lbs. per Square Inch.
$\frac{3}{16}$	$\frac{7}{16}$	$1\frac{3}{8}$	$\frac{15}{16}$	$\frac{5}{8}$	$1\frac{5}{8}$	·682	91,840	62,635
$\frac{1}{4}$	$\frac{1}{2}$	$1\frac{7}{16}$	$\frac{15}{16}$	$\frac{11}{16}$	$1\frac{3}{4}$	·652	"	59,880
$\frac{5}{16}$	$\frac{9}{16}$	$1\frac{1}{2}$	$\frac{15}{16}$	$\frac{3}{4}$	$1\frac{7}{8}$	·625	"	57,400
$\frac{3}{8}$	$\frac{11}{16}$	$1\frac{13}{16}$	$1\frac{1}{8}$	$\frac{7}{8}$	$2\frac{1}{4}$	·621	"	57,030
$\frac{7}{16}$	$\frac{3}{4}$	$1\frac{15}{16}$	$1\frac{3}{16}$	1	$2\frac{1}{2}$	·613	"	56,300
$\frac{1}{2}$	$\frac{13}{16}$	$2\frac{1}{16}$	$1\frac{1}{4}$	$1\frac{1}{8}$	$2\frac{3}{4}$	·606	"	55,650
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

RIVETED JOINTS FOR STEAM-BOILER WORK.

(44.) In riveting plates for girder-work, we have only to consider the proper proportions of area of rivets to area of plate between rivet-holes, but for steam-boiler joints we have further to consider the space, or distance between rivet-holes with reference to the pressure of steam, otherwise the joint may not be steam-tight. A riveted joint may be abundantly strong enough to resist the strain, but if the pitch of the rivets is too great, it will give trouble by leaking.

(45.) "*Space between Rivets.*"—An ordinary lap-joint, Fig. 16, is made steam-tight by caulking at C, and although the contraction of the rivet in cooling will draw the two plates together, still there will be a small space at D, sufficient to allow the steam to enter, being stopped in its passage by the caulking at C. The plate at E, between two rivets may therefore be regarded as a beam loaded all over by the pressure of the steam, and if that pressure exceeds a certain amount, the effect will be to cause the beam to spring or deflect slightly and thereby to leak at C.

We have now to consider the relations between the thickness of plate, distance between the insides of the rivet-holes, and the pressure of steam.

Let A, B, C, Fig. 17, be three beams, all of the same depth (or thickness) and breadth, but varying in length in the ratio 1, 2, 4; then by analogy these may be regarded as three steam joints having distances between insides of rivet-holes in the ratio 1, 2, 3, &c. Now, if we admit that a crack of any measurable amount will cause leakage, that amount will be the same in all three cases, so that the problem becomes this; to find what the respective loads must be, to give one and the same deflection in all the three cases. By the laws of deflection in (662) it is shown that $W = \frac{d^3 \times b \times \delta}{L^3 \times C}$. In our case d^3, b, δ, and C are constant, therefore W will be *inversely* proportional to L^3 simply, hence the lengths C, B, A, being in our case 1, 2, 4, the loads will be in the ratio 4^3, 2^3, 1^3, or 64, 8, 1. But in our case, the surfaces over which these loads are spread are also in the ratio 1, 2, 4, and our special object is to find the pressure or load *per square inch;* with A we have a load of 1 spread over a length of 4, hence $1 \div 4 = \frac{1}{4}$ per unit of length; with B, a load of 8 spread over a length of 2, or $8 \div 2 = 4$ per unit of length; and with C, a load of 64 spread over a length of 1, or 64 per unit of length. Thus, with lengths 4, 2, 1, we obtain pressures $\frac{1}{4}$, 4, 64; or in the ratio 1, 16, 256, which are inversely as the fourth power of the lengths, for 1^4, 2^4, 4^4 are 1, 16, 256, and we thus find that with constant thickness, the pressure tending to produce leaka[illegible] of steam will be inversely proportional to S^4, or the fourth [illegible] of the space, or distance between the insides of the rivet [illegible]

The formula in (662) shows that W is directly pro[illegible] to t^3, hence we have the rules:—

(46.) $$p = M_L \times t^3 \div S^4.$$

(47.) $$S = \sqrt[4]{M_L \times t^3 \div p}.$$

(48.) $$M_L = S^4 \times p \div t^3.$$

In which S = the space between insides of rivet [illegible]
t = thickness of plate in $\frac{1}{8}$ths of a[illegible]
p = working pressure of steam in lb[illegible]
M_L = a constant from practice = [illegible] iron; 6·2 for steel plate.

To find the value of M_L we may take a standard case, say $\frac{3}{8}$ plate, $\frac{11}{16}$ rivets, 2 inches pitch, $1\frac{5}{16}$ space, and 50 lbs. per square inch; these are common proportions, and have been proved to be satisfactory by universal experience.

We may find $1\frac{5}{16}{}^4$ by the *square of the square* of that number; thus $1\frac{5}{16}{}^2 = 1\cdot723$, and $1\cdot723^2 = 2\cdot969$, which is the fourth power of $1\frac{5}{16}$; then the rule $M_L = S^4 \times p \div t^3$, becomes $2\cdot969 \times 50 \div 27 = 5\cdot48$, say $5\cdot5$, the value of M_L.

(49.) Again: to find S for say 150-lb. steam with $\frac{1}{4}$-inch plate, the rule (47) becomes $S = \sqrt[4]{5\cdot5 \times 4^3 \div 150} = 1\cdot238$, or say $1\frac{1}{4}$ inch. For example, $5\cdot5 \times 64 \div 150 = 2\cdot35$; then we may obtain the 4th root of $2\cdot35$ by finding the square root of the square root of that number: thus $\sqrt{2\cdot35} = 1\cdot533$, and $\sqrt{1\cdot533} = 1\cdot238$ inch as before, this being the 4th root of $2\cdot35$. We should obtain the same result direct by the use of logarithms: thus the log. of $2\cdot35$ or $\cdot371068 \div 4 = \cdot092767$, the natural number due to which $= 1\cdot238$ inch as before.

Again: to find p for say $\frac{7}{16}$ plate with $\frac{3}{4}$ rivets, $1\frac{5}{16}$-inch space, therefore $\frac{3}{4} + 1\frac{5}{16} = 2\frac{1}{16}$ pitch, the rule (46) gives $p = 5\cdot5 \times 3\frac{1}{2}{}^3 \div 1\frac{5}{16}{}^4 = 80$-lb. steam. Thus $1\frac{5}{16}{}^2 = 1\cdot72$, and $1\cdot72^2 = 2\cdot96$, which is the 4th power of $1\frac{5}{16}$. Then $3\frac{1}{2}{}^3$ being $= 42\cdot87$, we obtain $p = 5\cdot5 \times 42\cdot87 \div 2\cdot96 = 80$-lb. steam as before.

The London and North-Western Railway Co. at Crewe, for their 4-foot locomotive boilers, use $\frac{13}{32}$-inch plates, $\frac{3}{4}$ rivets, $1\frac{3}{4}$ pitch, therefore 1-inch space; then $p = 5\cdot5 \times 3\frac{1}{4}{}^3 \div 1^4 = 189$-lb. steam; the actual ordinary working pressure is 120 lbs.; occasionally 150 lbs. per square inch.

(50.) Table 13 has been calculated by rule (46). It should be understood that these rules are approximate only, giving a fair working pressure. Possibly a pressure double or even treble that given by the rule, would not cause the joint to leak instantly, but in all probability it would eventually do so, and as it is *essential* that boilers should be *perfectly* steam-tight, it will be advisable that the working pressure should not much exceed those given by the rules, and Table 13. When a steam joint or anything else is overstrained, failure is always more or less a question of time.

TABLE 13.—Of the MAXIMUM PRESSURE of STEAM with RIVETED JOINTS, as governed by the Space between Rivet-holes.

Space between Rivet-holes.	Thickness of Plate in Inches.									
	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$
	Average Working Pressure of Steam: Lbs. per Square Inch.									
in.										
$\frac{5}{8}$	122	288	..	..	..	..	..	..	..	..
$\frac{11}{16}$	83	197	384	..	..	..	..	..	..	..
$\frac{3}{4}$	60	140	273	470	..	..	..	..	..	..
$\frac{13}{16}$	43	101	197	341	..	..	..	..	..	..
$\frac{7}{8}$	32	75	147	253	402	..	..	..	..	..
$\frac{15}{16}$	24	57	111	192	305	456	..	..	..	..
1	18	44	86	148	236	352	501	..	..	..
$1\frac{1}{16}$	15	35	68	117	185	276	393	..	..	..
$1\frac{1}{8}$	12	28	54	93	147	222	313	429	..	..
$1\frac{3}{16}$	9	22	43	75	119	177	252	346	460	..
$1\frac{1}{4}$	..	18	35	61	97	144	205	282	378	..
$1\frac{5}{16}$	..	15	29	50	80	119	169	232	300	400
$1\frac{3}{8}$	..	..	24	41	66	98	140	192	256	332
$1\frac{7}{16}$	..	..	20	35	55	82	117	161	214	278
$1\frac{1}{2}$	..	..	..	29	47	70	100	136	180	235
$1\frac{5}{8}$	..	..	..	21	32	50	72	100	131	170
$1\frac{3}{4}$	..	..	..	..	25	37	54	73	98	127
$1\frac{7}{8}$	..	..	..	..	..	28	41	55	74	96
2	..	..	..	..	..	..	31	43	57	74
$2\frac{1}{8}$	..	..	..	..	..	..	..	34	44	58
$2\frac{1}{4}$	..	..	..	..	..	..	..	..	36	4[illegible]

(51.) We have admitted in (28) that the diameter o
rivets shall be governed by the *thickness* of the plat
irrespective of the pressure of steam or other consid
and in (44) we have allowed that the space between
(and thereby the pitch of the rivets) shall be do
the pressure of the steam. But under these tw
it is impossible to secure that equality between
strain on the rivets and the tensile strain on the pl
an essential principle in riveting, as shown i
instance, in Table 14, the pitch is allowed t
whether the joints are single or double-riveted
if the area of the rivets is properly propc

former, they must have an excess of strength for the latter, because in one case each space is matched by one rivet, and in the other case by two.

(52.) This anomaly might be avoided if we allow that the diameter may be varied so as to adapt it to the strain, irrespective of the mere *thickness* of the plate. Say that we take $\frac{1}{2}$-inch plate for 50-lb. steam, = $1\frac{5}{8}$-inch space by Table 14: then the area of plate between two rivet-holes = $1\frac{5}{8} \times \frac{1}{2}$ = ·8125 square inch, giving in a single-riveted joint by col. 7 of Table 5, 34110 × ·8125 = 27710 lbs., and as rivets yield 49,280 lbs. per square inch (19), we have 27710 ÷ 49280 = ·562 square inch of rivet = say bare $\frac{7}{8}$-inch diameter, agreeing nearly with col. 2 of Table 6, which gives $\frac{13}{16}$ inch diameter for $\frac{1}{2}$-inch plate, showing that in a single-riveted joint the principle of equality between the strains on the rivet and plate is complied with.

But with double-riveted joints we have two rivets to each space, and 41,800 lbs. per square inch of plate by col. 7 of Table 5: then, we have 41800 × ·8125 = 33962 lbs. from the plate requiring 33962 ÷ 49280 = ·689 square inch, area of *two* rivets, or ·345 square inch each, = say, $\frac{11}{16}$ inch diameter, instead of $\frac{7}{8}$ inch, as for single-riveting; but by most practical men $\frac{11}{16}$ rivets would be deemed too light for $\frac{1}{2}$-inch plates.

(53.) Besides, there is this anomaly, that the higher the pressure of steam, the smaller the rivets become, this being due to the reduced space between rivets. Thus, for 350-lb. steam, and $\frac{1}{2}$-inch plates, the space = 1 inch by Table 13, hence $1 \times \frac{1}{2} = \frac{1}{2}$ square inch of metal, which in a single-riveted joint would give $34100 \times \frac{1}{2}$ = 17055 lbs., requiring 17055 ÷ 49280 = ·346 square inch of rivet = say $\frac{11}{16}$ inch diameter for 350-lb. steam, whereas for 50-lb. steam we obtained $\frac{7}{8}$ inch.

These calculated proportions are no doubt correct so far as the strains on the rivet and plate are concerned, but there are other considerations which render it inexpedient that they should be followed, and we must admit the practical dictum (28) that the diameter of the rivet shall be proportional to the thickness of the plate, as given by the rule (27).

(54.) "*Space between Rivets with Steel Plates.*"—By col. 4 of

Table 105, steel is stiffer than wrought iron, the difference being $1565 \div 1386 = 1 \cdot 13$ or 13 per cent., and the pressure of steam would be greater in that ratio; hence the rule (46) becomes:—

$$p = 6 \cdot 2 \times t^3 \div S^4.$$

But the difference of 13 per cent. is so small, and the rule such an approximate one, that we may safely admit that the working pressure with steel will be the same as for wrought iron as given by Tables 13, &c.

CHAPTER III.

COHESION APPLIED TO PIPES.

(55.) It is necessary to consider this subject under two different heads; namely, *thin* and *thick* pipes; the former being usually of wrought sheet metal, such as ordinary steam-boilers, and the latter of cast metals, such as strong water-pipes, hydraulic-press cylinders, &c. The strains in these two cases differ considerably from one another, the latter being much more complex than the former.

"*Thin Tubes.*"—Let Fig. 18 be a tube 1 inch square, and for the sake of illustration, say 1 inch deep, subjected to an internal fluid pressure of 100 lbs. per square inch, acting, of course, in all directions. Now the surface *c*, *d* having an area of 1 square inch, will exert a force of 100 lbs. in the direction of t[illegible] arrow *a*, and will be resisted by a similar force on the su[illegible] *e*, *f*, acting in the direction of *b*; hence we have a [illegible] strain of 50 lbs. on each of the sides *c*, *e* and *d*, *f*, tend[illegible] produce rupture say on the line B, B.

(56.) Let Fig. 19 be an octagonal tube 1 inch ins[illegible] 1 inch deep as before: we have first to find the d[illegible] of the sides of the polygon. The *half*-side *a*, *b* is [illegible] the tangent of the angle *a*, *d*, *b*, which being th[illegible] part of the circle will be $360 \div 16 = 25°$: then b[illegible] of natural tangents we find that with radius $1 \cdot 0$, [illegible]

of 25° = 0·4142, hence with radius $\frac{1}{2}$, as in our case, we have $a, b = 0 \cdot 2071$, and $a, c = 0 \cdot 4142$ inch. The pressure on a, c will act direct as a tendency to rupture on the points B, B, the force being $0 \cdot 4142 \times 100 = 41 \cdot 42$ lbs.; but the pressure on c, e and a, g will act obliquely. Thus the strain on c, e will of course be 41·42 lbs., as on a, c, but it will act in the direction of the arrow x, and must be resolved into two equivalent forces, one in the direction of the arrow y, which being at right angles to B, B, will tend to rupture on those points; the other in the direction of the arrow z being parallel to B, B will have no effect. By the well-known parallelogram of forces, Fig. 20 making the diagonal $D = 41 \cdot 42$ lbs., we have two equivalent strains, the direction and force of which are given by the two sides of the parallelogram E, F, each 29·29 lbs., F being a direct tensile strain on the points B, B in Fig. 19. Of course the side a, g will give 29·29 lbs. also, and the combined strain will be from $a, g = 29 \cdot 29$; $a, c = 41 \cdot 42$; and $c, e = 29 \cdot 29$ lbs., or $29 \cdot 29 + 41 \cdot 42 + 29 \cdot 29 = 100$ lbs., being precisely the same as with the tube, Fig. 18, 1 inch *square*.

Calculating in this way with a polygon of any number of sides we should obtain the same result, and a circle being regarded as a polygon with an infinite number of sides, we thus find that the strain on a cylindrical tube is the same as on a square one of the same dimensions. From this it follows that the strength of a cylinder of thin plate, such as an ordinary boiler, is simply and directly proportional to the thickness, and inversely as the diameter.

(57.) "*Lap-welded Tubes.*"—Say that we require the strength of a small boiler 24 inches diameter, $\frac{1}{4}$-inch plate, with welded joint, made of Staffordshire plates whose tensile strength, namely, that of a solid plate, is 20 tons per square inch. By Mr. Bertram's experiments at Woolwich the strength of a lap-welded joint may be taken at 65 per cent. of that of the solid plate: hence $20 \times \cdot 65 = 13$ tons, or 29,120 lbs. per square inch. In our case rupture strains $\frac{1}{2}$ a square inch (or $\frac{1}{4}$ inch at each side); hence $29120 \times \frac{1}{2} = 14560$ lbs., which on 24 inches gives $14560 \div 24 = 607$ lbs. per square inch

bursting pressure; with 6 for the Factor of safety (78) we obtain $607 \div 6 =$ say 100 lbs. per square inch, safe or working pressure. From this we have the general rules :—

(58.) For welded boilers: $P = 58200 \times t \div d$.

(59.) " " " $p = 9700 \times t \div d$.

In which t = thickness of plate in inches; d = inside diameter in inches; P = the bursting pressure, and p = the safe working pressure in lbs. per square inch: thus for the 24-inch boiler we have considered, the rule gives $9700 \times \frac{1}{4} \div 24 = 101$, say 100 lbs. per square inch working pressure, as before.

(60.) "*Steam-boilers with Riveted Joints.*"—Staffordshire plates are now so extensively used for boilers, that it will be expedient to take them as a basis for general rules, although, as shown by Table 5, their strength is inferior to the mean of British plate-iron, and still more inferior to Yorkshire iron.

We have shown in (44) that the pitch of rivets, and thereby the general proportions of joints in steam-boilers, is governed by the pressure of steam as affecting the tendency to leakage, irrespective of strength to resist bursting.

(61.) For the purpose of fixing general proportions, we may take as a "standard" case the working pressure of 50 lbs. per square inch, the proportions due to which will suffice for all lower pressures; and also with sufficient accuracy for practical purposes up to say 70 or 80 lbs. per square inch. The proportions for higher pressures should be found by special calculation (68) (76).

We have first to find the space between rivet-holes with the different thicknesses of plate for 50-lb. steam by Table 13; taking the nearest pressures in that Table we obtain col. 6 in Table 14. Thus, for $\frac{3}{8}$-inch plate we have for 50-lb. steam the space $= 1\frac{5}{16}$; Table 14 gives $\frac{11}{16}$ rivets, as in col. 3; hence the pitch $= 1\frac{5}{16} + \frac{11}{16} = 2$ inches, col. 4; the ratio of the metal between holes to the solid part of the plate $= 1\frac{5}{16} \div 2$, or $21 \div 32 = \cdot 656$, as in col. 7. The apparent strength in single-riveted joints of Staffordshire plates being 34,110 lbs. per square inch by col. 7 of Table 5, that on the solid part of the plate $= 34110 \times \cdot 656 = 22380$ lbs., as in col. 8 of Table 14.

TABLE 14.—Of the PROPORTIONS of RIVETED JOINTS in WROUGHT-IRON PLATES, for Steam-boilers, with 50-lb. Steam.

Thickness.	Diameter of Rivet-holes.		Pitch.	Space between Rivet-holes.	Working Pressure of Steam.	Ratio of Punched to Un-punched Plate.	Strain on Solid Part of Plate, in Lbs. per Square Inch.		Single Riveted Joints. Lap.	Double Riveted.	
	By Rule.	Practical Sizes.					Single Riveted.	Double Riveted.		Lap.	Between Rows of Rivets.
$\frac{1}{8}$	·344	$\frac{5}{16}$	$\frac{7}{8}$	$\frac{9}{16}$	55	·643	21,933	26,877	$\frac{7}{8}$	$1\frac{3}{8}$	$\frac{7}{16}$
$\frac{3}{16}$	·422	$\frac{7}{16}$	$1\frac{3}{16}$	$\frac{3}{4}$	60	·632	21,560	26,418	$1\frac{1}{8}$	$1\frac{3}{4}$	$\frac{5}{8}$
$\frac{1}{4}$	·500	$\frac{1}{2}$	$1\frac{7}{16}$	$\frac{15}{16}$	57	·652	22,240	27,254	$1\frac{1}{2}$	$2\frac{1}{8}$	$\frac{3}{4}$
$\frac{5}{16}$	·573	$\frac{9}{16}$	$1\frac{11}{16}$	$1\frac{1}{8}$	54	·667	22,760	27,880	$1\frac{3}{4}$	$2\frac{1}{2}$	$\frac{7}{8}$
$\frac{3}{8}$	·656	$\frac{11}{16}$	2	$1\frac{5}{16}$	50	·656	22,380	27,420	2	3	1
$\frac{7}{16}$	·734	$\frac{3}{4}$	$2\frac{3}{16}$	$1\frac{7}{16}$	55	·657	22,410	27,467	$2\frac{1}{8}$	$3\frac{1}{4}$	$1\frac{1}{16}$
$\frac{1}{2}$	·813	$\frac{13}{16}$	$2\frac{7}{16}$	$1\frac{5}{8}$	50	·667	22,760	27,880	$2\frac{3}{8}$	$3\frac{1}{2}$	$1\frac{1}{8}$
$\frac{9}{16}$	·891	$\frac{7}{8}$	$2\frac{5}{8}$	$1\frac{3}{4}$	54	·667	22,760	27,880	$2\frac{1}{2}$	$3\frac{3}{4}$	$1\frac{1}{4}$
$\frac{5}{8}$	·944	$\frac{15}{16}$	$2\frac{13}{16}$	$1\frac{7}{8}$	55	·667	22,760	27,880	$2\frac{3}{4}$	$4\frac{1}{8}$	$1\frac{3}{8}$
$\frac{11}{16}$	1·047	$1\frac{1}{16}$	$3\frac{1}{16}$	2	57	·653	22,280	27,295	$2\frac{7}{8}$	$4\frac{1}{2}$	$1\frac{1}{2}$
$\frac{3}{4}$	1·125	$1\frac{1}{8}$	$3\frac{1}{4}$	$2\frac{1}{8}$	58	·654	22,310	27,337	3	$4\frac{3}{4}$	$1\frac{5}{8}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)

With double-riveted joints, the apparent strength of metal between rivet-holes = 41,800 lbs. by col. 7 of Table 5, hence the strain on the solid part of the plate = $41800 \times \cdot 656 = 27420$ lbs. per square inch; col. 9. Calculating in this manner, we have obtained the general proportions in Table 14.

(62.) The mean strain on the solid part of the plate, when the metal between the rivet-holes is breaking, is with single-riveted joints 22,380 lbs., or say 10 tons, as in col. 8, and for double-riveted joints = 27,420 lbs.: col. 9.

"*General Rules.*"—We may now apply these results to practice, and may take for illustration a 48-inch boiler with $\frac{3}{8}$-inch plate, and for the purposes of calculation say 1 inch long. Now as we have $\frac{3}{8} \times 1$ inch *at each side*, this is evidently equal to $\frac{3}{4}$ square inch area of metal taken through the solid part of the plate, the reduced resistance of which in a single-riveted joint $= 22400 \times \frac{3}{4} = 16800$ lbs: this is the *total* strain on the whole surface with which the boiler would burst, which being spread over the diameter (56), or 48 inches, gives $16800 \div 48 = 350$ lbs. per square inch bursting pressure. Hence we have the rules:—

(63.) For single-riveted joints: $P = 44800 \times t \div d$.

(64.) " " " $p = 7466 \times t \div d$.

In which t = the thickness of plate in inches; d = inside diameter in inches; P = the bursting pressure, and p = the safe working pressure in lbs. per square inch:—thus in our case, $P = 44800 \times \frac{3}{8} \div 48 = 350$ lbs. per square inch as before.

With double-riveted joints, the mean reduced strain on the solid part of the plate = 27,420 lbs. per square inch, col. 9 of Table 14, or in our case $27420 \times \frac{3}{4} = 20565$ lbs. total bursting pressure on a circle 48 inches diameter, or $20565 \div 48 = 428$ lbs. per square inch: hence we have the rules:—

(65.) For double-riveted joints: $P = 54840 \times t \div d$.

(66.) " " " $p = 9140 \times t \div d$.

Thus in our case, $P = 54840 \times \frac{3}{8} \div 48 = 428$ lbs. per square inch bursting pressure.

[redacted]

TABLE 15.—Of the STRENGTH of CYLINDRICAL BOILERS made of STAFFORDSHIRE PLATES with RIVETED JOINTS, for internal Pressures in Pounds per Square Inch.

Thickness of Plate.	Inside Diameter of Boiler in Feet.													
	1½	2	2½	3	3½	4	4½	5	5½	6	6½	7	7½	8
	Bursting Pressure—Single-riveted Joints.													
inches.														
3/16	467	350	280	233	200	175	..	..	..	..	..	..	..	..
1/4	621	466	373	311	266	233	207	186	..	..	..	..	..	..
5/16	777	583	466	388	333	292	259	233	212	194	..	..	..	..
3/8	932	699	559	466	400	350	310	280	254	233	215	200	..	..
7/16	1090	816	652	544	466	408	362	326	297	272	251	233	218	204
1/2	..	932	746	621	533	466	414	373	339	310	287	267	249	233
9/16	..	..	839	700	600	524	466	419	381	350	322	300	280	262
	Bursting Pressure—Double-riveted Joints.													
3/8	1143	857	686	571	490	428	381	343	312	286	264	245	..	..
7/16	1333	1000	801	666	571	500	444	400	363	233	308	286	267	250
1/2	..	1142	915	762	653	571	508	457	415	381	352	324	305	286
9/16	..	..	1030	857	734	643	571	514	467	428	396	367	343	321
5/8	..	..	..	952	816	714	635	571	519	476	439	408	381	357
11/16	..	..	..	..	898	785	698	628	571	524	483	449	419	393
3/4	..	..	..	..	..	857	762	685	624	571	527	490	457	428
	Working Pressure—Single-riveted Joints: Factor 6.													
3/16	78	58	47	39	33	29	..	..	..	..	..	..	..	..
1/4	104	78	62	52	45	39	35	31	..	..	..	..	..	..
5/16	130	97	78	65	56	49	43	39	35	32	..	..	..	..
3/8	155	117	93	78	67	58	52	47	42	39	36	33	..	..
7/16	182	136	109	91	78	68	61	54	50	45	42	39	36	34
1/2	..	155	124	104	89	78	69	62	57	52	48	45	42	39
9/16	..	..	140	117	100	87	78	70	64	58	54	50	47	44
	Working Pressure—Double-riveted Joints: Factor 6.													
3/8	191	143	115	95	82	72	64	57	52	48	44	41	..	..
7/16	222	167	133	111	95	83	74	67	61	56	51	48	45	42
1/2	..	190	152	127	109	95	85	76	69	64	59	55	51	48
9/16	..	..	172	140	123	107	95	86	78	72	66	61	57	54
5/8	..	..	..	159	137	119	106	95	87	80	73	68	64	60
11/16	..	..	..	..	150	131	116	105	95	87	81	75	70	66
3/4	..	..	..	..	..	143	128	114	104	96	88	82	76	71

(67.) By these rules, Table 15 has been calculated for the bursting and safe working pressures: the former will enable the engineer to select a factor of safety to suit his case and to satisfy his judgment. For ordinary cases and moderate pressures Factor 6 should be used as in Table 15; but for very high pressures that factor would lead to excessive and almost impracticable thicknesses, and it becomes necessary to use a lower one, the risk being of course proportionally increased (78).

(68.) "*Boilers for very high Pressures.*"—Tables 14, 15 are strictly adapted for 50-lb. steam only, but may be used for higher pressures up to say 80 lbs. For higher pressures the proportions of the joints should be specially calculated in the manner illustrated in (61).

Say that we take the case of a boiler 27 inches internal diameter, for 300-lb. steam, this being the working pressure: for so high a pressure we may take the factor of safety at 4 (78). We will assume that the thickness shall be $\frac{11}{16}$ and double-riveted: then, by Table 13, the space between rivets = $1\frac{5}{16}$ inch, and the diameter of the rivets by col. 3 of Table 14 = $1\frac{1}{16}$ inch: hence the pitch = $1\frac{5}{16} + 1\frac{1}{16} = 2\frac{3}{8}$; the ratio of the area of the punched plate to that of the solid plate = $1\frac{5}{16} \div 2\frac{3}{8}$, or $21 \div 38 = \cdot 553$. The apparent strength of the metal between rivet-holes in a double-riveted joint of Staffordshire iron = 41,800 lbs. by col. 7 of Table 5, hence we have $41800 \times \cdot 533 = 23115$ lbs. per square inch on the solid part of the plate, and as we have $1\frac{3}{8}$ square inch of metal per inch run (or $\frac{11}{16}$ at each side) we obtain $23115 \times 1\frac{3}{8} = 31783$ lbs. bursting strain, or that on the whole of the internal surface of the 27-inch boiler, or $31783 \div 27 = 1177$ lbs. per square inch; then with Factor 4, we have $1177 \div 4 = 294$ lbs. safe working pressure per square inch, which is very nearly the actual pressure required, or 300 lbs.

(69.) We may now show the effect of erroneously calculating this boiler by the general Table 15, or rather by the rule (65) on which that Table is based, that rule and table being strictly correct for pressures of about 50 lbs. only (61). In our case the rule becomes $54840 \times \frac{11}{16} \div 27 = 1397$ lbs. per square inch bursting pressure. But by the correct calculation (68) we

obtained 1177 lbs. only, the difference being $1397 \div 1177 = 1 \cdot 19$, or 19 per cent.; this difference is due to the circumstance that to avoid leakage with so great a pressure as 300 lbs., the pitch of the rivets was reduced, with the result that the ratio of the metal between holes to the solid plate became ·553 with 300-lb. steam, instead of ·653 as for 50-lb. steam, by col. 7 of Table 14. It should be observed that both results are equally correct so far as the bursting strains only are concerned; the danger would be, that the joints having the pitch of rivets, &c., adapted for 50-lb. steam as given by Table 13, would in all probability leak more or less with 300 lbs., and in order to avoid that contingency it is expedient to sacrifice the 19 per cent. of strength involved in the case.

(70.) For such very high pressures it would be prudent and perhaps commercially economical to use the best Yorkshire iron, which, as shown by (24), has 25·6 per cent. greater strength than Staffordshire. Say for our 27-inch boiler and 300-lb. strain, we assume the thickness of Yorkshire plate at $\frac{9}{16}$ inch, for which Table 14 gives $\frac{7}{8}$-inch rivets, and Table 13, $1\frac{1}{8}$-inch space; hence the pitch $= \frac{7}{8} + 1\frac{1}{8} = 2$ inches; the ratio of punched to solid plate $= 1\frac{1}{8} \div 2$, or $9 \div 16 = \cdot 563$. The apparent strength of metal between rivet-holes in double-riveted joints of Yorkshire iron = 52,503 lbs. per square inch by col. 3 of Table 5; hence $52503 \times \cdot 563 = 29592$ lbs. per square inch on the solid part of the plate, and as we have $1\frac{1}{8}$ square inch of metal per inch run (namely $\frac{9}{16}$ at each side), we obtain $29592 \times 1\frac{1}{8} = 33291$ lbs. the bursting strain, or that on the whole of the internal surface of the 27-inch boiler, or $33291 \div 27 = 1233$ lbs. per square inch. Then with Factor 4 we have $1233 \div 4 = 308$ lbs. per square inch safe working pressure: by substituting Yorkshire plates for Staffordshire we have thus reduced the thickness from $\frac{11}{16}$ to $\frac{9}{16}$, and the weight from $1 \cdot 0$ to $9 \div 11 = \cdot 82$, or $100 - 82 = 18$ per cent. The effect of substituting steel for wrought-iron plate is shown by (76).

(71.) "*Longitudinal Strain on Boilers.*"—There are two distinct strains to which an ordinary cylindrical boiler is subjected, one acting circumferentially and the other longitudinally: the

former alone is considered in the various rules and tables we have so far given; we have now to investigate the latter.

Say we take a plain cylindrical boiler with either hemispherical or flat ends, but without any internal flue, $\frac{3}{8}$ inch thick, 48 inches internal, therefore $48\frac{3}{4}$ inches external diameter. Taking the apparent strength of single-riveted Staffordshire plates at 22,400 lbs. per square inch on the solid part of the plate, as in col. 8 of Table 14 and (62), the bursting pressure circumferentially $= 22400 \times \frac{3}{8} \times 2 \div 48 = 350$ lbs. per square inch, or the same as given by Table 15.

To find the strain on the two ends we have the area of $48\frac{3}{4} = 1868$, and of $48 = 1809$; hence the area of the annulus $= 1868 - 1809 = 59$ square inches, giving a total pressure of $22400 \times 59 = 1321600$ lbs. on the ends, and the internal area being 1809, we have $1321600 \div 1809 = 730$ lbs. per square inch bursting pressure longitudinally, or about *double* the circumferential bursting pressure, which we found to be 350 lbs.

Applying this reasoning to other diameters and thicknesses, it will be found that the ratio between the two strains is constant for all sizes; hence when a boiler is on the point of rupture circumferentially with the pressure given by the rules in this work, the longitudinal strain is only *half* the breaking weight in that direction.

In an ordinary Cornish boiler with one or two internal flues the longitudinal bursting pressure will be still greater, the flues adding greatly to the strength.

(72.) It is shown in (62) that with a single-riveted joint the strain on the solid part of the plate, when the joint is breaking through the rivet-holes, is 10 tons only, or half the normal strength of the iron, so that half the strength is lost. In order to avoid this loss, it has been proposed to roll the plates with extra thickness at the edges, as in Fig. 21: for example, if the thickness of the body of the plate at A is half that at the edge B, then when the metal between rivet-holes is breaking, the strain at A would become 20 tons per square inch, and the full strength of the iron would be utilised. Here, however, a difficulty seems to arise: the extra thickness at the edges could be

secured in one direction only, it being impracticable to roll a plate with thick edges *all round*. By a remarkable coincidence, however, the longitudinal strain is half only of the circumferential (71); hence only half the thickness of plate would be required in that direction. Thus, taking the example of the 48-inch boiler, with $\frac{3}{8}$ plates in (71), we found the bursting pressure circumferentially = 350 lbs. per square inch. Now, reducing the thickness of the body of the plate to $\frac{3}{16}$ inch, then the area of $48\frac{3}{8}$ = 1838, and of 48 inch = 1809; hence the area of the annulus = 1838 − 1809 = 29 square inches, giving $22400 \times 29 = 649600$ lbs. total pressure or $649600 \div 1809 =$ 359 lbs. per square inch longitudinally, being practically the same as the other. We thus obtain equality of strength in both directions, and a very considerable economy of material: it would, however, be inexpedient in most cases to carry this out literally for practical reasons; a $\frac{3}{16}$ plate would leave little margin for rust, &c.; moreover, the boiler would probably become deformed by its own weight and that of the water. Perhaps $\frac{1}{4}$ inch in the body of the plate and $\frac{3}{8}$ inch at the margin is the limit safely permissible in such a case.

"*Gusset-stays.*"—The longitudinal pressure in a boiler creates a heavy strain on the ends, and where those ends are flat, as they usually are in ordinary Cornish boilers, they require to be strengthened by gusset or other stays. This is quite a practical question, and may in most cases be left to the judgment of the boiler maker.

STEEL BOILERS.

(73.) "*Steam-boiler Joints for Steel Plates.*"—To obtain general rules for steel boilers it will be well to take a moderately high pressure, say 100 lbs. per square inch, which will serve for all lower pressures, and sufficiently well for higher ones, say up to 150 lbs.

Taking $\frac{3}{8}$-inch plate for 48-inch boiler, the rivets = $\frac{11}{16}$ inch diameter by col. 3 of Table 14, the space between rivet-holes for 100-lb. steam = say $1\frac{1}{16}$ by Table 13; hence the pitch $= \frac{11}{16} + 1\frac{1}{16} = 1\frac{3}{4}$ inch: the ratio of the metal between holes to the solid part of the plate $= 1\frac{1}{16} \div 1\frac{3}{4}$, or $17 \div 28 = \cdot 607$.

TABLE 16.—Of the PROPORTIONS of DOUBLE-RIVETED JOINTS in ANNEALED STEEL PLATES for STEAM-BOILERS with about 100-lb. Steam.

Thickness.	Diameter of Rivets.	Space between Rivet-holes.	Pitch.	Working Pressure of Steam.	Ratio of Punched to Solid Plate.	Lbs. per Square In. on Solid Part of Plate.	Distance between Rows of Rivets.	Lap.
$\frac{3}{16}$	$\frac{7}{16}$	$\frac{5}{8}$	$1\frac{1}{16}$	122	·588	54,000	$\frac{11}{16}$	$1\frac{3}{4}$
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{13}{16}$	$1\frac{5}{16}$	101	·619	56,850	$\frac{3}{4}$	2
$\frac{5}{16}$	$\frac{9}{16}$	$\frac{15}{16}$	$1\frac{1}{2}$	111	·625	57,400	$\frac{13}{16}$	$2\frac{1}{4}$
$\frac{3}{8}$	$\frac{11}{16}$	$1\frac{1}{16}$	$1\frac{3}{4}$	117	·607	55,750	$\frac{15}{16}$	$2\frac{5}{8}$
$\frac{7}{16}$	$\frac{3}{4}$	$1\frac{1}{4}$	2	97	·625	57,400	$1\frac{1}{16}$	3
$\frac{1}{2}$	$\frac{13}{16}$	$1\frac{3}{8}$	$2\frac{3}{16}$	98	·629	57,770	$1\frac{3}{16}$	$3\frac{1}{4}$
$\frac{9}{16}$	$\frac{7}{8}$	$1\frac{1}{2}$	$2\frac{3}{8}$	100	·632	58,040	$1\frac{1}{4}$	$3\frac{1}{2}$
$\frac{5}{8}$	$\frac{15}{16}$	$1\frac{5}{8}$	$2\frac{9}{16}$	100	·634	58,230	$1\frac{3}{8}$	$3\frac{3}{4}$
$\frac{11}{16}$	$1\frac{1}{16}$	$1\frac{3}{4}$	$2\frac{13}{16}$	98	·622	57,120	$1\frac{1}{2}$	$4\frac{1}{8}$
$\frac{3}{4}$	$1\frac{1}{8}$	$1\frac{7}{8}$	3	96	·625	57,400	$1\frac{5}{8}$	$4\frac{1}{2}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

The apparent strength of the metal between rivet-holes in steel joints = 91,840 lbs. per square inch by (41); hence we have 91840 × ·607 = 55747 lbs. per square inch on the solid part of the plate, and as we have $\frac{3}{4}$ square inch of metal per inch run (or $\frac{3}{8}$ on each side) we obtain 55747 × $\frac{3}{4}$ = 41800 lbs. on the whole area, or 41800 ÷ 48 = 871 lbs. per square inch bursting pressure. With 6 for the factor of safety (78) we have 871 ÷ 6 = 145 lbs. per square inch working pressure. Calculating in this way we have obtained Table 16: the *mean* strain in col. 7 = 57,000 lbs. per square inch. Hence for double-riveted steel boilers with *annealed* plates we have the general rules:—

(74.) $$P = 114000 \times t \div d.$$

(75.) $$p = 19000 \times t \div d.$$

In which t = the thickness of plate in inches; d = inside diameter in inches; P = the bursting, and p = the safe working pressure in lbs. per square inch. Thus in our case P = 114000 × $\frac{3}{8}$ ÷ 48 = 890 lbs.; and p = 19000 × $\frac{3}{8}$ ÷ 48

= 148 lbs. per square inch. The general Table 17 has been calculated by these rules, and will apply for all ordinary pressures, not exceeding say 100 to 150 lbs. steam: for higher pressures the case should be specially calculated.

(76.) "*Steel Boilers for extreme Pressures.*"—As an illustration of an extreme case, say that we require a steam-boiler 27 inches diameter for a working pressure of 450 lbs. per square inch: we will assume $\frac{1}{2}$-inch plates and $\frac{13}{16}$ rivets as per col. 2 of Table 16. Then by Table 13 the space between rivet-holes $= \frac{15}{16}$; hence the pitch $= \frac{13}{16} + \frac{15}{16} = 1\frac{3}{4}$ inch: the ratio of the metal between rivet-holes to the solid part of the plate $= \frac{15}{16} \div 1\frac{3}{4}$ or $15 \div 28 = \cdot 536$; and the apparent strength of metal between holes in a steel joint being 91,840 lbs. per square inch (41), we have $91840 \times \cdot 536 = 49220$ lbs. per square inch on the solid part of the plate. We have 1 square inch of metal per inch run of solid plate (or $\frac{1}{2}$ inch at each side), hence we obtain $49220 \times 1 = 49220$ lbs. on the whole of the diameter, or $49220 \div 27 = 1823$ lbs. per square inch bursting pressure. Taking 4 for the value of the factor of safety (78), we have $1823 \div 4 = 456$ lbs. per square inch safe or working pressure, or nearly 450 lbs., as required.

Now, if we had attempted to solve this question by Table 16, which is strictly adapted for 100-lb. steam only, col. 7 gives for $\frac{1}{2}$-inch plate $57770 \div (27 \times 4) = 535$ lbs. per square inch working pressure, instead of 456 lbs. Both results, however, are equally correct so far as the strain on the metal is concerned; but then the joint whose pitch, &c., was adapted for 100-lb. steam would most likely leak sooner or later with 450 lbs.

(77.) "*Limitations.*"—In applying these rules and tables for *very* low pressures, it will be found that the thicknesses come out much too light to satisfy practical considerations, although undoubtedly sufficient to resist the internal pressure. For example, with a boiler $6\frac{1}{2}$ feet or 78 inches diameter and a pressure of 6 lbs. per square inch, the rule (64) gives a thickness of $6 \times 78 \div 7466 = \cdot 0626$, or $\frac{1}{16}$ inch only, which obviously is excessively too light; in fact, if it were possible to construct the boiler with that thickness it would not be able to sustain its

TABLE 17.—Of the STRENGTH of STEEL BOILERS with DOUBLE-RIVETED JOINTS.

Thickness.	Inside Diameter of Boiler, in Feet.													
	1½	2	2½	3	3½	4	4½	5	5½	6	6½	7	7½	8
	Bursting Pressure in Pounds per Square Inch.													
¼	1584	1187	950	792	679	594	528	475	..	..	..	..	..	..
5/16	1980	1485	1188	990	848	742	660	594	540	496	..	..	..	..
⅜	2376	1782	1426	1188	1018	891	792	713	648	594	548	509	..	..
7/16	2772	2079	1663	1386	1188	1039	924	831	756	693	640	594	554	519
½	3168	2376	1901	1584	1358	1188	1055	950	864	792	731	679	634	594
9/16	3564	2673	2138	1782	1526	1336	1185	1069	972	891	822	763	713	668
⅝	3960	2970	2376	1980	1698	1485	1320	1188	1080	990	914	849	792	743
11/16	4356	3267	2614	2178	1866	1633	1450	1307	1188	1089	1005	933	871	817
¾	4752	3564	2851	2375	2036	1782	1580	1425	1296	1188	1096	1018	950	890
	Working Pressure: Factor 6.													
¼	264	198	158	132	113	99	88	76	..	..	..	..	..	..
5/16	331	247	198	165	141	124	110	99	90	83	..	..	..	..
⅜	396	297	238	198	170	149	132	119	108	99	91	84	..	..
7/16	463	346	277	231	198	173	154	139	126	115	107	99	92	86
½	528	396	317	264	226	198	176	158	144	132	122	113	106	99
9/16	594	446	356	297	254	223	197	178	162	149	137	127	119	111
⅝	660	495	396	330	283	248	220	198	180	165	152	141	132	124
11/16	726	544	436	363	311	272	242	218	198	181	168	156	145	136
¾	792	594	475	396	339	297	263	238	216	198	183	170	158	148

own weight and that of the water contained by it. The rules have, therefore, certain limitations: in the first place, we should not usually make use of plates less than $\frac{3}{16}$ inch thick for steam-boiler work, whatever the pressure or diameter; and, secondly, with thicknesses of

$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$ inches

the diameters should not in ordinary cases exceed

4	5	6	7	8 feet,

the corresponding working pressures being

29	31	32	33	34 lbs.

per square inch, as per Table 15, which is carried out in accordance with these limitations. Thus for our $6\frac{1}{2}$-foot boiler the thickness would be between $\frac{5}{16}$ and $\frac{3}{8}$ inch; $\frac{5}{16}$ would suffice for such a case, and this, it should be observed, is five times the theoretical thickness necessary for the pressure.

(78.) "*Factor of Safety for Boilers.*"—It is shown in (886) that with ordinary structures of wrought iron and steel the factor of safety for dead loads may be 3, and there appears to be no good reason why that factor should not suffice for new boilers constructed on sound principles. But boilers are subject to great deterioration from corrosion, &c., and for that reason, perhaps, the factor used by Mr. Fairbairn and most practical men is 6, and this value is admitted in Table 15, &c., and should be followed for ordinary cases and moderate pressures of steam. But with Factor 6 the thicknesses for very high pressures come out excessive and almost impracticable, and engineers have been compelled to use a lower factor, and they do so apparently with safety. Thus the L. & N. W. Railway Co. at their Crewe works use best Yorkshire plates $\frac{13}{32}$ inch thick for 4-foot locomotive boilers, with single-riveted joints, $\frac{3}{4}$ rivets, $1\frac{3}{4}$ pitch, therefore 1 inch between rivets. By Mr. Fairbairn's experiments in col. 3 of Table 5, Yorkshire plates in single-riveted joints break with 42,847 lbs. per square inch of *metal between rivet-holes;* hence we have $42847 \times \frac{13}{32} \times 1 \div 1\frac{3}{4} = 9946$ lbs. per inch *run* of joint, or 19,892 lbs. on the two sides. With a boiler 48 inches diameter we have $19892 \div 48 = 414$ lbs. per

square inch bursting pressure of steam, and the ordinary working pressure being 120 lbs., the factor is $414 \div 120 = 3 \cdot 45$: occasionally the pressure is 150 lbs., or even more, and the factor becomes $414 \div 150 = 2 \cdot 76$.

From all this we may admit that in ordinary cases the factor should be 6, but for exceptional cases it may be 4, as in (68), &c., or even 3 with comparatively new and sound boilers: but this is a matter which must be left to the judgment of the engineer.

STRENGTH OF THICK PIPES.

(79.) The strength of a pipe in resisting internal pressure is not simply proportional to the thickness of the metal; the material stretches under a tensile strain, the result being that the metal inside is more strained than that outside, and that thick pipes are weaker in proportion to their thickness than thin ones.

To illustrate this, let Fig. 22 be a 10-inch pipe, 5 inches thick, therefore 20 inches outside, and let an internal pressure be exerted until the inside diameter becomes $10\frac{1}{8}$ inches: now if the metal at A were strained in the same proportion as at B, it would be extended or stretched in the same proportion, and the outside diameter would become $20\frac{1}{4}$ inches. But obviously the cross-sectional area must be the same in all cases, 20 inches being $= 314 \cdot 16$, and 10 inches $= 78 \cdot 54$, the area of the annulus must be $314 \cdot 16 - 78 \cdot 54 = 235 \cdot 62$ square inches, therefore the outside diameter in Fig. 23 will be found by adding the area of $10\frac{1}{8}$, or $80 \cdot 516$ to $235 \cdot 62$, and we thus obtain $80 \cdot 516 + 235 \cdot 62 = 316 \cdot 136$, the diameter due to which $= 20\frac{1}{16}$ inches instead of $20\frac{1}{4}$ inches, and if we admit that the strains are proportional to the extensions, the metal at A is strained to $\frac{1}{4}$ only of that at B: for instance, if the strain at $A = 4$ tons per square inch, that at B will be 1 ton only, and between A and B we have an infinite series of strains progressively diminishing from 4 to 1 ton per square inch.

(80.) It will now be seen that the strain is inversely proportional to the *square* of the distance from the centre: in our case the strain at B being 4, that at A will be $4 \times 5^2 \div 10^2$

= 1 ton, &c. Let Fig. 24 be the section of a 10-inch pipe with various thicknesses up to 10 inches: we will assume that the strain at C, where it is a maximum, is 7 tons per square inch, this being nearly the breaking weight for ordinary cast iron (4), the extension due to which by rule (605) is:—

$$E = (\cdot00015 \times 7) + (\cdot0000122 \times 7^2) = \cdot0016487.$$

This being at 5 inches from the centre, that at D, or 6 inches, will be $\cdot0016487 \times 5^2 \div 6^2 = \cdot001144$, the strain due to which by the rule (606) becomes:—

$$W = \left\{\frac{\cdot001144}{\cdot0000122} + 37\cdot8\right\}\sqrt{\ } - 6\cdot15 = 5\cdot32 \text{ tons}$$

per square inch, as per col. 3 of Table 18; hence the *mean* strain on the ring A is $(7 + 5\cdot32) \div 2 = 6\cdot16$ tons as in col. 4, and as we have 2 square inches of metal per inch *run* (namely 1 inch at each side) we obtain $6\cdot16 \times 2 = 12\cdot32$ ton bursting pressure on the whole diameter, or $12\cdot32 \div 10 = 1\cdot232$ ton per square inch as in col. 5. Calculating in this way we obtain the strains and pressures in cols. 4, 5 of Table 18: thus for 10-inch pipes, 5 inches thick, the *mean* strain throughout the section becomes

$$\left(6\cdot16 + 4\cdot75 + 3\cdot775 + 3\cdot07 + 2\cdot54\right) \div 5 = 4\cdot059 \text{ tons}$$

per square inch of metal as in col. 4. Then as we have 10 square inches of metal per inch *run* (or 5 inches at each side) we have $4\cdot059 \times 10 = 40\cdot59$ tons on the whole diameter, or $40\cdot59 \div 10 = 4\cdot059$ tons internal pressure per square inch, col. 5. If the whole cross-sectional area had yielded the maximum strain of 7 tons per square inch, we should have had $7 \times 10 = 70$ tons on the whole diameter, or $70 \div 10 = 7$ tons pressure per square inch instead of $4\cdot059$ as per col. 5.

We should obtain nearly the same results by the following rules:—

(81.) $$p = \frac{S \times (R^2 - r^2)}{R^2 + r^2}.$$

(82.) $$S = \frac{p \times (R^2 + r^2)}{R^2 - r^2}.$$

In which p = the internal pressure per square inch in tons, lbs., &c., dependent on the value of S.

S = the maximum tensile strain, or that at the inside of the pipe, in tons, lbs., &c., per square inch.

R = the external, and r = the internal radius of the pipe in inches.

Thus for example, with a 10-inch pipe 5 inches thick, R = 10, and $r = 5$ inches: taking S = 7 tons, which is nearly the ultimate or breaking tensile strength of ordinary cast iron, we get $p = \frac{7 \times (10^2 - 5^2)}{10^2 + 5^2} = 4 \cdot 2$ tons per square inch: calculating in this way we obtain col. 6 of Table 18.

Again: say that with a cylinder 12 inches bore, 5 inches thick, and an internal pressure of 2 tons per square inch, we require the maximum strain on the metal or that at the inside of the cylinder. Then R being 11, $r = 6$, rule (82) becomes $S = \frac{2 \times (11^2 + 6^2)}{11^2 - 6^2} = 3 \cdot 7$ tons per square inch of metal.

(83.) The ordinary proportions adopted almost universally by practical engineers for hydraulic-press cylinders, is to make the thickness equal to the internal radius, and it is supposed that those proportions will allow a working pressure of 4 tons per square inch, or say 3 tons per circular inch. But Table 18 shows by cols. 5 or 6, that with ordinary cast iron these are really *bursting* pressures. It is shown in (883) that cast iron will sustain for years a strain very nearly equal to the breaking weight, but it is not safe to trust to that fact, and in most cases the working pressure should not exceed say half the ultimate pressure, or in our case 2 tons per square inch with ordinary iron. Many presses, however, may be found which *seem* to bear much heavier pressures than that as shown by the safety-valve, but as usually constructed a safety-valve is a very unreliable indicator of pressure, the breadth of the conical seat being great, and the acting or *effective* area uncertain. A better form is shown by Fig. 202: the valve V is of hardened steel formed like a hollow punch, the cutting edge imbeds itself in the hard gun-

TABLE 18.—Of the STRAINS in a 10-INCH CAST-IRON PIPE, with DIFFERENT THICKNESSES of METAL.

Thickness in Inches.	Tensile Strain on the Metal: Tons per Square Inch.			Bursting Pressure in Tons per Square Inch.		Allowing for Reduced Tensile Strength of Thick Metal.				
						Ratios of Strength.	Ordinary Iron, 2nd Melting.		Strong Iron, 4th Melting.	
	Max.	Min.	Mean.	By Analysis.	By Rule.		Max. Strain.	Bursting Pressure.	Max. Strain.	Bursting Pressure.
1	7·0	5·320	6·160	1·232	1·26	1·000	7·00	1·232	18·26	3·201
2	"	4·180	5·455	2·182	2·27	·740	5·04	1·571	13·15	4·100
3	"	3·370	4·895	2·937	3·07	·620	4·34	1·832	11·32	4·735
4	"	2·770	4·439	3·551	3·70	·547	3·83	1·941	10·00	5·070
5	"	2·310	4·059	4·059	4·20	·497	3·48	2·017	9·08	5·265
6	"	1·956	3·738	4·486	4·60	·459	3·21	2·055	8·38	5·367
7	"	1·676	3·463	4·849	4·93	·429	3·00	2·075	7·83	5·418
8	"	1·451	3·226	5·162	5·20	·405	2·83	2·084	7·40	5·450
9	"	1·268	3·019	5·434	5·40	·385	2·69	2·089	7·03	5·458
10	"	1·117	2·837	5·703	5·60	·367	2·57	2·089	6·73	5·484
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)

metal seat and forms it own bed, giving a precise area, and thereby a certain pressure. Thus for $\frac{3}{8}$ inch diameter, the area $= \cdot 11$ inch, requiring for say 1 ton per square inch $2240 \times \cdot 11 = 246$ lbs. strain, and with a leverage of say 20 to 1, we have $246 \div 20 = 12 \cdot 3$ lbs. weight on the lever per ton pressure. The knife-edges at A and B, also the key K must be of hardened steel, and in order to adjust the level of the lever and compensate for wear (which is a practical necessity) the upper edge of the key should be wedge-shaped, and at an angle adapted to its seat in the *slotted* recess prepared for it.

(84.) The actual load on the ram of a hydraulic press is not often known with accuracy, but in the presses used for raising the Conway and Britannia bridges we have more precise information. For the Conway tube, a ram $18\frac{3}{8}$ inches diameter, or 265 square inches area, was used at each end; the gross weight of the tube, &c., was about 1300 tons, or 650 tons at each end: hence we have $650 \div 265 = 2 \cdot 45$ tons per square inch. The cylinder was 20 inches diameter internally, and 10 inches thick: hence $R = 20$, $r = 10$, and the rule (82) gives $S = \dfrac{2 \cdot 45 \times (20^2 + 10^2)}{20^2 - 10^2} = 4 \cdot 08$ tons tensile strain per square inch of metal; this being the maximum strain, or that at the inside of the cylinder (80).

(85.) With the Britannia tube, the two Conway presses were used at one end and a large one with 20-inch ram at the other. The gross weight of the tube, &c., was about 1640 tons, or 820 tons at each end: then the 20-inch ram being 314 square inches area, we have $820 \div 314 = 2 \cdot 61$ tons pressure per square inch. The cylinder was 22 inches internal diameter and 11 inches thick, hence $R = 22$, and $r = 11$, and rule (82) gives $S = \dfrac{2 \cdot 61 \times (22^2 + 11^2)}{22^2 - 11^2} = 4 \cdot 35$ tons maximum tensile strain per square inch of metal.

It is probable that this pressure, $2 \cdot 61$, and strain $4 \cdot 35$ tons per square inch, were very nearly the breaking weights, indeed one cylinder failed by the bottom blowing off, which would have led to most disastrous results, but for the wise precaution taken of blocking up the tube inch by inch as it was raised by the

press. It would appear from this, that the tensile strength of cast iron in great masses 10 and 11 inches thick is much below the normal strength, or that for small thicknesses, namely, 7 tons per square inch.

(86.) "*Tensile Strength of Thick Cast Iron.*"—Experiments have shown (933) that the specific *transverse* strength of cast iron is not the same for castings of all sizes, but that large castings, or rather castings with great thicknesses, are specifically weaker than small ones, so that bars 1 inch, 2 inch, and 3 inch square have specific transverse strengths in the ratio 1·0, 0·7184, 0·6195, and may be found approximately by the rule (934) or $R = 1 \div \sqrt[2\cdot 3]{t}$. How far the *tensile* strength of cast iron is affected by the thickness of the casting is not known experimentally, but admitting the same law as for transverse strength we obtain the ratios given by col. 7 of Table 18, from which we obtain col. 8, also col. 9 from col. 5: thus for 5 inches thick, we get $4\cdot 059 \times \cdot 497 = 2\cdot 017$ tons bursting pressure per square inch, &c.

(87.) By repeated re-melting the tensile strength of cast iron may be greatly increased as shown in (5), where metal of the fourth melting (pig iron being the first) gave as much as 18·26 tons per square inch in small thicknesses; applying to this iron the ratio in col. 7 we obtain the tensile strengths in col. 10, and finally from col. 5 the bursting pressures in col. 11. Thus for 5 inches thick we have $4\cdot 059 \times 18\cdot 26 \div 7 \times \cdot 497 = 5\cdot 265$ tons, &c. But we have seen (7) that there is great uncertainty in this method of increasing the strength of cast iron; the safest course where heavy pressures are required, is to test the iron selected by direct experiments on its tensile strength.

(88.) Table 18 seems to show by cols. 9 and 11, that there is no sensible advantage from great thicknesses of metal, the pressure remaining practically constant with all thicknesses from 5 to 10 inches, &c. Of course this rests on the assumption that the tensile strength follows the ratio in col. 7, but as that is derived from limited experiments on *transverse* strength, where the thickness did not exceed 3 inches, the results are not absolutely reliable, but are the best we can give in the present state of our knowledge.

The great uncertainty as to the important data connected with this subject, should lead to the adoption of large diameters permitting low values for the strain S and pressure p. For example, in the case of the Britannia press, say that the ram shall be 30 inches diameter = 707 square inches area, giving $820 \div 707 = 1 \cdot 16$ ton pressure per square inch. The cylinder might be 32 inches internal diameter, and say 5 inches thick, therefore $R = 21$ and $r = 16$; for strong iron of the fourth melting 5 inches thick $S = 9 \cdot 08$ tons by col. 10 of Table 18: then rule (81) becomes $p = \dfrac{9 \cdot 08 \times (21^2 - 16^2)}{21^2 + 16^2} = 2 \cdot 4$ tons bursting pressure per square inch, or double the working pressure, $1 \cdot 16$ ton; thus leaving a fair margin for contingencies.

(89.) Another advantage of these proportions would be that the weight of the cylinder is reduced nearly to half, despite the increase in diameter; thus with the original sizes, 22 inches diameter = 380 area, and 44 inches = 1520 area, hence $1520 - 380 = 1140$ square inches, the area of the annulus. With the enlarged cylinder, 32 inches diameter = 804 area, and $42 = 1385$ area, hence the annulus $= 1385 - 804 = 581$ square inches, or about half.

(90.) "*Cylinders Hooped with Wrought Iron.*"—When large diameters are inadmissible and heavy pressures a necessity, the best course is to abandon dependence on the strength of cast iron altogether, and to rely on wrought-iron hoops shrunk *hot*, on a comparatively thin cast-iron shell, as in Fig. 25. In that case, the cast-iron cylinder may be regarded as a *padding* adding nothing to the strength of the combination, because when the hoops are shrunk on they exert a powerful *compressive* strain on the cylinder, which will be partially or perhaps wholly relieved when the internal pressure comes on: if wholly relieved the cylinder will be simply restored to its primitive state, being unstrained either way. It is rather difficult to calculate the strength under these conditions; if we take it as a wrought-iron cylinder 26 inches diameter, 5 inches thick, we have $R = 18$, $r = 13$, and the ultimate strength of wrought bar-iron or S being about 25 tons per square inch by Table 1,

the rule (81) becomes $p = \frac{25 \times (18^2 - 13^2)}{18^2 + 13^2} = 7 \cdot 86$ tons per square inch bursting pressure. But we have here taken the pressure as acting on 26 inches diameter, whereas it really acts on 22 inches only: hence $7 \cdot 86 \times 26 \div 22 = 9 \cdot 3$ tons per square inch. As applied to the Britannia tube, where the actual pressure due to the weight of the tube was $2 \cdot 61$ tons as shown by (85), we have $9 \cdot 3 \div 2 \cdot 61 = 3 \cdot 5$ as the "factor of safety."

It is shown in (625) that whatever the initial strain on wrought iron may be, the permanent working load with a fixed length cannot exceed 8 or 9 tons per square inch: in our case it is $25 \times 2 \cdot 61 \div 9 \cdot 3 = 7$ tons only.

(91.) Care must be taken that the *longitudinal* pressure does not blow the bottom out: the area of $26 = 531$, and of $22 = 380$, hence of the annulus $531 - 380 = 151$ square inches, giving with 7 tons per square inch of metal, $151 \times 7 = 1057$ tons, or $1057 \div 380 = 2 \cdot 78$ tons per square inch bursting pressure, or about $\frac{3}{10}$ only of $9 \cdot 3$ tons the bursting pressure circumferentially. This difficulty is easily overcome by the construction shown by Fig. 25; the bottom of the cylinder, supported all over by the sole plate of the press, B, is entirely relieved of the bursting pressure.

(92.) "*Wrought-iron Press-pipe.*"—Wrought-iron drawn tubes are commonly used for hydraulic-press work, &c., where the pressures are very heavy; the ordinary sizes are 1 inch diameter outside, $\frac{1}{2}$-inch bore, therefore $\frac{1}{4}$ inch thick. A series of experiments was made on pipes of this kind with a pressure of 3 tons per circular inch, or $3 \div \cdot 7854 = 3 \cdot 82$ tons per square inch; the pressure was obtained by a steel plunger 1 inch diameter, loaded with 3 tons of *direct* weights; there was therefore no uncertainty as to the real pressure. The result was that nearly all the pipes tested in this way bore the strain satisfactorily, the faulty ones alone failing. The maximum strain on the metal by the rule (82), with $R = \frac{1}{2}$ inch, and $r = \frac{1}{4}$ inch, becomes $S = \frac{3 \cdot 82 \times (\frac{1}{2}^2 + \frac{1}{4}^2)}{\frac{1}{2}^2 - \frac{1}{4}^2} = 6 \cdot 37$ tons per square inch, which is about $\frac{1}{4}$ only of $25 \cdot 7$ tons, the mean tensile strength of wrought iron by Table 1, &c.

Other experiments were made on lighter pipes of the same kind 1 inch external and $\frac{5}{8}$ inch internal diameter; most of these failed at once under the pressure of 3·82 tons per square inch. In this case, $R = \frac{1}{2}$ inch, $r = \frac{5}{16}$ inch, and rule (82) gives $S = \frac{3\cdot 82 \times (\frac{1}{2}^2 + \frac{5}{16}^2)}{\frac{1}{2}^2 - \frac{5}{16}^2} = 8\cdot 7$ tons per square inch of metal, if the pressure had been borne, which it was *not*. We may take the ultimate or breaking tensile strain of wrought iron in drawn tubes at 7 tons per square inch, which is $7 \div 25\cdot 7 = \cdot 27$ or 27 per cent. only of the strength of ordinary bar iron.

(93.) "*Wrought-iron Gas-pipe.*"—Ordinary drawn wrought-iron gas tubing is also extensively used for steam-pipes, water-mains, &c., where the pressure is considerable and a knowledge of its strength becomes important. Taking 2-inch pipe as an example, the thickness would be about $\frac{3}{16}$ inch: hence $R = 1\frac{3}{16}$, $r = 1$ inch, and taking $S = 7$ tons or 15,680 lbs. per square inch of metal, rule (81) gives $p = \frac{15680 \times (1\frac{3}{16}^2 - 1^2)}{1\frac{3}{16}^2 + 1^2}$ $= 2667$ lbs. per square inch bursting pressure for constant steady load equivalent by (909) to $2667 \times \frac{2}{3} = 1780$ lbs. for intermittent load.

The proper value of the "factor of safety" will depend on circumstances, and must be fixed with judgment; for a water-pipe where the pressure is not only intermittent but where the sudden closing of a cock may create heavy shocks, whose effect cannot be calculated, but must be provided for by the use of a high factor, we may take it at 15, giving in our case $2667 \div 15 = 178$ lbs. per square inch, or $178 \times 2\cdot 3 = 410$ feet of water, safe working pressure.

(94.) "*Lead Pipe.*"—Mr. Jardine made two experiments on ordinary drawn lead pipes, the pressure being obtained by a force pump. One pipe $1\frac{1}{2}$ inch internal diameter, $\frac{1}{5}$ inch thick, bore without apparent alteration a head of 1000 feet of water; with 1200 feet it began to swell, and with 1400 feet, or $1400 \div 2\cdot 3 = 606$ lbs. pressure per square inch, it burst. We *may find from this* the maximum breaking strain on the metal

by the rule (82), which with $R = \cdot 95$, $r = \cdot 75$, becomes $S = \frac{606 \times (\cdot 95^2 + \cdot 75^2)}{\cdot 95^2 - \cdot 75^2} = 2611$ lbs. per square inch.

In the other experiment, the pipe 2 inches internal diameter, $\frac{1}{5}$ inch thick, bore without alteration 800 feet of water, and burst with 1000 feet, or $1000 \div 2 \cdot 3 = 435$ lbs. pressure per square inch; then R being $= 1 \cdot 2$ and $r = 1$, we obtain $S = \frac{435 \times (1 \cdot 2^2 + 1^2)}{1 \cdot 2^2 - 1^2} = 2412$ lbs. per square inch of metal.

The mean of the two, or 2510 lbs., may be taken as a basis for the strength of lead pipes of all sizes. Table 19 gives the thickness, weights, and safe pressures for standard sizes of lead pipes; they are commonly rated by the weight per 15-foot length up to 1 inch diameter, and per 12-foot length for larger sizes.

The weight per foot being given, the thickness may be calculated by finding the external diameter by the rule:—

$$(95.) \qquad (D^2 - d^2) = W \div 3 \cdot 86.$$

In which D = the external, and d = the internal diameter in inches, W = the weight per foot run in lbs. Thus, say we take $\frac{3}{4}$-inch pipe, 32 lbs. per 15 feet, therefore $2 \cdot 133$ lbs. per foot: then $2 \cdot 133 \div 3 \cdot 86 = \cdot 5503$, which is $D^2 - d^2$; then $\cdot 5503 + \frac{3}{4}^2$ or $\cdot 5503 + \cdot 5625 = 1 \cdot 1128$, the square root of which or $1 \cdot 055 = d$; hence the thickness $= (1 \cdot 055 - \cdot 75) \div 2 = \cdot 1525$ inch, as per col. 2 of Table 19.

Having thus found the thickness, and thereby R, the bursting pressure will be given by the rule:—

$$(96.) \qquad p_w = \frac{5800 \times (R^2 - r^2)}{R^2 + r^2}.$$

In which p_w = the bursting pressure in feet of water, and the rest as in (82): thus with the $\frac{3}{4}$-inch pipe, $\cdot 1525$ inch thick as in (95) $r = \cdot 375$, $R = \cdot 375 + \cdot 1525 = \cdot 5275$, and the rule gives

$$p_w = \frac{5800 \times (\cdot 5275^2 - \cdot 375^2)}{\cdot 5275^2 + \cdot 375^2} \text{ or, } \frac{5800 \times (\cdot 2782 - \cdot 1406)}{\cdot 2782 + \cdot 1406},$$

$$\text{or } \frac{5800 \times \cdot 1376}{\cdot 4188} = 1906 \text{ feet}$$

of water, as in col. 5 of Table 19, &c.

TABLE 19.—Of the STRENGTH of DRAWN LEAD PIPES of the ordinary Standard Weights.

Diameter.	Thickness.	Weight in Lbs.		Pressure in Feet of Water.		
		Per 15 Feet.	Per Foot.	Bursting.	Working Head.	
					Ordinary.	With Shock.
½	·0881	12	0·800	1700	170	85
"	·1067	15	1·000	1978	198	100
"	·1356	20	1·330	2367	236	118
"	·1486	22	1·467	2525	252	126
⅝	·1060	18	1·200	1649	165	82
"	·1264	22	1·467	1898	190	95
"	·1503	27	1·800	2169	217	108
¾	·1105	22	1·467	1466	147	73
"	·1236	25	1·667	1610	161	80
"	·1365	28	1·867	1745	174	87
"	·1525	32	2·133	1906	191	96
"	·1695	36	2·400	2068	207	104
"	·1810	39	2·600	2173	217	109
1	·1370	36	2·400	1378	138	69
"	·1570	42	2·800	1545	155	78
"	·2010	56	3·733	1888	189	95
		Per 12 Feet.				
1¼	·1610	42	3·500	1307	131	66
"	·1945	52	4·333	1535	154	77
"	·2300	63	5·250	1760	176	88
1½	·1625	50	4·167	1123	112	56
"	·1800	56	4·667	1228	123	62
"	·2250	72	6·000	1488	149	75
"	·2580	84	7·000	1672	167	84
1¾	·1940	70	5·833	1146	115	58
"	·2220	81	6·750	1290	129	65
"	·2435	90	7·500	1396	140	70
2	·2055	84	7·000	1067	107	54
"	·2320	96	8·000	1193	119	60
"	·2670	112	9·333	1347	135	68
(1)	(2)	(3)	(4)	(5)	(6)	(7)

To find the safe working pressure we have to determine the value of the factor of safety (880), which requires care and judgment: Table 137 shows that for lead with a perfectly dead pressure, the safe load may be ⅓ of the bursting, or the factor = 3, but this is seldom the case in practice; by the sudden closing

of a cock, &c., heavy shocks are common, producing the well-known knocking sound, moreover the pressure is frequently intermittent as in all the lead service-pipes of a town supplied on the ordinary system. When a ball-cock is used, which shuts off the water gradually, the strain may be taken as an intermittent dead load, for which by col. 2 of Table 141, the ratio is $\frac{1}{3}$ of the equivalent constant dead load. Taking the factor of safety at 3, we obtain $\frac{1}{3} \div 3 = \frac{1}{9}$ of the dead pressure, as the intermittent working pressure. But where the pipe is of considerable length, and is subjected to heavy shocks from the sudden closing of a cock, the strain becomes an intermittent dynamic one, for which col. 3 gives the ratio $= \frac{1}{6}$, or with factor 3, $\frac{1}{6} \div 3 = \frac{1}{18}$ of the ultimate dead load. Taking these values ($\frac{1}{9}$ and $\frac{1}{18}$) for the sake of round numbers, at $\frac{1}{10}$ and $\frac{1}{20}$ respectively and applying them to col. 5 of Table 19, we obtain cols. 6 and 7, &c.

ORDINARY WATER AND GAS PIPES: CAST IRON.

(97.) With very low pressures, such as for gas and low-service water-pipes, the rules we have given will not apply without correction. The first question is to determine the minimum thickness with which it is practicable to cast them; here we have nothing but experience to guide us, and from that we obtain the empirical rule:—

(98.) $$t = \frac{\sqrt{d}}{10} + \cdot 15.$$

In which d = diameter of the pipe in inches, and t = the thickness when the pressure is practically nothing; thus for a 9-inch pipe $t = \frac{\sqrt{9}}{10} + \cdot 15 = \cdot 45$ inch; col. 3 of Table 20 has been calculated by this rule, and will apply for gas and low pressures in water say up to 50 feet head.

For higher pressures such as occur in ordinary water-mains the rules become:—

(99.) $$t = \left(\frac{\sqrt{d}}{10} + \cdot 15\right) + \left(\frac{H_w \times d}{25000}\right).$$

(100.) $$H_w = \left\{t - \left(\frac{\sqrt{d}}{10} + \cdot 15\right)\right\} \times 25000 \div d.$$

TABLE 20.—Of the THICKNESS and WEIGHT of CAST-IRON SOCKET-PIPE to BEAR SAFELY DIFFERENT PRESSURES of WATER.

Diameter in. Inches.	Length exclusive of Socket.		For Gas, &c.				100 Feet.				250 Feet.				500 Feet.				750 Feet.				1000 Feet.			
	ft.	in.	thick.	cwt.	qrs.	lbs.	thick.	cwt.	qrs.	lbs.	thick.	cwt.	qrs.	lbs.	thick.	cwt.	qrs.	lbs.	thick.	cwt.	qrs.	lbs.	thick.	cwt.	qrs.	lbs.
1½	6	0	·27	0	1	3	·28	0	1	4	·29	0	1	5	·30	0	1	7	·31	0	1	8	·33	0	1	10
2	6	0	·29	0	1	17	·30	0	1	19	·31	0	1	20	·33	0	1	23	·35	0	1	26	·37	0	2	2
2½	6	0	·30	0	2	1	·31	0	2	3	·33	0	2	7	·35	0	2	11	·37	0	2	14	·40	0	2	20
3	9	0	·32	0	3	18	·33	0	3	21	·35	1	0	3	·38	1	0	9	·41	1	0	19	·44	1	1	0
4	9	0	·35	1	1	7	·37	1	1	15	·39	1	1	24	·43	1	2	13	·47	1	3	1	·51	1	3	18
5	9	0	·37	1	2	23	·39	1	3	5	·42	1	3	21	·47	2	0	19	·52	2	1	16	·57	2	2	14
6	9	0	·39	2	0	16	·42	2	1	6	·45	2	1	25	·51	2	3	6	·57	3	0	15	·63	3	1	24
7	9	0	·41	2	2	13	·44	2	3	8	·48	3	0	9	·55	3	2	4	·62	4	0	0	·69	4	1	21
8	9	0	·43	3	0	14	·46	3	1	10	·51	3	2	23	·59	4	1	4	·67	4	3	13	·75	5	1	22
9	9	0	·45	3	2	18	·48	3	3	17	·53	4	1	7	·63	5	0	14	·72	5	3	12	·81	6	2	10
10	9	0	·47	4	0	26	·51	4	2	10	·57	5	0	15	·67	6	0	4	·77	6	3	21	·87	7	3	9
12	9	0	·49	5	1	2	·54	5	3	6	·61	6	2	6	·73	7	3	11	·85	9	0	15	·97	10	2	0
15	9	0	·53	7	1	0	·59	8	0	6	·68	9	1	4	·83	11	1	9	·98	13	1	14	1·13	15	2	0
18	9	0	·57	9	1	0	·64	10	1	16	·75	12	1	0	·93	15	0	11	1·11	18	0	0	1·29	21	0	0
21	9	0	·60	11	0	11	·69	12	3	12	·81	15	0	18	1·02	19	1	7	1·23	23	1	0	1·44	27	0	10
24	9	0	·64	13	2	0	·73	15	2	2	·88	18	3	2	1·12	23	2	4	1·36	28	2	9	1·60	33	2	14
30	9	0	·69	18	0	14	·81	21	1	3	1·00	26	1	2	1·29	34	3	4	1·59	42	2	18	1·89	50	2	5
36	9	0	·75	23	2	16	·89	28	0	6	1·11	35	0	0	1·47	46	1	10	1·83	57	3	0	2·19	69	1	12

In which d = diameter of the pipe in inches, t = the thickness in inches, and H_w = safe working head of water, in feet.

Thus, the thickness for an 8-inch pipe for 250 feet of water becomes by the rule (99) $t = \left(\frac{\sqrt{8}}{10} + \cdot 15\right) + \left(\frac{250 \times 8}{25000}\right) =$ ·513 inch. Again, to find the safe head of water for a 9-inch pipe ·63 inch thick, the rule (100) gives $H_w = \left\{\cdot 63 - \left(\frac{\sqrt{9}}{10} + \cdot 15\right)\right\} \times 25000 \div 9 = 500$ feet head.

(101.) Table 20, taken from the author's 'Practical Hydraulics,' has been calculated by the rule (98), and gives the thickness and approximate weights of cast-iron socket-pipes for pressures ranging from gas-pipes up to 1000 feet of water. The usual practice of engineers is to specify the weights of pipes, rather than the thickness, leaving the founder to determine that for himself, which long practice enables him to do with considerable precision; of course absolute accuracy cannot be attained and should not be expected: a margin for unavoidable variations should be allowed, say 1 lb. to the inch either way, so that, for instance, a 7-inch pipe for 250 feet head, specified to weigh 3 cwt. 0 qr. 9 lbs. as per Table 20, should not be rejected if its real weight is between 3 cwt. 0 qr. 2 lbs. and 3 cwt. 0 qr. 16 lbs., &c., 14 lbs. being thus allowed for variation in a 7-inch pipe. Founders consider this to be a fair allowance.

CHAPTER IV.

STRENGTH OF CHAIN, ROPES, ETC.

(102.) The strength of chain is not equal to that of a straight bar of the same material; experiments at Woolwich have shown that a straight bar being 1·0, that of a chain = 1·622 instead of 2·0. Table 21 gives the result of direct experiments on the strength of chain, which shows that wrought iron is the best material. Steel seems entirely to lose its superior tensile

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TABLE 21.—Of EXPERIMENTS on the STRENGTH of CHAIN.

COMMON CLOSE-LINKED CRANE-CHAIN.

No.	Material.	Diameter.	Breaking-weight in Tons. Max.	Min.	Mean. Tons.	Mean. Per Square Inch.	No. of Experiments.	Authority.
1	Wrought-iron	$\frac{1}{4}$	1·8	1·40	1·60	16·32	2	Hawks and Crawshay.
2	"	$\frac{3}{8}$	4·4	3·00	3·78	17·12	4	"
3	"	$\frac{7}{16}$	6·8	6·15	6·48	21·55	2	"
4	"	$\frac{1}{2}$	8·4	7·50	7·91	20·15	4	"
5	"	$\frac{5}{8}$	13·0	11·20	12·10	19·72	3	"
6	"	$\frac{11}{16}$	14·9	14·00	14·45	18·46	2	"
7	"	$\frac{3}{4}$	16·5	15·25	15·87	17·96	2	"
8	"	$\frac{13}{16}$	21·4	19·5	20·60	19·86	3	"
9	"	$\frac{7}{8}$	27·5	21·0	25·14	20·90	32	"
10	"	1	38·6	26·0	31·81	20·15	61	"
11	"	$1\frac{1}{16}$	35·0	28·5	31·30	17·66	3	"
12	"	$1\frac{1}{4}$	52·0	35·0	46·19	18·82	3	"
13	"	$1\frac{1}{2}$	63·5	55·5	60·62	17·15	4	"
..	..	..	..	..	..	18·92	Mean.	"

[redacted]

STUD-LINKED CABLE-CHAIN.

14	„	$\frac{5}{8}$	..	..	9·58	15·60	6	Woolwich D.Y.
15	„	$\frac{3}{4}$	..	..	13·51	15·30	6	„
16	„	$\frac{3}{4}$	18·0	15·5	16·75	18·91	2	Hawks and Co.
17	„	$\frac{7}{8}$	27·0	20·0	22·75	18·91	6	„
18	„	$\frac{7}{8}$	..	..	20·38	16·90	20	Woolwich D.Y.
19	„ annealed ..	$\frac{7}{8}$	20·25	19·0	19·65	16·84	10	„
20	„ not annealed	$\frac{7}{8}$	21·75	20·5	21·10	17·54	10	„
21	„	1	37·5	32·5	34·20	21·77	5	„
22	„	1	..	..	24·25	15·40	6	„
23	„	1	34·5	26·0	30·70	19·54	3	Hawks and Co.
24	„	$1\frac{1}{8}$	..	..	29·54	14·90	6	Woolwich D.Y.
25	„ Lowmoor ..	$1\frac{1}{8}$	..	..	41·25	20·75	..	„
26	„ Trinity ..	$1\frac{1}{8}$	..	..	40·38	20·31	..	„
27	„ „ ..	$1\frac{1}{8}$	..	..	41·50	20·87	..	„
28	„	$1\frac{1}{2}$	..	..	59·58	16·90	6	„
29	„	$1\frac{3}{4}$	..	..	74·12	15·40	6	„
30	„	$1\frac{3}{4}$	94·0	83·0	88·50	18·40	9	„
31	„	$1\frac{3}{4}$	90·7	82·5	84·50	17·56	6	„
32	„	$1\frac{3}{4}$	83·6	72·2	80·10	16·65	12	Portsmouth D.Y.
33	„	$1\frac{3}{4}$	100·0	74·0	84·53	17·57	7	Hawks and Co.
34	„	$1\frac{7}{8}$	..	..	92·88	16·80	6	Woolwich D.Y.
35	„	2	..	..	99·54	15·80	3	„
36	„	$2\frac{1}{8}$	123·0	100·7	113·90	16·60	8	Portsmouth D.Y.
37	„	$2\frac{1}{4}$	133·0	119·5	125·20	15·74	3	„
38	Puddled steel, Firth's ..	$1\frac{1}{8}$	..	..	41·00	20·62	..	Woolwich D.Y.
39	„ Howell's ..	$1\frac{1}{8}$	..	..	39·75	20·00	..	„
40	„ Mersey ..	$1\frac{1}{8}$	..	..	29·75	14·96	..	„
41	Mild steel	$1\frac{1}{8}$	..	..	37·75	18·98	..	„
42	Cast steel, Muschet's ..	$1\frac{1}{8}$	..	..	33·0	16·60	..	„
43	„ Bessemer's ..	$1\frac{1}{8}$	..	..	35·0	17·61	..	„

strength when made into chain, which is due no doubt to the welding process. Table 1 shows that *bar* steel is superior to bar iron to the extent of $47 \cdot 84 \div 25 \cdot 7 = 1 \cdot 86$, or 86 per cent.; but when welded, iron is superior to steel, $21 \cdot 1 \div 20 \cdot 4 = 1 \cdot 034$, or $3 \cdot 4$ per cent.

When made into stud-linked chain the mean strength of steel by Nos. 38 to 43 of Table 21 is $= 18 \cdot 13$ tons per square inch, iron being $= 17 \cdot 44$ tons; practically, therefore, there is little or no difference in the two materials.

There are two kinds of chain in common use, the short-linked or crane-chain used for most purposes on land; and the stay or stud-linked cable-chain for naval purposes.

(103.) "*Short-linked Crane-chain.*"—Table 21 shows that the mean strength of crane-chain from $\frac{1}{4}$ inch to $1\frac{1}{2}$ inch diameter is 19 tons per square inch, and it appears to be about the same for all the sizes between those extremes. A chain made of 1-inch iron would therefore break with $19 \times \cdot 7854 \times 2 = 29 \cdot 84$, or say 30 tons, and for short-linked crane-chain we have the rules:—

(104.) Mean breaking weight in pounds, $w = d^2 \times 1050$.
" " tons $W = d^2 \times \cdot 47$.

(105.) Government proof-strain in pounds, $p = d^2 \times 420$.
" " tons, $P = d^2 \times \cdot 1875$.

In which d = the diameter of the iron in 8ths of an inch, &c.: thus for 1-inch chain we have $W = 64 \times \cdot 47 = 30$ tons, and $P = 64 \times \cdot 1875 = 12$ tons, &c.; cols. 2 and 3 in Table 22 have been calculated by these rules. It will be observed that the ratio of the proof strain to the breaking weight is 1 to $1050 \div 420 = 2 \cdot 5$. With lifts, cranes, &c., where life is jeopardised, the safe working load should not exceed $\frac{1}{5}$th of the breaking weight, but for many less critical cases it may be 50 per cent. more than that, or $\frac{3}{10}$ths of the breaking weight: we thus obtain cols. 4 and 5 in Table 22. The weights per fathom (or 6 feet) in col. 6, from $\frac{1}{4}$ inch to 1 inch, were found by weighing given lengths; the rest were calculated from the 1-inch chain.

(106.) "*Stud-linked or Cable-chain.*"—Table 21 shows that the strength of stud-chain is not so great as is commonly

TABLE 22.—Of the STRENGTH and WEIGHT of SHORT-LINKED CRANE-CHAIN.

Diameter	Breaking Weight.	Admiralty Proof-strain.	Maximum Safe Strain.	Working Strain for Cranes, &c.	Weight per Fathom.
inches.	tons.	tons.	tons.	tons.	lbs.
$\frac{1}{4}$	1·87	·75	·56	·37	4·5
$\frac{5}{16}$	2·93	1·17	·88	·58	6·0
$\frac{3}{8}$	4·22	1·69	1·26	·84	10·5
$\frac{7}{16}$	5·74	2·30	1·72	1·15	12·0
$\frac{1}{2}$	7·50	3·00	2·25	1·50	18·0
$\frac{9}{16}$	9·49	3·80	2·84	1·90	21·0
$\frac{5}{8}$	11·72	4·69	3·51	2·34	27·9
$\frac{11}{16}$	14·18	5·67	4·25	2·83	31·3
$\frac{3}{4}$	16·87	6·75	5·06	3·37	36
$\frac{13}{16}$	19·80	7·92	5·94	3·96	42
$\frac{7}{8}$	22·97	9·19	6·89	4·59	50
$\frac{15}{16}$	26·37	10·55	7·91	5·27	57
1	30·00	12·00	9·00	6·00	65
$1\frac{1}{16}$	33·87	13·54	10·16	6·77	73
$1\frac{1}{8}$	37·97	15·18	11·39	7·59	82
$1\frac{3}{16}$	42·30	16·92	12·69	8·46	91
$1\frac{1}{4}$	46·87	18·75	14·06	9·37	101
$1\frac{5}{16}$	51·68	20·67	15·50	10·33	110
$1\frac{3}{8}$	56·72	22·68	17·01	11·34	120
$1\frac{7}{16}$	62·00	24·80	18·60	12·40	130
$1\frac{1}{2}$	67·50	27·00	20·25	13·50	140
Ratios =	5·0	2·0	1·5	1·0	..
(1)	(2)	(3)	(4)	(5)	(6)

supposed: the mean of the twenty-four experiments, Nos. 14 to 37 on *iron* chains from $\frac{5}{8}$ inch to $2\frac{1}{4}$ inches diameter is 17·43 tons per square inch, or $1\frac{1}{2}$ ton *less* than that of ordinary crane-chain. This is contrary to the current notion on the subject, but is the clear result of experiment: cable-chain has, however, some important advantages, principally in its non-liability to *kink* or become entangled, which for naval purposes is all-important: moreover it is lighter, as shown by col. 5 of Table 23.

Admitting 17·43 tons per square inch as the mean strength of cable-chain, we have $17·43 \times ·7854 \times 2 = 27·37$ tons, the

TABLE 23.—Of the STRENGTH and WEIGHT of STUD-LINKED CABLE-CHAIN.

Diameter.	Breaking Weight.	Admiralty Proof-strain.	Maximum Safe Strain.	Weight per Fathom.
inches.	tons.	tons.	tons.	lbs.
$\frac{1}{2}$	6·75	4·50	2·25	15
$\frac{9}{16}$	8·54	5·75	2·84	19
$\frac{5}{8}$	10·55	7·03	3·51	24
$\frac{11}{16}$	12·76	8·52	4·25	28
$\frac{3}{4}$	15·18	10·10	5·06	32
$\frac{13}{16}$	17·82	11·9	5·94	37
$\frac{7}{8}$	20·67	13·9	6·89	44
$\frac{15}{16}$	23·73	15·8	7·91	49
1	27·00	18·0	9·00	58
$1\frac{1}{8}$	30·48	22·9	11·45	72
$1\frac{1}{4}$	42·19	28·1	14·05	90
$1\frac{3}{8}$	51·05	34·1	17·05	110
$1\frac{1}{2}$	60·75	40·6	20·30	125
$1\frac{5}{8}$	71·30	47·6	23·8	145
$1\frac{3}{4}$	82·68	55·4	27·7	170
$1\frac{7}{8}$	94·92	63·3	31·6	195
2	108·00	72·0	36·0	230
$2\frac{1}{8}$	121·92	81·3	40·6	256
$2\frac{1}{4}$	136·68	91·2	45·6	285
$2\frac{3}{8}$	141·75	101·7	50·8	320
$2\frac{1}{2}$	168·75	112·5	56·2	360
Ratios=	3	2	1	..
(1)	(2)	(3)	(4)	(5)

breaking weight of 1-inch chain: taking 27 tons for round numbers, we have for stud-linked chain, the rules:—

(107.) Mean breaking weight in pounds, $w = d^2 \times 945$.
,, ,, tons, $W = d^2 \times \cdot 423$.

(108.) Admiralty proof-strain in pounds, $p = d^2 \times 630$.
,, ,, tons, $P = d^2 \times \cdot 282$.

In which d = the diameter of the iron in 8ths of an inch: thus for 1-inch cable $W = 64 \times \cdot 423 = 27$ tons, and $P = 64 \times \cdot 282 = 18$ tons, &c.: cols. 2 and 3 in Table 23 have been calculated by these rules.

The ratio of the proof strain to the breaking weight is 1 to $945 \div 630 = 1 \cdot 5$, which is a very severe test, but it is maintained that the object of testing, namely, to discover faulty

links, would not be answered without a heavy strain, and that experiments at Portsmouth Dockyard have shown that the strength of a chain is not seriously or even sensibly impaired by repeated strains almost equal to the breaking weight. In ordinary cases, however, it is not desirable to load chain-cables to more than half the Admiralty proof-strain, or $\frac{1}{3}$rd the breaking weight, and from this we obtain col. 4 in Table 23.

(109.) "*Annealing Chain.*"—It is shown in (14) that the effect of annealing wrought iron in the form of bars, plates, or chains is a loss of tensile strength. The effect on chain is very clearly shown by Nos. 19, 20, in Table 21, where the matter was submitted to direct experiment: ten specimens of $\frac{7}{8}$-inch annealed chain gave a mean strength of 16·34 tons per square inch; and ten similar ones not annealed, gave 17·54 tons, showing $16 \cdot 34 \div 17 \cdot 54 = \cdot 93$, or 7 per cent. loss. Comparing the maximum strengths in col. 4, or the minimum ones in col. 5, we obtain similar results: thus with the former we have $20 \cdot 25 \div 21 \cdot 75 = \cdot 93$, or 7 per cent.; and with the latter $19 \div 20 \cdot 5 = \cdot 93$, or 7 per cent. also.

But if in the course of manufacture, the iron is cold-hammered, which not unfrequently occurs in practice, a loss in strength of about 30 per cent. may accrue as shown in (14), and in order to avoid that great loss, it is expedient to submit to the 7 per cent. loss due to annealing.

It has been found by experience that by long-continued use, chains become brittle and break with a comparatively small impulsive strain, such as very often occurs with cranes, &c., by the load slipping or otherwise: thus a $\frac{5}{8}$-inch chain has been known to break in one case with $3\frac{1}{4}$ tons, and in another with 5·9 tons, although the breaking load by col. 2 of Table 22 should not have been less than 11·72 tons. It should be observed, however, that in both cases the strain exceeded the safe working load given in col. 5, namely, 2·34 tons.

It is very desirable that all chains should be periodically re-annealed and re-tested: the failure of chains is the source of most of the serious accidents in our factories, which might be avoided to a great extent by more frequent annealing and testing.

(110.) "*Strength of Ropes.*"—The experiments of Captain Huddart have shown that hand-laid ropes such as are commonly made in small rope-works are not so strong as those made with register and press-block. He also found in the latter greater uniformity of strength among the various sizes: in the hand-laid ropes the smaller sizes were proportionately stronger than the larger, ranging from 560 lbs. per circular inch in 3-inch girth, to 421 lbs. in the 8-inch. Those made with the register were nearly uniform in strength, being = 820 lbs. per circular inch in both the 3-inch and 8-inch ropes.

From the deterioration by age and moisture to which ropes are subjected the safe working load should not exceed $\frac{1}{4}$ of the breaking weight for such cases as cranes and pulley-blocks. But where life and limb depend absolutely on the strength of ropes, as in hoists or lifts, &c., and where moreover there is considerable wear-and-tear by constant passing over pulleys, &c., the working load should not exceed $\frac{1}{6}$th of the breaking weight.

The proof strain is commonly taken at half the breaking weight, but this seems to be too high in most cases: we have taken it at $\frac{1}{3}$ in Table 24, which gives the breaking weights by Captain Huddart's experiments, also the proof strain and working load in accordance with varying circumstances.

It is frequently expedient to use two ropes of equivalent strength rather than one large one, and this is commonly done in the teagles or hoists used in the factories of the north of England. Thus, where the weight of cage and load = 30 cwt., we might use by col. 6, one $6\frac{1}{2}$-inch, or preferably two $4\frac{1}{2}$-inch ones, &c.

(111.) "*Flat Ropes.*"—The continual bending of ropes over pulleys is found to be very destructive, especially with small pulleys, and of course ropes of large size suffer the most. For this reason flat ropes are better adapted for such cases; their strength may be found from that of the round ones of which they are composed: thus a flat rope $1\frac{1}{2} \times 6$ inches, composed of four round ropes each $1\frac{1}{2}$ inch diameter or $4\frac{1}{2}$ inches girth, will by col. 6 of Table 24 give a working load = $15 \cdot 3 \times 4 =$ 61 cwt., &c.

(112.) "*Rigidity of Ropes.*"—When a rope passes over a

[redacted]

TABLE 24.—Of the STRENGTH and WEIGHT of HEMPEN ROPES.

Girth in Inches.	Weight per Fathom.	Breaking Weights.	Proof Strain.	Safe loads: Cwts. Ordinary.	Safe loads: Cwts. Hoists, &c.
		Ropes made with Register.			
	lbs.	cwts.	cwts.		
1½	0·50	16·5	5·5	4·1	2·4
2	0·88	29·3	9·8	7·3	4·9
2½	1·38	45·7	15·2	11·4	7·6
3	1·98	66	22	16·5	11·0
3½	2·70	90	30	22·5	15·0
4	3·52	117	39	29·2	19·5
4½	4·46	148	49	37·0	24·7
5	5·50	183	61	45·7	30·5
5½	6·66	221	70	55·2	37·0
6	8·00	263	88	66	44·0
6½	9·3	309	103	77	51·5
7	10·8	358	119	89	59·7
7½	12·4	412	137	103	69
8	14·1	468	156	117	78
9	17·8	593	198	148	99
10	22·0	732	244	183	122
11	26·6	886	295	221	144
12	31·7	1054	351	264	176
		Hand-laid Ropes.			
1½	0·50	11·3	3·8	2·8	1·9
2	0·88	20·0	6·7	5·0	3·3
2½	1·38	31·3	10·4	7·8	5·2
3	1·98	45·1	15·0	11·3	7·5
3½	2·70	60·6	20·2	15·1	10·1
4	3·52	78	26·0	19·5	13·0
4½	4·46	92	30·7	23·0	15·3
5	5·50	118	39·3	29·5	19·7
5½	6·66	138	46	34·5	23·0
6	8·00	162	54	40·5	27·0
6½	9·3	183	61	45·7	30·5
7	10·8	205	68	51·2	34·1
7½	12·4	223	74	56·0	37·2
8	14·1	240	80	60·0	40·0
Ratios	..	6	2	1·5	1
(1)	(2)	(3)	(4)	(5)	(6)

pulley, the strain upon it is not only that due to the weight lifted, but also that due to the stiffness or rigidity of the rope itself, and still further to the friction of the pin. Coulomb, Navier, and others have investigated this subject, and the following rule is based on their results:—

$$(113.)\quad w = \frac{\Big\{[\cdot 0257 + (\cdot 0212 \times x)] \times x\Big) + \Big(\cdot 014273 \times x \times W\Big\}}{D}.$$

In which $x = d^2 \times 48$ for white hempen rope; d = diameter of rope in inches; D = diameter of pulley in inches measured at the centres of the ropes; W = the statical weight on the rope in lbs. without motion; and w = the extra weight to overcome the stiffness of the rope and produce motion.

(114.) Thus, let A in Fig. 26 be a pulley $14\frac{3}{4}$ inches diameter, with a rope 1 inch diameter having equal weights W, W of 1100 lbs. suspended on each side, the diameter at the centres of the ropes will then be $15\frac{3}{4}$ inches. Now, with no rigidity in the rope or friction of axle, the addition of the smallest weight to one side would cause motion, but the rigidity of the rope will require a considerable extra weight to overcome it; in our case:—

$$w = \frac{\Big\{[\cdot 0257 + (\cdot 0212 \times 48)] \times 48\Big) + \Big(\cdot 014273 \times 48 \times 1100\Big\}}{15\frac{3}{4}} = 51 \text{ lbs.}$$

Table 25 has been calculated by the rule (113), col. A by rule A = $\cdot 014273 \times x$: and col. B by rule B = $[0257 + (0212 \times x)] \times x$. To use this Table; multiply the number in col. A opposite the given size of rope by the statical strain or weight W; add the number in col. B and divide the sum by the diameter of the pulley in inches, measured at centres of ropes; the quotient is the extra weight w required to overcome the rigidity and produce motion. Thus for a rope 3 inches girth, or 1 inch diameter, on a 7-inch pulley, or 8 inches centres, and a weight of 1000 lbs., we have from col. A, $\cdot 6247 \times 1000 = 624\cdot 7$, and from col. B, $41\cdot 74$, from which we obtain $w = (624\cdot 7 + 41\cdot 74) \div 8 = 83$ lbs.: motion would therefore ensue with 1000 lbs. at one side, and 1083 at the other, if there was no friction from the pin.

TABLE 25.—Of CONSTANTS for Loss of POWER by RIGIDITY of HEMP ROPES.

Size of Rope.		A	B	Size of Rope.		A	B
Girth.	Diameter.			Girth.	Diameter.		
$1\frac{1}{2}$	·477	·1556	2·80	6	1·91	2·501	655
2	·636	·2767	8·50	$6\frac{1}{2}$	2·07	2·850	851
$2\frac{1}{2}$	·796	·4343	20·41	7	2·23	3·404	1212
3	·955	·6247	41·74	$7\frac{1}{2}$	2·39	3·912	1600
$3\frac{1}{2}$	1·114	·8495	76·66	8	2·55	4·453	2072
4	1·27	1·103	128·6	9	2·86	5·604	3278
$4\frac{1}{2}$	1·43	1·255	205·9	10	3·18	6·927	5005
5	1·59	1·733	315·6	11	3·50	8·392	7330
$5\frac{1}{2}$	1·75	2·097	461·3	12	3·82	10·000	10415
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)

(115.) The advantage of large diameters for the pulleys will now be apparent, the loss of effect being inversely proportional to the diameter: thus the extra weight due to rigidity for 4, 8, and 16-inch pulleys to raise 1000 lbs. with a 3-inch rope would be 166, 83, and 41·5 lbs., the strain becoming $1000 + 166 = 1166$; $1000 + 83 = 1083$, and $1000 + 41·5 = 1041·5$ lbs. respectively, irrespective of friction.

(116.) "*Loss by Friction.*"—The effect of friction may be illustrated by Fig. 26: the strain on the pin is in that case $1100 + 1151 = 2251$ lbs.; taking friction at $\frac{1}{6}$th of the weight, we have $2251 \div 6 = 375$ lbs. friction at the surface of the pin. Taking 2 inches for the diameter of the pin and $15\frac{3}{4}$ for the *effective* diameter of the pulley, we have $375 \times 2 \div 15·75 =$ 48 lbs. at the rope. Thus to raise 1100 lbs. we require a strain of $1100 + 51 + 48 = 1199$ lbs.: hence $1100 \div 1199 = ·918$, or say 92 per cent. of the power is utilised, 8 per cent. being lost by friction and rigidity of the rope.

(117.) "*Strains in Common Sheave-blocks.*"—The strains on a rope in common sheave-blocks are very complicated, the effect of rigidity and friction *accumulating* throughout with every

additional pulley. Say that we take the case of a pair of 4 and 3-sheave blocks, Fig. 27, with a rope 3 inches in girth and 7-inch pulleys, or 8-inch centres, having pins $1\frac{1}{4}$ inch diameter.

Assuming 800 lbs. on the rope h, the extra strain for rigidity on the pulley r by Table 25 and (113) will be $\frac{(\cdot 6247 \times 800) + 41\cdot 74}{8} = 68$ lbs., and the tension on the rope g would be 868 lbs. if there were not a further loss by friction of the pin. The weight on the pin of the pulley r = $800 + 868 = 1668$ lbs., the friction, $1668 \div 6 = 278$ lbs. at the surface of the pin, which is reduced to $278 \times 1\frac{1}{4} \div 8 =$ 43 lbs. at the rope: the tension on g thus becomes $868 + 43 =$ 911 lbs.

Similarly, the extra strain from rigidity on the pulley p, is $\frac{(\cdot 6247 \times 911) + 41\cdot 74}{8} = 76$ lbs., which increases the strain on the rope f to $911 + 76 = 987$ lbs.: the weight on the pin becomes $911 + 987 = 1898$ lbs.: friction $(1898 \times 1\frac{1}{4}) \div (6 \times 8) = 48$ lbs. at the rope, thus increasing the tension on f to $987 + 48 = 1035$ lbs., &c., &c. The strains on all the ropes in Fig. 27 have been thus calculated, and from that figure we may now obtain data for any number of pulleys from 1 to 7.

(118.) Thus with a single pulley r, we have $800 \div 911 =$ $\cdot 878$, or say 88 per cent. of the power utilised, hence 12 per cent. is lost: we should obtain the same result from the pulley k, namely, $1727 \div 1960 = \cdot 881$.

With 1 and 2 sheaves, o, p, r, the weight lifted at W is equal to the sum of the strains on the ropes f, g, h, or $1035 +$ $911 + 800 = 2746$ lbs. But the mechanical power of the combination being 3 to 1, the tension at e should have raised $1177 \times 3 = 3531$ lbs. at W; hence we have $2746 \div 3531 =$ $\cdot 78$, or 78 per cent. utilised, and 22 per cent. lost.

With 2 and 3 sheaves, m, n, o, p, r, the weight lifted is the sum of the strains on the ropes d, e, f, g, h, or 5260 lbs., but the weight due to the strain of 1518 at c is $1518 \times 5 =$ 7590 lbs., hence $5260 \div 7590 = \cdot 69$, or 69 per cent. is utilised, and 31 per cent. lost.

With 3 and 4 sheaves, the weight lifted is the sum of the strains on the seven ropes, b, c, d, e, f, g, h, or 8507 lbs.; but the weight due to 1960 lbs. at a, is $1960 \times 7 = 13720$ lbs.: hence $8507 \div 13720 = \cdot 62$, or 62 per cent. is utilised, and 38 per cent. wasted.

(119.) These ratios may be applied generally with approximate accuracy to average pulley-blocks with ropes of other sizes, and we have thus obtained Table 26: thus taking the working strain on common hand-laid ropes from col. 5 of Table 24, we obtain col. 4, and from that, and the theoretical power, combined with the duty, we obtain cols. 5 to 8 in Table 26. Thus for a rope 2 inches girth without any pulley the direct safe load by col. 4 = 5 cwt.: with 1 pulley, $5 \times \cdot 88 = 4 \cdot 4$ cwt.; with 2 and 1 sheaves, $5 \times 3 \times \cdot 78 = 11 \cdot 7$ cwt.; with 3 and 2 sheaves, $5 \times 5 \times \cdot 69 = 17 \cdot 2$ cwt.; and with 4 and 3 sheaves, $5 \times 7 \times \cdot 62 = 21 \cdot 7$ cwt., &c.

TABLE 26.—Of the SAFE LOAD for PULLEY-BLOCKS, allowing for RIGIDITY of the ROPE, and FRICTION of PIN.

Size of Rope.		Diameter of Pulley.	No Pulley. Duty 100 Per Cent.	1 Pulley. Theoretical Power 1 to 1; Duty 88 Per Cent.	2 and 1 Sheaves. Theoretical Power 3 to 1; Duty 78 Per Cent.	3 and 2 Sheaves. Theoretical Power 5 to 1; Duty 69 Per Cent.	4 and 3 Sheaves. Theoretical Power 7 to 1; Duty 62 Per Cent.
Girth.	Diameter.		Working Loads, in Cwts.				
		inches.					
1½	·48	3·5	2·8	2·5	6·5	9·7	12·1
2	·64	4·7	5·0	4·4	11·7	17·2	21·7
2½	·80	5·8	7·8	7	18	27	34
3	·96	7·0	11·3	10	26	39	49
3½	1·11	8·2	15·1	13	35	52	66
4	1·27	9·3	19·5	17	47	67	85
4½	1·43	10·5	23·0	20	54	79	100
5	1·59	11·7	29·5	26	69	101	128
5½	1·75	12·8	34·5	30	81	119	150
6	1·91	14·0	40·5	36	95	140	176
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

(120.) When pulley-blocks can be so arranged that the *power* shall act upwards instead of downwards, the last pulley can be dispensed with and economy of power effected. For example, if in Fig. 27 the rope b is continued to Q, the pulley k is useless, all the strains on the different ropes remain as before, therefore the weight lifted is the same, but instead of 1960 lbs. as at a, we now require 1727 lbs. only, the weight due to which is $1727 \times 7 = 12089$ lbs.; hence $8507 \div 12089 = \cdot 70$, or 70 per cent., is utilised, and 30 per cent. lost, whereas in (118) we had 62 per cent. utilised: 8 per cent. being thus saved.

(121.) "*Strength of Wire Rope.*"—Wire ropes are very extensively used for winding purposes in collieries, &c., where the principal objection to them, namely, their great rigidity, is easily overcome by the use of very large pulleys. The breaking weight and safe working load of round and flat iron-wire ropes shown by Table 27 are given by Messrs. Newall and Co.: it will be observed that they fix the working load of round ropes for inclined planes and other ordinary work at $\frac{1}{7}$th of the breaking weight, and for flat ropes (111) used in pits, hoists, &c., where life depends absolutely on the strength of the rope,

TABLE 27.—Of the STRENGTH of IRON WIRE ROPES.

Circumference.	Breaking Weight.	Working Load.	Weight per Fathom.	Circumference.	Breaking Weight.	Working Load.	Weight per Fathom.
	cwts.	cwts.	lbs.		cwts.	cwts.	lbs.
1	40	6	1	$4\frac{3}{8}$	640	96	16
$1\frac{5}{8}$	80	12	2	$4\frac{5}{8}$	800	120	20
$1\frac{7}{8}$	120	18	3				
$2\frac{1}{8}$	160	24	4	Sizes.	Flat Wire-ropes.		
$2\frac{3}{8}$	200	30	5				
$2\frac{5}{8}$	240	36	6	$2\frac{1}{4} \times \frac{1}{2}$	400	44	11
$2\frac{7}{8}$	280	42	7	$2\frac{3}{4} \times \frac{5}{8}$	540	60	15
$3\frac{1}{8}$	320	48	8	$3\frac{1}{4} \times \frac{5}{8}$	640	72	18
$3\frac{1}{2}$	400	60	10	$3\frac{3}{4} \times \frac{11}{16}$	800	88	22
$3\frac{3}{4}$	480	72	12	$4\frac{1}{4} \times \frac{3}{4}$	1000	112	28
4	560	84	14	$4\frac{5}{8} \times \frac{3}{4}$	1200	136	34

only $\frac{1}{9}$th of the breaking weight, agreeing with (923) and Table 141, which gives in col. 3, for an intermittent dynamic load $\frac{1}{3}$ of the statical breaking weight, which, with Factor 3, becomes $\frac{1}{3} \div 3 = \frac{1}{9}$th.

(122.) "*Strength of Pump-rods.*"—By Table 1 the mean tensile strength of *welded* wrought iron $= 47{,}266$ lbs. per square inch breaking weight with dead load, equivalent by the "ratios" in col. 3 of Table 141 to $47266 \times \frac{1}{3} = 15755$ lbs. intermittent dynamic breaking weight. Table 137 gives factor of safety $= 3$, hence we obtain $15755 \div 3 = 5252$ lbs. per square inch working strain: taking it in round numbers at 5000 lbs., we obtain col. 2 of Table 28. Fig. 28 gives the form and proportions of socket-joints for pump-rods, which have been found to work well in practice, and the table gives the sizes of socket, &c., &c., for different diameters from $\frac{5}{8}$ inch to 2 inches.

TABLE 28.—Of the PROPORTIONS of SOCKET-JOINTS for WROUGHT-IRON SINGLE-ACTING PUMP-RODS.

Diam. of Rod.	Working Strain in Lbs.	Letters of Reference. Fig. 28.											
		A	B	C	D	E	F	G	H	I	J	K	L
5/8	1,534	5/8	3/4	1	1 3/8	1/4	1 3/4	1 5/8	5/16	5/16	3/4	7/8	3/8
3/4	2,208	3/4	7/8	1 1/8	1 1/2	1/4	2	1 7/8	3/8	3/8	7/8	1 1/8	5/8
7/8	3,006	7/8	1 1/8	1 3/8	1 7/8	1/4	2 3/8	2 1/8	7/16	7/16	1	1 1/4	3/4
1	3,927	1	1 1/4	1 5/8	2 1/8	5/16	2 5/8	2 3/8	1/2	1/2	1 1/8	1 1/2	7/8
1 1/8	4,970	1 1/8	1 3/8	1 3/4	2 1/4	5/16	2 3/4	2 1/2	9/16	9/16	1 1/4	1 3/4	1
1 1/4	6,136	1 1/4	1 1/2	1 7/8	2 1/2	3/8	3	2 3/4	5/8	5/8	1 1/4	1 7/8	1 1/8
1 1/2	8,836	1 1/2	1 3/4	2 1/4	3	7/16	3 1/2	3 1/4	3/4	3/4	1 1/2	2 1/8	1 3/16
1 3/4	12,026	1 3/4	2	2 1/2	3 1/2	1/2	4	3 3/4	3/4	7/8	1 3/4	2 3/8	1 1/4
2	15,708	2	2 1/4	2 7/8	4	9/16	4 3/4	4 1/2	3/4	1	2	2 3/4	1 1/2

Table 29 gives the strain on single-acting pump-rods in practice: col. 10 shows that it seldom exceeds 5000 lbs. per square inch, but at Trafalgar Square it was 6600 lbs., the result being that fractures were frequent.

For double-acting pump-rods the strain is not only intermittent but alternate also, and being accompanied by more or

TABLE 29.—Of the TENSILE STRAIN on PUMP-RODS in Practice.

Diam. of Pump.	Head of Water.	Socket Joint.		Screw: Diam. of Thread.		Reduced Area.	Material.	Strain in Lbs.		Kind of Pump.	Place, &c.
		Diam.	Thickness of Key.	Top.	Bottom.			Total.	Per Square Inch of Metal.		
in.	ft.					sq. in.					
18	200	$2\frac{1}{2}$	$\frac{5}{8}$	..	..	3·34	Iron	22,000	6600	S.-acting	Trafalgar Square, broke often.
$12\frac{3}{4}$	160	..	..	$1\frac{7}{8}$	$1\frac{5}{8}$	2·074	"	8,930	4310	D.-acting	Lord Carlisle.
$11\frac{3}{4}$	164	$1\frac{3}{4}$	$\frac{3}{8}$	..	..	1·749	Copper	7,695	4393	3-throw	Crystal Palace.
$10\frac{1}{4}$	250	..	..	$1\frac{3}{4}$	$1\frac{1}{2}$	1·77	Iron	8,980	5070	D.-acting	Tonbridge Wells.
9	100	..	..	$1\frac{1}{2}$	$1\frac{1}{4}$	1·227	"	2,760	2250	D.-acting	Tonbridge.
$8\frac{1}{2}$	150	$1\frac{1}{8}$	$\frac{1}{4}$	..	..	·714	Copper	3,688	5165	3-throw	Lowestoft.
8	156	$1\frac{1}{4}$	$\frac{5}{16}$	..	..	·875	"	3,403	3888	"	Dover Castle.
8	156	..	..	1	$\frac{7}{8}$	·600	"	3,403	5670	"	" "
$6\frac{1}{2}$	140	$1\frac{1}{8}$	$\frac{1}{4}$	..	..	·714	"	2,025	2836	"	Colney Hatch.
5	160	1	$\frac{5}{16}$	..	..	·473	"	1,374	2905	"	Brewery, Chiswick.
4	90	$\frac{7}{8}$	$\frac{1}{4}$	..	..	·3823	"	490	1282	"	Lord Rivers.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	

less violent shocks from the motion of the water, the factor of safety = 3, by col. 5 of Table 141 becomes $3 \div \frac{1}{6} = 18$; hence we have $47266 \div 18 = 2626$ lbs. per square inch working load.

CHAPTER V.

ON THE SHEARING STRAIN.

(123.) "*Single and Double Shear.*"—When two plates are connected by a rivet or pin, as at A in Fig. 6, and the rivet is severed by a tensile strain applied to the plates, we have a case of single-shearing, and it has been found that the strain is simply proportional to the area sheared, being independent of the form of the pin in cross-section, whether round, square, &c.

In Fig. 8, or at C in Fig. 6, we have two side plates and one central one: it is obvious that to shear the pin a double area has to be severed requiring double strain for the double shear.

Mr. E. Clark made direct experiments on the resistance of $\frac{7}{8}$-inch rivet-iron to single and double shearing: he found that the

Maximum	Minimum	Mean

single-shearing strain by four experiments was

26·1	23·9	24·14

tons per square inch. Double-shearing gave as the result of eight experiments:—

22·9	21·6	22·1

tons. The mean of the two = 23·12 tons per square inch: the direct tensile strength of the same iron was 24 tons, from which it appears that the shearing and tensile strains are practically equal to one another, and this is admitted as a general rule: it requires, however, some modification as applied to rivets in joints. It appears that in the process of riveting red-hot in the

usual way the metal is damaged and its strength reduced by the maltreatment experienced: thus by Table 1, Mr. Kirkaldy gives 26 tons per square inch as the mean tensile strength of rolled rivet-iron; but Mr. Fairbairn found it to be 22 tons only in single-riveted joints of boiler-plate (19); hence we have $22 \div 26 = \cdot 846$, or say 85 per cent. realised; therefore 15 per cent. is lost by riveting hot.

With steel rivets the loss is very great, as shown by (42); the tensile strength of bar steel is $47 \cdot 84$ tons per square inch, but that of steel in riveted joints is $23 \cdot 77$ tons only: hence $23 \cdot 77 \div 47 \cdot 84 = \cdot 50$, or 50 per cent. only, is realised, and 50 per cent. is lost by riveting hot.

With treenails of English oak, commonly used in ship-building, the shearing strain across the grain by experiments at H.M. Dockyard was 4000 lbs. per square inch, and as by Table 79 the *mean* tensile strength of oak = 12,332 lbs., the ratio is $3 \cdot 1$ to $1 \cdot 0$.

(124.) "*Rectangular Bars.*" — Experiments recorded by the Institute of Mechanical Engineers show that in shearing flat bars, the shearing strain is nearly the same whether the bar is flat or on edge; thus bars $\frac{1}{2}$ inch by 3 inches gave on the flat $22 \cdot 3$ and on edge $23 \cdot 1$ tons per square inch. Others 1 inch by 3 inches gave $23 \cdot 1$ and $22 \cdot 7$ tons per square inch respectively: in these experiments the shear blades were parallel.

(125.) "*Oblique Shearing.*"—When the blades are fixed at an angle so as to shear a plate obliquely, the strain is less than with parallel shearing to an extent which varies with the angle of the blade and the thickness of the plate. Say, *for illustration*, that Fig. 29 is a blade 12 inches wide, with four steps in it, each 3 inches wide, and B the plate to be sheared, the thickness of the plate and the height of the steps being $\frac{1}{2}$ inch. Now, it will be observed that the steps act one after the other, thus C will have done its work and passed through the plate before D begins to act, &c., hence the strain is $\frac{1}{4}$th only of that due to a parallel blade 12 inches wide, but of course the travel is 4 times as much; therefore the mechanical power is the same in either case. The line E, F, G at a slope of 1 to $12\frac{1}{2}$ or 1 to 8 would

evidently give the same result as a blade with steps, that is to say, with a ½-inch plate as in our case. But the slope would vary with the thickness; thus for ¼-inch plate it might be 1 to 16; with 1-inch plate 1 to 4, &c.; the strain being then ⅛th of that with a parallel blade. For ordinary and general purposes a slope of 1 in 8 to 1 in 12 is commonly used.

"*Strain for Punching.*"—An ordinary punch may be regarded as a circular cutter or shearing blade whose length is equal to the circumference: then by Table 1 the *mean* strength of plate-iron is 48,454 lbs., or 21·63 tons per square inch: hence a punch 1 inch diameter with a plate 1 inch thick would require

TABLE 30.—Of the STRAIN for PUNCHING RIVET-HOLES in PLATE-IRON and STEEL.

Thickness.	Diameter of Rivet-hole in Inches.											
	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8
	Punching Strain in Tons: Wrought-iron Plate.											
1/8	1·6	2·1	2·7	3·2	3·7	4·3	4·8	5·3	5·8	6·4	6·9	7·4
3/16	..	3·2	4·0	4·8	5·6	6·4	7·2	8·0	8·8	9·5	10	11
1/4	..	..	5·3	6·4	7·4	8·5	9·5	11	12	13	14	15
5/16	..	..	..	8·0	9·3	10·6	12	13	15	16	17	19
3/8	..	..	..	..	11·2	13	14	16	18	19	21	22
7/16	..	..	..	..	..	15	17	19	20	22	24	26
1/2	..	..	..	..	..	..	19	21	23	25	28	30
9/16	..	..	..	..	..	..	..	24	26	29	31	33
5/8	..	..	..	..	..	..	..	..	29	32	35	37
11/16	..	..	..	..	..	..	..	..	..	35	38	41
3/4	..	..	..	..	..	..	..	..	..	..	41	45
	Punching Strain in Tons: Steel Plate.											
1/8	2·8	3·7	4·7	5·6	6·6	7·5	8·5	9·4	10	11	12	13
3/16	..	5·6	7·0	8·5	10	11	13	14	16	17	18	20
1/4	..	..	9	11	13	15	17	19	21	23	24	26
5/16	..	..	..	14	17	19	21	24	26	28	31	33
3/8	..	..	..	..	20	23	25	28	31	34	37	40
7/16	..	..	..	..	..	26	30	33	36	40	43	46
1/2	..	..	..	..	..	..	34	38	41	45	49	53

a force of $21 \cdot 63 \times 3 \cdot 1416 = 68$ tons, and we have for wrought-iron plates the rule:—

(126.) $$S_P = d \times t \times 68.$$

In which $d =$ the diameter of punch, and $t =$ the thickness of the plate, both in inches; S_P being the punching strain in tons: thus, for example, with $\frac{3}{4}$-inch punch and $\frac{1}{2}$-inch plate we obtain $S_P = \frac{3}{4} \times \frac{1}{2} \times 68 = 25 \cdot 5$ tons.

With steel plates the *mean* tensile and shearing strength by Table 1 = 85,977 lbs., or $38 \cdot 38$ tons per square inch; hence a punch 1 inch diameter with 1-inch plate requires $38 \cdot 38 \times 3 \cdot 1416 = 120$ tons, and we have for steel plates the rule:—

(127.) $$S_P = d \times t \times 120.$$

Thus for say $\frac{1}{2}$-inch punch and $\frac{1}{4}$-inch steel plate the punching strain becomes $S_P = \frac{1}{2} \times \frac{1}{4} \times 120 = 15$ tons. Table 30 has been calculated by these rules.

DETRUSION.

(128.) This term has been applied to the shearing strength of timber in the direction of the fibres. Experiments have shown that, 1st, This is practically the same as the tensile strength *perpendicular to the grain* which is given at the end of Table 1; and, 2nd, That both are very small and very variable: with

Poplar	Oak	Larch	Scotch Fir	Memel

the mean resistance to detrusion with the grain and tensile strain across the grain is:—

1782	2316	1335	562	690 lbs.

per square inch. Taking the ordinary mean tensile strength at

7200	12,332	9560	12,200	15,370 lbs.

per square inch, we have the ratios:—

4·04	5·3	7·2	21·7	22·3 to 1·0

In practice, simple detrusion is easily avoided by bolts through the bar, hence the great weakness of some kinds of timber to *that particular* strain is a matter of small importance.

CHAPTER VI.

ON THE CRUSHING STRAIN.

(129.) We are indebted to Mr. Hodgkinson for almost the whole of our exact knowledge of the strength of materials in resisting crushing strains, and from his experimental investigations we obtain the following laws:—

1st. That for specimens whose height is between $1\frac{1}{2}$ and 3 times the diameter or side of square, the crushing strain is simply proportional to the area.

2nd. In that case the plane of rupture is inclined at an angle with the base, and therefore with the axis, which angle is constant for the same material, but is different for different materials.

3rd. That for heights less than $1\frac{1}{2}$ times the diameter, the crushing strain becomes greater irregularly with the reduction in height (130).

4th. For very great heights, the specimen becoming a pillar of considerable length in proportion to the diameter, failure takes place by lateral flexure, with a load very much less than that necessary to crush the material (306).

5th. For intermediate heights, the pillar fails with an intermediate load, partly by flexure, and partly by *incipient* crushing (163).

(130.) "*Cast Iron.*"—The effect of height is well illustrated by some experiments by Mr. Hodgkinson on cylinders $\frac{1}{2}$ inch diameter, the heights being

$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{3}{4}$ $1\frac{1}{2}$ 2 3·78

inches. The crushing strains were

13·6 12·5 11·8 10·8 10·5 10·5 9·7 6·7

tons; the equivalent strains per square inch were

69·3 63·5 60·0 55·0 53·3 53·3 49·6 34·4

tons. It appears from this that when the height is equal to the diameter the resistance to crushing is $55 \div 53 \cdot 3 = 1 \cdot 032$, or $3 \cdot 2$ per cent. greater than when the height is between $1\frac{1}{2}$ and 2 with diameter $1 \cdot 0$.

(131.) Table 31 gives the general results of Mr. Hodgkinson's experiments on the crushing strength of cast-iron cylinders $\frac{3}{4}$ inch diameter; those in col. 1 were $1\frac{1}{2}$ inch in height, or double the diameter; those in col. 4 were $\frac{3}{4}$ inch high, and they show an excess of 5·8 per cent. over those in col. 1.

Most of the old experiments on the resistance of materials to crushing by Rennie, Bramah, and others, were made on *cubes*, and it has been objected that this fact vitiates their results, but we have seen that in cast iron at least the difference is from 3·2 to 5·8 per cent. only, so that the earlier experiments on cubes may be accepted as correct enough for practice.

TABLE 31.—Of the TENSILE and CRUSHING STRENGTH of CAST IRON, in Tons per Square Inch.

Kind of Iron.	Crushing: Height Double the Diameter. C.	Tensile. T.	Ratio. C to T.	Crushing: Height Equal to Diameter.
Lowmoor, No. 1	25·198	5·667	4·446	28·809
„ No. 2	41·219	6·901	5·973	44·430
Clyde, No. 1	39·616	7·198	5·503	41·459
„ No. 2	45·549	7·949	5·729	49·102
„ No. 3	46·821	10·477	4·469	47·855
Blaenavon, No. 1 ..	35·964	6·222	5·780	40·562
„ No. 2 ..	45·717	7·466	6·123	52·502
„ No. 2, 2nd	30·594	6·380	4·795	30·606
Calder, No. 1	33·921	6·131	5·532	32·229
Coltness, No. 3	45·460	6·820	6·665	44·723
Brymbo, No. 1	33·784	6·440	5·246	33·399
„ No. 3	34·356	6·923	4·963	33·988
Bowling, No. 2	33·028	6·032	5·476	33·987
Devon, No. 3, hot-blast	64·92	9·75	6·638	..
Buffery, No. 1 „	38·56	6·00	6·431	..
„ „ cold-blast	41·67	7·80	5·346	..
Coed-Talon, No. 2, H.B.	36·92	7·45	4·961	..
„ „ C.B.	36·50	8·40	4·337	..
Carron, No. 2, H.B. ..	51·20	6·00	8·493	..
„ „ C.B. ..	49·65	7·45	6·668	..
„ No. 3, H.B. ..	59·56	7·90	7·515	..
„ „ C.B. ..	51·53	6·35	8·129	..
Lowmoor „ C.B. ..	49·00	7·39	7·554	..
	(1)	(2)	(3)	(4)

(132.) Admitting the experiments on specimens whose height is double the diameter, col. 1, as the more correct, the mean resistance of cast iron to crushing may be taken at 43 tons, or 96,320 lbs. per square inch, and the mean tensile strength, in col. 2, being 7·142 tons, or 16,000 lbs., the ratio becomes practically 6 to 1.

It will be observed that there is great variation in the crushing strength of cast iron, as shown by col. 1, Devon being 64·92 and Lowmoor 25·198 tons, giving a ratio of 2·57 to 1·0. The mean crushing strength being 100, the maximum = 156, and the minimum = 61; the effect of re-melting is shown by col. 3 of Table 2.

With the tensile strength the variation is much less, ranging from 10·477 tons in Clyde iron to 5·667 with Lowmoor, the variation being 1·85 to 1·0: the mean tensile strength being 100, the maximum = 147, and the minimum = 79. Table 147.

(133.) "*Wrought Iron and Steel.*"—It is shown in (503) that there is great difficulty in determining the ultimate or absolute crushing strength of all malleable metals such as wrought iron, which in short specimens *flow* or spread out laterally under the pressure rather than crush or break. Wrought iron practically fails entirely with about 12 tons per square inch, the extensions and compressions with greater strains becoming excessive, as shown by the diagram, Fig. 215. Experiments on the transverse strength (520) seem to show 24 tons as the absolute crushing strain, but with pillars of different kinds 19 tons per square inch agrees the best with the results of experiment (201), from which it appears that the resistance of wrought iron is 24 ÷ 19 = 1·26, or 26 per cent. greater in beams than in pillars. The wrinkling strain shows similar differences, namely, 104 ÷ 80 = 1·30, or 30 per cent. greater in beams than in pillars (322).

With steel, the apparent crushing strength under transverse strains seems to be 61·48 tons per square inch (507), but with steel *pillars*, 52 tons agrees better with experiment (268), the difference being 61·48 ÷ 52 = 1·18, or 18 per cent.

This difference of resistance to crushing in beams and pillars is remarkable, but admits of explanation. In a *short* pillar

every part of the cross-section is equally strained or nearly so, but in a beam the strain is a maximum at the edge of the section, and is supposed to diminish in arithmetical ratio toward the neutral axis, where it becomes nil, as shown in (494) and Fig. 164. But when a wrought-iron bar is deflected by the transverse strain, the malleable nature of the metal causes it to yield so much under the maximum pressure at the remote edge that heavier strains are thrown on the rest of the section. For example, Fig. 30 is the section below the neutral axis of a bar of any material whose maximum resistance to crushing at A = 19 tons per square inch, therefore $9\frac{1}{2}$ tons at B, $4\frac{3}{4}$ at C, &c., the mean of the whole being = $9\frac{1}{2}$ tons. Let Fig. 31 be a similar beam where the maximum = 24 tons, &c., the mean of the whole being 12 tons. Now let Fig. 32 be another beam whose resistance at B = $14\frac{1}{2}$ tons: if, therefore, the resistance is proportional to the distance from the neutral axis, it should be 29 tons at A, but if we allow that the metal there compresses excessively, as in diagram, Fig. 215, until it is reduced to 19 tons, we then have a double series of strains as in the figure, the mean of the whole being 12, as in Fig. 32. It will now be observed that Fig. 32 gives the same mean crushing strain with 19 tons maximum, as Fig. 31 gave with 24 maximum: the *apparent* maximum strain in Fig. 32 is 24 tons, although the real maximum = 19 tons only: see (504).

In confirmation of this reasoning it should be observed that with cast iron, which maintains comparative uniformity in its compression under crushing strain, as shown by the diagram, Fig. 215, the crushing strength is the same in pillars as in beams, namely, 43 tons per square inch.

(134.) "*Timber.*"—Table 32 gives Mr. Hodgkinson's experiments on the crushing strength of various kinds of timber: the results in col. 2 were a mean of about 3 experiments on cylinders 1 inch diameter and 2 inches high, with flat ends, the woods being moderately dry or in the ordinary state. Col. 1 were specimens turned to the sizes and kept drying in a warm place for two months: the lengths of these specimens were in some cases 1 inch only, being equal to the diameter, which would *increase the* strength a little (129).

TABLE 32.—Of the STRENGTH of TIMBER to RESIST CRUSHING STRAINS, in Lbs. and Tons per Square Inch.

Kind of Timber.	Maximum Dry. Lbs.	Minimum Ordinary State. Lbs.	Mean.		Ratio of Col. 1 to Col. 2.
			Lbs.	Tons.	
Alder	6,960	6831	6,896	3·08	1·02
Ash	9,363	8683	9,023	4·03	1·08
Bay-wood	7,518	7518	7,518	3·36	1·00
Beech	9,363	7733	8,548	3·81	1·21
Birch, English	6,402	3297	4,850	2·16	1·94
" American	11,663	8970*	10,316	4·60	1·30
Box	9,971	7670*	8,820	3·94	1·30
Cedar	5,863	5674	5,768	2·58	1·03
Crab-tree	7,148	6499	6,824	3·05	1·10
Deal, red	6,586	5748	6,167	2·75	1·15
" white	7,293	6781	7,037	3·14	1·08
Elder	9,973	7451	8,712	3·89	1·34
Elm	10,331	7950*	9,140	4·08	1·30
Fir, Spruce	6,819	6499	6,659	2·97	1·05
Hornbeam	7,289	4533	5,911	2·64	1·60
Mahogany	8,198	8198	8,198	3·66	1·00
Oak, English	10,058	6484	8,271	3·69	1·55
" Quebec	5,982	4231	5,106	2·28	1·41
" Dantzic	7,731	5950*	6,840	3·05	1·30
Pine, pitch	6,790	6790	6,790	3·03	1·00
" yellow	5,445	5375	5,410	2·41	1·01
" red	7,518	5395	6,457	2·88	1·40
Plum	10,493	8241	9,367	4·18	1·27
Poplar	5,124	3107	4,116	1·84	1·65
Sycamore	9,207*	7082	8,144	3·67	1·30
Teak	12,101	9310*	10,706	4·78	1·30
Larch	5,568	3201	4,385	1·96	1·74
Walnut	7,227	6063	6,645	2·97	1·19
Willow	6,128	2898	4,513	2·02	2·11
	(1)	(2)	(3)	(4)	(5)

* Calculated from the general ratio of the experiments in columns 1 and 2, which is 1·3 to 1·0.

The effect of the drying process on most kinds of wood is to increase the crushing strength, varying from nothing with bay, mahogany, and pitch-pine to 2·11 with willow, col. 5: the mean increase for the 29 kinds of timber is 1·30, or 30 per cent. In several cases indicated by a * the experiments were made with the wood in one of the states only: in those cases the strength in the other state was calculated by the general ratio 1 to 1·3, &c.

TABLE 33.—Of the STRENGTH of STONE, &c., to RESIST a CRUSHING STRAIN in Lbs. and Tons per Square Inch.

Kind of Stone, &c.	Maximum. Lbs.	Minimum. Lbs.	Mean. Lbs.	Mean. Tons.	Authority.
Granite, Herm	16,240	13,507	14,873	6·64	2 Bramah.
„ Dartmoor	14,493	10,080	12,275	5·48	„
„ Peterhead, Red ..	11,760	10,080	10,931	4·88	„
„ Aberdeen, Blue ..	11,267	9,520	10,394	4·64	„
„ Peterhead, Blue ..	10,204	9,206	9,766	4·36	„
„ Penrhyn	8,736	6,356	7,271	3·25	„
„ Peterhead	..	..	8,283	3·70	Rennie.
„ Cornish	..	..	6,356	2·84	„
„ Aberdeen, Blue ..	..	..	10,911	4·87	„
Cragleith (Sandstone) ..	7,661	5,645	6,653	2·97	Bramah.
„ „ ..	8,688	5,487	7,000	3·12	3 Rennie.
Portland (Oolite)	4,571	3,729	4,150	1·85	2 „
Purbeck	..	..	9,160	4·10	„
York paving (Sandstone) ..	..	..	5,713	2·55	„
„ Cromwell Bottom	9,900	7,773	8,825	3·94	2 Bramah.
Humbie (Sandstone) ..	5,040	4,189	4,614	2·06	2 „
Whitby „	2,509	2,240	2,374	1·06	2 „
Marble, White Italian ..	10,326	8,960	9,632	4·30	2 „
„ Veined „ ..	10,640	7,840	9,251	4·13	„
„ Statuary	..	..	3,216	1·436	Rennie.
„ White Italian ..	..	..	9,681	4·322	„
„ Black Brabant ..	..	..	9,220	4·116	„
„ Red Devonshire ..	..	..	7,433	3·318	„
Red Sandstone	3,925	1,085	2,185	·9754	E. Clark.
Chalk	..	..	501	·2237	Rennie.
Slate, Valentia	13,375	10,820	12,062	5·385	4 Bramah.
Bramley Fall Sandstone ..	..	..	6,059	2·700	Rennie.
Limestone, compact	..	..	7,713	3·443	„
Derby Grit	4,344	3,145	3,744	1·671	„
Glass	38,825	20,775	30,047	13·41	8 Fairbairn
Portland Cement, neat, 30 days old	2,453	2,074	2,264	1·01	B. White.
Do., 1 part, Sand 2 parts, 52 days old	..	..	1,244	·55	„
Do., 1 part, Sand 3 parts, 52 days old	..	..	691	·31	„
Roman Cement, neat, 30 days old	829	746	788	·35	„
Do., 1 part, Sand 2 parts, 52 days old	..	..	83	·037	„
Concrete Stone, Portland Cement 1 part, Sand 2 parts, 48 days old ..	..	..	664	·296	„

TABLE 33.—Of the STRENGTH of STONE, &c.—*continued.*

Kind of Stone, &c.	Maximum. Lbs.	Minimum. Lbs.	Mean. Lbs.	Mean. Tons.	Authority.
Concrete Stone, Portland Cement 1 part, Sand 2 parts, 270 days old ..	..	..	1,763	·79	B. White.
Do., Portland Cement 1 part, Sand 3 parts, 70 days old	..	..	442	·20	"
Do., Portland Cement 1 part, Shingle 10 parts, 30 days old..	..	..	276	·12	"
Brick Cubes, in Portland Cement, neat, 30 days old	..	..	442	·20	"
Do., Portland Cement 1 part, Sand 2 parts, 52 days old	..	..	608	·27	"
Do., Portland Cement 1 part, Sand 3 parts, 30 days old	..	..	726	·324	"
Do., Roman Cement, neat, 30 days old	..	..	829	·37	"
Do., Roman Cement 1 part, Sand 2 parts, 52 days old	..	..	386	·17	"
Ordinary Mortar	..	..	480	·21	Claudel.
Do., 14 years old	..	..	260	·12	Vicat.
Brick, 9-in. Cubes, in Cement	613	417	521	·2326	E. Clark.
" pale red	..	..	562	·251	Rennie.
" red	..	..	808	·361	"
" Pavior's	..	..	1,000	·4464	"
" " Burnt extra	..	..	1,441	·647	"
" Fire, Stourbridge	..	..	1,718	·77	"

(135.) "*Stone, &c.*"—Table 33 gives a summary of experiments on the crushing strength of stone: those by Bramah were on cubes from 4 to 6 inches square, and were obtained by means of a 12-inch hydraulic press, with which very accurate results could hardly be expected (83); however, they are perhaps correct enough for practical purposes.

CHAPTER VII.

ON THE STRENGTH OF PILLARS.

(136.) "*Theory of Pillars.*"—The theory of the strength of pillars will be most easily understood by analogy with the transverse strength and stiffness of a beam of the same sizes and material. In Fig. 33 let A be a beam 10 feet long supported at each end and deflected $\frac{1}{2}$ foot by a central load w = 20 lbs: now obviously, if that load is removed, the elasticity of the beam will cause it to react with a force of 20 lbs. in the direction of the arrow B, and the question becomes, what horizontal forces in the direction of the arrows C, C will counterbalance the vertical strain B.

Let D, E, in Fig. 34 be two rods similar to A in Fig. 33 and resting on the plane f: for the purposes of illustration we may suppose them to be jointed by pins at a, b, c, d. With a vertical load of 200 lbs. at W, we have to find the horizontal strain at c, d; by the "parallelogram of forces": drawing the straight lines $a\,c$, $c\,b$, $a\,d$, $d\,b$, we obtain a parallelogram, and, as is well known, the ratio of the length of the vertical diagonal $a\,b$, to that of the horizontal one $c\,d$, is also the ratio of the vertical strain to the horizontal strain. Thus, in our case, the vertical strain being 200, we divide $a\,b$ into 200 equal parts and thus obtain a scale, which applied to $c\,d$, gives 20 lbs. as the horizontal strain on that line, that is to say, a Salter's balance between $c\,d$, showing a strain of 20 lbs., would support the vertical load W = 200 lbs.

In Fig. 35, the two pillars are separated simply for the purpose of analysis, and we have two similar pillars F, G, &c., each loaded with 100 lbs., or half the former load. The strain shown by the Salter's balance at $c\,d$, acted in *both* directions, therefore substituting weights w, w, as in Fig. 35, we shall require 20 lbs. for *each*. We thus find that a beam, as in Fig. 33, 10 feet long, deflected $\frac{1}{2}$ foot by 20 lbs. will, when unloaded, react with 20 lbs., and cause a horizontal strain of 100 lbs. in the direction of the arrows C, C, and as shown at W, W in the figure.

(137.) From this we obtain a general law connecting the vertical with the horizontal strains, and we find that if we take a bar of any given length resting on end-supports, and observe the central deflection with a certain transverse central load, we can calculate the equivalent load acting longitudinally and straining the same bar as a pillar by multiplying the transverse load by the length of the beam, and dividing the product by four times the deflection. Thus, in our case, $20 \times 10 \div (\frac{1}{2} \times 4) = 100$: hence we have the general rule:—

(138.) $$W = w \times l \div (\delta \times 4).$$

In which W = the load on the pillar in lbs., tons, &c.

w = the transverse load in the centre of the same bar and in the same terms as W.

δ = the deflection in inches produced by w.

l = the length of the pillar and beam, in inches.

(139.) As an example of the application of the rule, we may take a pillar of Dantzic oak, say 1 inch square and 12 inches long. Mr. Hodgkinson gives as the result of his experiments the rule $24542 \times S^4 \div L^2 = W$ for pillars of that material with both ends flat. Here 24,542 lbs. is the theoretical breaking weight of a pillar 1 inch square and 1 foot long, as due by flexure, neglecting incipient crushing (163). Now, by Table 67, a *beam* of Dantzic oak 1 inch square, 1 foot between supports, loaded transversely with the safe or working load of 71 lbs., deflects ·026 inch, which by our rule (138) is equivalent to $71 \times 12 \div (\cdot 026 \times 4) = 8192$ lbs. longitudinally, straining the bar as a pillar, this strain being in the centre line, or the pillar having both ends pointed, and is that due to flexure only, as in Mr. Hodgkinson's rule. By (149) we obtain for the same pillar with both ends *flat* $8192 \times 3 = 24576$ lbs., which is almost exactly 24,542 lbs. as given by Mr. Hodgkinson.

(140.) But it will be observed that we have taken the *safe* transverse load of 71 lbs., and the corresponding deflection, whereas Mr. Hodgkinson's rule gives the *breaking* weight. It is, however, a theoretical law with pillars, that a load which will produce the smallest deflection will equally produce a greater, sufficient to break the pillar by flexure.

Thus, let us take a beam of any material whose elasticity is

perfect, that is to say, one in which the deflections are simply proportional to the strains, and say 5 feet or 60 inches long, deflected 1 inch by 10 lbs. in the centre:—then by the rule (138) the equivalent load as a pillar will be $10 \times 60 \div (1 \times 4) = 150$ lbs. It would be the same with any other transverse load and corresponding deflection; for instance, with 20 lbs. in the centre, the deflection would evidently be 2 inches, and W would be $20 \times 60 \div (2 \times 4) = 150$ lbs. as before. If we take an extremely small deflection, say $\frac{1}{1000}$th of an inch, the transverse load producing that deflection would evidently be $\frac{1}{100}$th of a pound, and the rule would give $\cdot 01 \times 60 \div (\cdot 001 \times 4) = 150$ lbs. as before.

(141.) This fact conducts us to two remarkable laws:—1st. As the smallest possible deflection of this pillar requires a longitudinal strain of 150 lbs. to produce it, it follows that less than 150 lbs. would not produce any deflection whatever, but the pillar would be perfectly rigid and unyielding until that load was laid upon it. 2nd. That as 150 lbs. will with equal ease produce a deflection of $\frac{1}{1000}$th of an inch—or 1 inch —or any other amount, it follows that when 150 lbs. are laid on, the pillar will not only bend, but will go on increasing in flexure until it breaks.

(142.) Such is the theory; Mr. Hodgkinson found, however, by experiments on various materials, that these laws do not hold good in practice, and that instead of a pillar showing no signs of bending until a certain load is laid on, and then suddenly bending and breaking, he found that there is no weight, however small, that does not produce a slight flexure, which increases progressively as the load is increased until the breaking-point is attained.

(143.) Another remarkable result of Mr. Hodgkinson's experiments was, that the deflection of a pillar on the point of breaking by flexure is very much less than that of the same bar broken by a transverse strain. For instance, a pillar of Dantzic oak $1\frac{3}{4}$ inch square, and $5 \cdot 04$ feet long, broke with a deflection of $\cdot 48$ inch only. Calculating the ultimate deflection with a transverse load by the rules in (695) and taking the value of M from col. 2 of Table 67 at $\cdot 198$, we obtain $5 \cdot 04^2 \times \cdot 198 \div 1 \cdot 75$

$= 2 \cdot 9$ inches, or about 6 times the ultimate deflection of the same bar as a pillar.

A pillar of wrought iron, 10 feet long, and practically 3×1 inches, failed with a deflection of $\cdot 6$ inch only. Calculating the deflection under a transverse load for the "crippling" strain only (373) we obtain $10^2 \times \cdot 035 \div 1 = 3 \cdot 5$ inches, or about six times the ultimate deflection as a pillar. But, in fact, the ultimate deflection of all pillars is very irregular and uncertain, for example, with two pillars $7\frac{1}{2}$ feet long, 3×1 inches, although the breaking weights were nearly the same, one failing with 29,572 lbs., and the other with 29,666 lbs., the ultimate deflection was $\cdot 39$ inch in one case, and $\cdot 08$ inch only in the other, the ratio for two precisely similar pillars being about 5 to 1.

With cylindrical cast-iron pillars the same anomalies were found to prevail, the ultimate deflection being very small, and very irregular.

(144.) With materials whose elasticity is imperfect (688) the ultimate deflection, or that with the breaking weight, is much greater in proportion to the load than the deflection with a small load such as would occur in practice, as is shown by col. 8 in Table 67, which, combined with the fact that the ultimate deflection of pillars is very small, seems to show that in calculating the strength of a pillar from the transverse strength and stiffness, a small load and corresponding deflection should be taken as a basis, rather than the ultimate deflection with the transverse breaking weight. The connection between the strength of pillars, and the transverse strength and deflection of the same materials, will be considered more at large in (296).

(145.) "*Effect of Diameter and Length.*"—We may now search for the laws by which the diameter and length of pillars govern their strength.

1st for the length:—say we take the same beam as before (140), but of double length, namely, 10 feet, or 120 inches. It is shown in (659) that the deflection of a beam loaded transversely with a constant weight, is directly proportional to the cube of the length, or L^3:—in our case, the length being doubled, and 2^3 being $= 8$, we shall have 8 inches deflection, or eight times the deflection due to the length of 1 foot. Then,

by the rule in (138) the equivalent longitudinal strain as a pillar will be $10 \times 120 \div (8 \times 4) = 37 \cdot 5$ lbs., which is $\frac{1}{4}$th of the load borne by the pillar 5 feet long. Again, the deflection of a beam of half the length, or 30 inches, would, by the same reasoning, be $\frac{1}{8}$ inch, and the strength as a pillar $10 \times 30 \div (\frac{1}{8} \times 4) = 600$ lbs., which is four times the strength of the 5-foot pillar, and 16 times the strength of the 10-foot pillar. We thus find that the strength of pillars is *inversely* proportional to the *square* of the length:—thus with lengths in the ratio 1, 2, 4, the strengths are in the ratio 1, $\frac{1}{4}$, $\frac{1}{16}$.

(146.) Searching now for the power of the diameter (or side of square pillars):—say we take a beam 5 feet long, loaded as before with 10 lbs., &c., but 2 inches square. Then, by (659), the deflection with a constant transverse load is *inversely* as $d^3 \times b$, or in our case, $2^3 \times 2 = 16$, hence the deflection of the 1-inch beam, being 1 inch, that of the 2-inch beam will be $\frac{1}{16}$th of an inch, and the strength as a pillar $10 \times 60 \div (\frac{1}{16} \times 4) = 2400$ lbs., which is 16 times 150 lbs., the strength of a 1-inch square pillar of the same length.

Again, with a beam of the same length, but 3 inches square, we have $3^3 \times 3 = 81$, and instead of 1 inch deflection, as with a beam 1 inch square, we have $\frac{1}{81}$st part of an inch with the 3-inch beam, and the strength as a pillar becomes $10 \times 60 \div (\frac{1}{81} \times 4)$ or $10 \times 60 \times 81 \div 4 = 12150$ lbs., which is $12150 \div 150 = 81$ times the strength of the same pillar with a length of 1 foot. We thus find that the strength of pillars is directly proportional to the fourth power of the diameter, or side of square, for 1^4, 2^4, and $3^4 = 1, 16$, and 81, and this, as we have shown, is the ratio of the strengths of the pillars of those respective sizes.

(147.) Combining these results we find that the strength of pillars is proportional to $d^4 \div L^2$. By the same reasoning the strength of rectangular pillars will be proportional to $d^3 \times b \div L^2$, in which d is the depth, or smaller dimension and $b =$ the breadth or greater dimension of the pillar.

These theoretical laws should be correct for all materials, but the experimental researches of Mr. Hodgkinson have shown that timber pillars alone follow those laws exactly. Thus, with cast-iron pillars, he found the strength to be proportional to

$d^{3 \cdot 76} \div L^{1 \cdot 7}$ in the case of pillars with both ends pointed, and to $d^{3 \cdot 55} \div L^{1 \cdot 7}$ in those with both ends flat:—for practical purposes a mean between these extremes may be taken for all modes of bearing at the ends, and we obtain the law $d^{3 \cdot 6} \div L^{1 \cdot 7}$. For wrought-iron and steel pillars, the experimental law is $d^{3 \cdot 6} \div L^2$.

(148.) The effect of this divergence of the experimental from the theoretical law is very considerable:—thus, if the strength of a pillar 1 inch diameter $= 1 \cdot 0$, then another, of the same length, &c., but 6 inches diameter, would, by theory, have a strength of $6^4 = 1296$, whereas by the experimental ratio it would be $6^{3 \cdot 6} = 633$ only, or about half.

Again, as to the length:—say we have a pillar 10 feet long whose strength $= 1 \cdot 0$, then the same pillar with a length of 1 foot would, by theory, have a strength of $10^2 = 100$, but by the experimental ratio it would be $10^{1 \cdot 7} = 50$ only.

It will be seen from this that it is impossible to give general "ratios" for the strengths of pillars of different materials, which will be correct for all diameters and lengths. Mr. Hodgkinson has given a series of numbers as the ratios of strength for cast-iron, wrought-iron, steel, and timber pillars, but these are simply misleading, for if they are correct for a particular diameter and length, they must, of necessity, be incorrect for all other dimensions.

(149.) "*Effect of Form at the Ends.*"—One of the remarkable results of Mr. Hodgkinson's experimental researches was to show that the strength of pillars is potentially governed by the character of the bearings at their ends: it was found that a *long* pillar of any material with both ends flat and well bedded, being pressed between two perfectly parallel planes, had a strength 3 times that of a similar pillar with both ends pointed or rounded, so that the strain was exactly in the axis. It was also found that with one end flat and the other pointed, the strength was an arithmetical mean between the other two.

We have, therefore, the ratios 1, 2, 3 for the strength of three similar pillars—with both ends pointed—one end pointed and one flat—and both ends flat respectively. That these ratios are practically correct may be shown by the tables in this chapter.

Thus, in Table 44, pillars of wrought iron $7 \cdot 56$ feet long $1 \cdot 02$ inch diameter failed with 1825, 3355, and 5280 lbs.

respectively, the ratio being pretty nearly 1, 2, 3. Others 5·04 feet long and *about* 1·02 inch diameter, failed with 3938, 8137, and 12,990 lbs. respectively, which is rather in excess of the ratio 1, 2, 3.

With steel, Nos. 28 to 30 in the same table, we have in col. 6 numbers nearly in the ratio 1, 2, 3, which would have followed the law almost exactly but for the fact that Nos. 29 and 30 required correction for incipient crushing (163).

For Dantzic oak, in col. 7 of Table 57, we have 3197, 6109, and 9625 lbs., which are nearly in the ratio 1, 2, 3.

With cast-iron pillars Table 38 shows similar results: thus comparing Nos. 1 and 13, we have in col. 6, 143 and 487 lbs., where the ratio should be 1 to 3:—Again, in Nos. 2 and 14, we have 1902 and 6238 lbs. where the same ratio should have prevailed. In these cases, however, the diameters of the pillars compared with one another are not precisely identical, which may account in part for the divergence of the experiments from the standard ratio 1 to 3.

(150.) It will be evident from all this, that it is highly expedient, wherever possible, to secure flat ends for pillars:—with cast iron this is easily done by casting sole-plates at both ends, but even then great care should be taken that they are well bedded and guarded from the effects of unequal settlement of foundations, &c.

Connecting-rods, with jointed ends as usual, must be regarded as pillars with both ends pointed (204).

The piston-rod of a steam-engine may be taken as a pillar, flat at one end, where it is connected to the piston, and further steadied by the gland: the upper end being jointed at the cross-head, is assimilated to a pillar with pointed end.

(151.) "*Cast-iron Pillars.*"—Mr. Hodgkinson's experiments have supplied very full information on the strength of pillars of cast iron; for solid cylindrical pillars we have the following general rules:—

(152.) $$F = M_P \times D^{3\cdot6} \div L^{1\cdot7}.$$

(153.) $$D = \sqrt[3\cdot6]{(F \times L^{1\cdot7} \div M_P)}.$$

(154.) $$L = \sqrt[1\cdot7]{(M_P \times D^{3\cdot6} \div F)}.$$

(155.) $$M_P = F \times L^{1\cdot7} \div D^{3\cdot6}.$$

In which F = the breaking weight of the pillar by flexure in lbs., tons, &c., dependent on M_P.

D = the diameter of the pillar at the centre, in inches.

L = the length of the pillar in feet.

M_P = constant multiplier, the value of which is given in Table 34.

TABLE **34.**—Of the VALUE of M_P, being the THEORETICAL BREAKING WEIGHT OF PILLARS, 1 Foot long.

Material.		Cylindrical Pillars.			Square and Rectangular.		
		Both Ends Pointed.	One Flat, One Pointed.	Both Ends Flat.	Both Ends Pointed.	One Flat, One Pointed.	Both Ends Flat.
Cast Iron ..	lbs.	33,000	66,000	99,000	56,100	112,200	168.300
	tons	14·73	29·46	44·19	25	50	75
Wrought Iron	lbs.	95,848	197,700	299,620	162,900	336,000	498,500
	tons	42·79	88·26	133·8	72·74	150	223
Steel	lbs.	108,500	217,000	325,500	184,400	368,900	553,300
	tons	48·44	96·88	145·3	82·35	164·7	247
Dantzic Oak..	lbs.	6,000	12,000	18,000	9,000	18,000	27,000
	tons	2·68	5·36	8·04	4·02	8·04	12·06
Red Deal ..	lbs.	5,333	10,666	16,000	8,000	16,000	24,000
	tons	2·38	4·76	7·14	3·57	7·14	10·71
Teak	lbs.	11,150	22,300	33,450	16,730	33,460	50,190
Red Pine ..	"	8,520	17,040	25,560	12,780	25,560	38,340
Canadian Oak	"	8,360	16,720	25,080	12,540	25,080	37,620
Deal	"	7,933	15,866	23,800	11,900	23,800	35,700
Ash	"	7,773	15,546	23,319	11,660	23,320	34,980
Beech	"	6,222	12,444	18,666	9,333	18,666	28,000
Pitch Pine ..	"	5,600	11,200	16,800	8,403	16,806	25,209
English Oak	"	5,440	10,880	16,320	8,160	16,320	24,480
Riga Fir ..	"	5,200	10,400	15,600	7,800	15,600	23,400
Larch.. ..	"	4,108	8,216	12,324	6,162	12,324	18,486
Memel Deal	"	4,087	8,174	12,261	6,130	12,260	18,390
Elm	"	3,154	6,308	9,462	4,731	9,462	14,193
Willow ..	"	2,600	5,200	7,800	3,902	7,804	11,706
Cedar	"	2,247	4,494	6,741	3,370	6,740	10,110
		(1)	(2)	(3)	(4)	(5)	(6)

(156.) "*Hollow Cast-iron Pillars.*"—For hollow pillars, instead of $D^{3.6}$, we have $D^{3.6} - d^{3.6}$, in which D = the external,

TABLE 35.—Of the POWERS of NUMBERS for PILLARS, &c.

N	$N^{1\cdot6}$	$N^{2\cdot6}$	$N^{3\cdot6}$	N	$N^{1\cdot6}$	$N^{2\cdot6}$	$N^{3\cdot6}$
..	..	..	..	5 5/8	15·8	89·1	502
1/4	·1088	·0272	·0068	5 3/4	16·4	94·1	543
3/8	·2082	·0781	·0293	5 7/8	17·0	100	587
1/2	·3299	·1650	·0825	6	17·6	105	633
5/8	·4714	·2946	·1842	6 1/8	18·2	111	685
3/4	·6311	·4733	·3550	6 1/4	18·8	117	733
7/8	·8077	·7067	·6184	6 3/8	19·3	123	787
1·0	1·00	1·00	1·00	6 1/2	20·0	130	844
1 1/8	1·21	1·36	1·53	6 5/8	20·6	136	904
1 1/4	1·43	1·79	2·20	6 3/4	21·2	143	967
1 3/8	1·66	2·28	3·15	6 7/8	21·8	150	1033
1 1/2	1·91	2·87	4·30	7	22·5	157	1102
1 5/8	2·18	3·54	5·74	7 1/8	23·2	165	1175
1 3/4	2·45	4·28	7·50	7 1/4	23·8	172	1251
1 7/8	2·73	5·13	9·61	7 3/8	24·5	181	1330
2	3·03	6·06	12·1	7 1/2	25·1	188	1413
2 1/8	3·34	7·08	15·1	7 5/8	25·8	196	1500
2 1/4	3·66	8·23	18·5	7 3/4	26·4	205	1590
2 3/8	4·00	9·50	22·5	7 7/8	27·1	214	1685
2 1/2	4·33	10·8	27·1	8	27·8	223	1783
2 5/8	4·68	12·3	32·3	8 1/8	28·5	232	1882
2 3/4	5·04	13·8	38·2	8 1/4	29·2	241	1992
2 7/8	5·42	15·6	44·8	8 3/8	30·0	251	2102
3	5·80	17·4	52·2	8 1/2	30·6	260	2218
3 1/8	6·20	19·4	60·5	8 5/8	31·4	270	2338
3 1/4	6·60	21·4	69·6	8 3/4	32·2	281	2462
3 3/8	7·00	23·6	79·8	8 7/8	32·9	292	2591
3 1/2	7·42	26·0	90·9	9	33·6	302	2724
3 5/8	7·84	28·4	103·0	9 1/8	34·4	313	2863
3 3/4	8·29	31·1	116	9 1/4	35·1	325	3007
3 7/8	8·73	33·8	131	9 3/8	35·9	337	3158
4	9·19	36·7	147	9 1/2	36·7	348	3310
4 1/8	9·64	39·7	164	9 5/8	37·4	360	3470
4 1/4	10·1	42·9	183	9 3/4	38·2	373	3634
4 3/8	10·6	46·4	203	9 7/8	39·0	385	3805
4 1/2	11·1	49·8	225	10	39·8	398	3981
4 5/8	11·6	53·6	248	10 1/4	41·3	423	4351
4 3/4	12·1	57·6	273	10 1/2	42·9	452	4746
4 7/8	12·6	61·5	300	10 3/4	44·7	480	5165
5	13·1	65·7	328	11	46·4	510	5611
5 1/8	13·6	69·7	359	11 1/4	48·1	541	6083
5 1/4	14·2	74·5	391	11 1/2	49·8	573	6584
5 3/8	14·7	79·1	426	11 3/4	51·5	605	7115
5 1/2	15·3	83·9	463	12	53·3	640	7675

and d = the internal diameter, and the rest being as before, we have the rules :—

(157.) $$F = M_P \times (D^{3 \cdot 6} - d^{3 \cdot 6}) \div L^{1 \cdot 7}.$$

(158.) $$L = \sqrt[1 \cdot 7]{\left\{M_P \times (D^{3 \cdot 6} - d^{3 \cdot 6}) \div F\right\}}.$$

(159.) $$M_P = F \times L^{1 \cdot 7} \div (D^{3 \cdot 6} - d^{3 \cdot 6}).$$

It should be clearly understood, that these rules give the breaking load of long flexible pillars, or those whose length is so great in proportion to their diameter, that they will fail by bending simply. Short pillars require correction for "Incipient Crushing," as explained and illustrated more fully in (163).

(160.) Tables 35, 36 give the 3·6 and 1·7 powers of numbers to facilitate calculations of the strength of solid and hollow pillars of cast iron, wrought iron, and steel:—Thus, say we

TABLE 36.—Of the 1·7 POWER of NUMBERS for CALCULATING the STRENGTH of PILLARS.

Length. L.	$L^{1 \cdot 7}$	Length. L.	$L^{1 \cdot 7}$	Length. L.	$L^{1 \cdot 7}$
0·3	·129	4	10·56	15	99·8
0·4	·211	4¼	11·70	16	111·4
0·5	·308	4½	12·90	17	123·5
0·6	·420	4¾	14·14	18	136·1
0·7	·545	5	15·43	19	149·2
0·8	·684	5½	18·14	20	162·8
0·9	·836	6	21·03	21	176·9
1·0	1·00	6½	24·10	22	191·5
1¼	1·46	7	27·33	23	206·5
1½	1·99	7½	30·73	24	222
1¾	2·59	8	34·30	25	238
2	3·25	8½	38·02	26	254
2¼	3·97	9	41·90	28	286
2½	4·75	9½	45·94	30	324
2¾	5·58	10	50·10	32	362
3	6·47	11	58·94	34	401
3¼	7·42	12	68·33	36	442
3½	8·41	13	78·29	38	485
3¾	9·46	14	88·80	40	529

TABLE 37.—Of the STRENGTH of SOLID and HOLLOW PILLARS
Bed-plates, or by Sole-plates

Diameter at the Centre, in Inches.		Thickness of Metal.	Crushing Weight due to Area, in Tons.	Length of the … 5		6		7		8	
				Breaking Weight,							
Outside.	Inside.			By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.
1½	..	..	76	12·4	12·4	9·0	9·0	7·0	7·0	5·6	5·6
1¾	..	..	103	21·4	21·4	15·7	15·7	12·2	12·2	9·7	9·7
2	..	..	135	34·5	34·5	25·3	25·3	19·7	19·7	15·7	15·7
2¼	..	..	171	53·0	50·3	38·7	38·7	30·0	30·0	24·0	24·0
2½	..	..	211	77·4	69·7	56·7	56·4	44·0	44·0	35·0	35·0
2¾	..	..	255	..	..	80·0	75·6	62·0	62·0	49·7	49·7
3	..	..	304	..	..	..	..	85·0	83·1	67·3	67·3
3	2	½	169	..	..	..	..	65·4	57·7	52·0	49·4
3½	2½	½	203	..	..	..	..	104	68·7	83·0	71·4
4	3	½	236	..	..	..	..	154	110·0	123	96·6
4½	3½	½	270	..	..	..	..	..	..	173	124
4½	3	¾	380	..	..	..	..	..	..	224	168
5	4	½	303	..	..	..	..	..	..	234	154
5	3½	¾	429	..	..	..	..	..	..	307	209
6	5	½	374	..	..	..	..	..	..	395	218
6	4½	¾	533	..	..	..	..	..	..	528	303
6	4	1	677	..	..	..	..	..	..	629	375
7	5¾	⅝	537	..	..	..	..	..	..	724	345
7	5½	¾	655	..	..	..	..	..	..	827	411
7	5	1	812	..	..	..	..	..	..	1003	504
8	6¾	⅝	624	..	..	..	..	..	..	..	..
8	6½	¾	735	..	..	..	..	..	..	..	..
8	6	1	946	..	..	..	..	..	..	..	..
9	7½	¾	835	..	..	..	..	..	..	..	..
9	7	1	1080	..	..	..	..	..	..	..	..
9	6½	1¼	1307	..	..	..	..	..	..	..	..
10	8½	¾	937	..	..	..	..	..	..	..	..
10	8	1	1212	..	..	..	..	..	..	..	..
10	7½	1¼	1475	..	..	..	..	..	..	..	..
12	10½	¾	1135	..	..	..	..	..	..	..	..
12	10	1	1484	..	..	..	..	..	..	..	..
12	9½	1¼	1810	..	..	..	..	..	..	..	..
12	9	1½	2124	..	..	..	..	..	..	..	..

of Cast Iron, the ends being Flat and well supported by Iron cast on both ends.

Pillar, in Feet.

9		10		12		14		16		18		20	
in Tons.													
By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.
..	..	..	..	..	..	..	..	..	..	..	..	..	..
..	..	..	..	..	..	..	..	..	..	..	..	..	..
12·7	12·7	10·6	10·6	..	..	..	..	..	..	..	..	..	..
19·4	19·4	16·2	16·2	..	..	..	..	..	..	..	..	..	..
28·4	28·4	23·8	23·8	..	..	..	..	..	..	..	..	..	..
40·0	40·0	33·6	33·6	..	..	..	..	..	..	..	..	..	..
55·0	55·0	46·0	46·0	33·8	33·8	..	..	..	..	..	..	..	..
42·0	42·0	35·2	35·2	26·0	26·0	..	..	..	..	..	..	..	..
67·0	62·3	56·0	54·8	41·3	41·3	31·6	31·6	25·3	25·3	..	..	..	..
99·0	84·3	83·0	75·1	60·7	60·7	46·9	46·9	37·6	37·6	..	..	..	..
140	110	118	99·2	87·0	80·7	66·0	66·0	53·2	53·2	..	..	..	..
181	151	152	133	112·0	108	86·0	86·0	68·6	68·6	..	..	..	..
190	136	159	125	117	103	89·0	85·8	71·8	71·8	..	..	..	..
248	187	208	169	153	138	117	114	93·9	93·9	..	..	..	..
319	199	268	182	197	154	151	131	121	112	..	..	..	..
427	275	359	252	264	212	202	178	162	153	..	..	..	..
508	338	428	309	315	259	240	217	193	186	..	..	..	..
585	318	490	245	362	254	276	218	222	191	..	..	..	..
670	378	563	350	413	300	316	256	253	223	..	..	..	..
811	464	682	429	501	367	383	313	308	273	..	..	..	..
..	..	718	378	528	331	403	289	323	255	264	225	222	201
..	..	824	440	607	386	464	335	372	296	304	261	253	232
..	..	1012	556	743	484	568	421	457	371	372	326	310	288
..	..	1224	543	855	482	651	425	524	380	427	339	356	303
..	..	1427	689	1050	610	802	537	643	478	524	424	438	379
..	..	1654	943	1219	725	931	639	748	567	608	501	507	446
..	..	1551	645	1140	579	871	519	698	467	572	412	476	378
..	..	1932	823	1421	738	1087	659	871	593	711	532	593	478
..	..	2777	993	1660	885	1270	782	1018	709	831	633	693	568
..	..	2578	835	1892	783	1448	715	1160	654	948	598	792	547
..	..	3250	1106	2390	1014	1826	922	1464	843	1195	768	997	701
..	..	3841	1337	2825	1223	2160	1110	1730	1013	1412	923	1178	841
..	..	4357	1556	3205	1418	2448	1287	1963	1171	1602	1062	1337	969

have a cast-iron pillar 6 inches diameter externally, and 5 inches internally, therefore $\frac{1}{2}$ inch thick, and 14 feet long, with both ends flat.

From Table 34 the value of $M_P = 44 \cdot 19$, say 44 tons; from Table 35, col. 4, the value of $6^{3 \cdot 6} = 633$, and of $5^{3 \cdot 6} = 328$; from Table 36 we obtain 88·8, say 89, for the value of $L^{1 \cdot 7}$ or $14^{1 \cdot 7}$. Then the breaking weight by flexure by rule (157) becomes $44 \times (633 - 328) \div 89 = 151$ tons, which being due to flexure only, will require correction for incipient crushing as shown by (168). Table 37 gives the breaking weight of solid and hollow pillars of cast iron from $1\frac{1}{2}$ to 12 inches diameter, and from 5 to 20 feet long, calculated in the way we have illustrated, the result being there entered as due to flexure, which is corrected for incipient crushing in the next column when necessary. The breaking weight due to flexure is thus given separately in order to adapt the table to conditions other than those where the pillar is flat at both ends: thus, the pillar which we have found to have a strength of 151 tons when both ends were flat, would bear only $151 \div 3 = 50$ tons with both ends rounded, and $50 \times 2 = 100$ tons when one end is flat, and the other rounded, &c., correction being made for incipient crushing in all cases where necessary (163).

(161.) Table 38 gives a selection of all the more important experiments of Mr. Hodgkinson on solid and hollow pillars of cast iron, and in order to show the correctness of the rules in (151), col. 9 has been calculated by them, the value of M_P being taken from Table 34. In col. 7 these results are corrected where necessary for incipient crushing by the method explained in (163), the value of C, or the crushing strain being taken at 49 tons, or 109,760 lbs. per square inch, this being the strength of the particular iron used by Mr. Hodgkinson, as found by him from direct experiment. The *mean* crushing strength of British cast-iron is 43 tons, as shown in (132), and this value should be used in ordinary cases. In col. 8 we have given the error or difference between the calculated and experimental results:—the sum of all the + errors is 163, and of all the − errors, 141·7; hence we have as a general average result of the forty experiments (163 − 141·7)

$\div 40 = +0 \cdot 532$, or $\frac{1}{2}$ per cent. only. It will also be observed that the *range* of the error is nearly equal, the greatest + error being $+ 19 \cdot 8$, and the greatest − error is $- 22 \cdot 1$ per cent. (959).

(162.) As an example of the application of the rules in (157) to cases where exact results are required, as in Table 38, we will take No. 35, in which $D = 2 \cdot 01$ inches, $d = 1 \cdot 415$ inch, and $L = 7 \cdot 395$ feet. We require logarithms for working these rules; then for $D^{3 \cdot 6}$ we have, log. of $2 \cdot 01 = 0 \cdot 303196 \times 3 \cdot 6 = 1 \cdot 091505$, the natural number due to which, or $12 \cdot 35$, is the $3 \cdot 6$ power of $2 \cdot 01$. Similarly, for $d^{3 \cdot 6}$, we have log. of $1 \cdot 415 = 0 \cdot 150756 \times 3 \cdot 6 = \cdot 542721$, the natural number due to which, or $3 \cdot 49$, is the $3 \cdot 6$ power of $1 \cdot 415$. For the $1 \cdot 7$ power of L, we have log. of $7 \cdot 395 = 0 \cdot 868938 \times 1 \cdot 7 = 1 \cdot 477194$, the natural number due to which is 30. The value of M_{P} from Table 34, for pillars with both ends flat, as in our case, is 99,000: with these data, the rule in (157) gives $99{,}000 \times (12 \cdot 35 - 3 \cdot 49) \div 30 = 29230$ lbs. as in cols. 9 and 7; correction for incipient crushing not being necessary in this case.

INCIPIENT CRUSHING.

(163.) If we calculate the strength of a series of pillars with a progressively diminishing length, the calculated strain increases as the length is reduced until it eventually becomes greater than the absolute crushing strength of the material. Obviously, the pillar cannot sustain a load greater than the crushing strain due to the area of the section:—there is therefore a limit to the shortness of pillars, beyond which the rules in (151) do not apply without correction. It might be supposed, that down to a certain length, the pillar would break simply by flexure with the strain given by the rules, and that with any length less than that, the breaking weight would be simply the crushing strain due to the area of the section and the specific strength of the material, irrespective, therefore, of any further reduction of length. But Mr. Hodgkinson found that long before that length was reached there was a falling off in the strength of long pillars, and he was led to the following

TABLE 38.—Of EXPERIMENTS on SOLID and

No. of Experiment.	Form of Ends.	Diameter. External.	Diameter. Internal.	Length. Feet.
1	Two ends round ..	0·5	..	5·04
2	"	0·99	..	"
3	"	1·52	..	"
4	"	1·96	..	"
5	"	0·5	..	2·52
6	"	0·99	..	"
7	"	1·52	..	"
8	"	0·5	..	1·26
9	"	0·99	..	"
10	"	0·497	..	0·63
11	"	0·77	..	"
12	"	0·5	..	0·315
13	Two ends flat ..	0·51	..	5·04
14	"	0·997	..	"
15	"	1·56	..	"
16	"	0·5	..	2·52
17	"	1·01	..	"
18	"	0·51	..	1·26
19	"	1·00	..	"
20	"	0·5	..	0·63
21	"	0·777	..	"
22	"	0·5	..	0·315
23	"	0·52	..	0·1667
24	"	0·52	..	0·08333
25	Two ends round ..	1·78	1·21	7·56
26	"	2·01	1·415	"
27	"	2·24	1·735	"
28	"	2·49	1·89	"
29	"	2·74	2·155	"
30	"	3·01	2·48	"
31	"	3·36	2·63	"
32	"	1·78	1·21	4·75
33	"	1·85	1·36	2·583
34	Two ends flat ..	1·78	1·21	7·395
35	"	2·01	1·415	"
36	"	2·23	1·54	"
37	"	1·26	0·767	2·5208
38	"	1·16	0·7705	1·917
39	"	1·16	0·932	1·2604
40	"	1·13	·91	0·7333
(1)	(2)	(3)	(4)	(5)

HOLLOW CYLINDRICAL PILLARS of CAST IRON.

Breaking Weight.			Calculated Breaking Weight by Flexure. F.	Calculated Crushing Strain due to Area. C_P.	No. of Experiment.
By Experiment. Lbs.	By Calculation.				
	Lbs.	Error per Cent.			
143	172	+19·8	172	21,550	1
1,902	2,036	+ 7·1	2,036	84,490	2
10,861	9,531	−12·2	9,531	199,200	3
24,291	23,800	− 2·0	23,800	331,200	4
539	558	+ 3·5	558	21,550	5
6,105	6,613	+ 8·3	6,613	84,490	6
32,531	30,960	− 4·8	30,960	199,200	7
1,904	1,812	− 4·8	1,812	21,550	8
19,752	21,390*	+ 8·3	21,480	84,490	9
5,262	5,703*	+ 8·4	5,842	21,300	10
22,948	21,685*	− 5·5	28,250	51,110	11
15,107	11,770*	−22·1	19,440	21,550	12
487	560	+15·0	560	22,420	13
6,238	6,260	+ 0·3	6,260	85,690	14
28,962	31,370	+ 8·3	31,370	209,800	15
1,662	1,672	+ 0·1	1,672	21,550	16
20,310	21,300	+ 4·9	21,300	87,930	17
6,764	5,915	−12·5	5,915	22,420	18
40,250	43,800*	+ 8·8	66,780	86,210	19
11,255	11,250*	0·0	17,650	21,550	20
32,007	35,990*	+12·7	87,485	52,040	21
17,468	16,870*	− 3·4	58,270	21,550	22
22,867	21,420*	− 6·3	197,500	23,310	23
24,616	22,996*	− 6·6	1,281,000	23,310	24
5,585	6,340	+13·5	6,340	148,000	25
8,357	9,386	+12·3	9,386	175,700	26
13,341	11,610	−13·0	11,610	173,100	27
19,855	17,800	−10·4	17,800	226,500	28
27,883	23,090	−17·2	23,090	246,900	29
26,707	28,090	+ 5·2	28,090	319,400	30
50,477	48,710	− 3·5	48,710	377,000	31
13,693	13,970	+ 2·0	13,970	148,000	32
33,763	32,030	− 7·9	32,030	135,500	33
17,840	19,750	+10·7	19,750	148,000	34
28,353	29,230	+ 3·4	29,230	175,700	35
40,569	43,580	+ 7·4	43,580	224,200	36
33,679	32,380*	− 3·8	39,330	85,600	37
30,383	30,450*	+ 0·2	43,070	64,830	38
26,729	27,480*	+ 2·8	62,120	41,120	39
34,037	32,100*	− 5·7	141,060	38,690	40
(6)	(7)	(8)	(9)	(10)	(1)

reasoning as to the cause. Considering the pillar as having two functions, one to support the direct crushing weight, and the other to resist flexure; when the pressure necessary to break the pillar is very small because of its great length in proportion to its diameter, then the whole strength of the material may be considered as employed in resisting flexure. When the breaking weight is half of that required to crush the material, one half only of the strength may be considered as available for resistance to flexure, the other half being employed in resisting crushing. When, through the shortness of the pillar, the breaking weight is nearly equal to the crushing strain, we may consider that no part of the strength of the pillar is applied to resist flexure, &c. It was found by experiment, that when the load on a pillar was $\frac{1}{4}$ only of the crushing strain, there was a sensible falling off in the strength as calculated by the rules in (151), due therefore to "Incipient" rather than *absolute* Crushing.

As the combined result of reasoning and experiment, Mr. Hodgkinson gives the rule:—

(164.) $$P_C = F \times C_P \div (F + \tfrac{3}{4} C_P).$$

In which F = the breaking weight by flexure as due by the rules in (151) (156), &c.

C_P = the crushing strain due to the area of the section and the specific strength of the material.

P_C = the reduced actual breaking weight; all in the same terms.

This rule requires some caution in its application; where F is less than $\frac{1}{4} C_P$, the effect of it would be to make the calculated strength *greater* than F. Now, the strength of a pillar can never be greater than is due to flexure, hence there is a limit beyond which the rule must not be applied:—when F is exactly $\frac{1}{4} C_P$, the effect of the rule is nil: when F is greater than $\frac{1}{4} C_P$, the rule is necessary, and will reduce the calculated strength of the pillar as due by flexure:—when F is less than

$\frac{1}{4}$ C_P the rule will give the erroneous result of making P_c greater than F.

Thus, say $C_P = 80$ and $F = 20$, or exactly $\frac{1}{4}$ C_P; then, $80 \times \cdot 75 = 60$, and the rule (164) gives $P_c = 20 \times 80 \div (20 + 60) = 20$, or the same value as F, the effect of the rule being nil. Again, say $C_P = 80$, $F = 10$, then $P_c = 10 \times 80 \div (10 + 60) = 11 \cdot 43$, which is *greater* than 10, or the value of F, and is impossible, showing that the rule has been applied in a case where it was not admissible. Again, say $F = 30$, and $C_P = 80$ as before:—then the rule gives $P_c = 30 \times 80 \div (30 + 60) = 26 \cdot 67$ tons, which is less than 30, the value of F, and is a correct result.

(165.) We may now search for the lengths of pillars, with which the correction given by this rule becomes nil, which will happen when the length is such that F or the breaking weight by flexure is $\frac{1}{4}$ C_P.

The mean crushing strain of cast iron is 43 tons per square inch as given in (132); a pillar 1 inch diameter will be crushed with $43 \times \cdot 7854 = 33 \cdot 73$ tons, and the required length of pillar will be that which breaks by flexure with $33 \cdot 77 \div 4 = 8 \cdot 44$ tons. For pillars with both ends pointed, the value of M_P as given by Table 34, is $14 \cdot 73$ tons, and the rule (154), namely, $L = \sqrt[1 \cdot 7]{(M_P \times D^{3 \cdot 6} \div W)}$, becomes in our case $(14 \cdot 73 \times 1 \div 8 \cdot 44)^{\frac{1}{1 \cdot 7}} = 1 \cdot 387$ feet, or $16 \cdot 64$ inches. The length with which the correction becomes nil is therefore in this case $16 \cdot 64$ times the diameter. Similarly, with one end flat, and the other pointed, the value of M_P being in that case $29 \cdot 46$ tons by Table 34; we have $(29 \cdot 46 \times 1 \div 8 \cdot 44)^{\frac{1}{1 \cdot 7}} = 2 \cdot 086$ feet, or 25 inches, the length being thus 25 times the diameter. With both ends flat $M_P = 44 \cdot 19$ tons, and the length comes out $(44 \cdot 19 \times 1 \div 8 \cdot 44)^{\frac{1}{1 \cdot 7}} = 2 \cdot 648$ feet, or $31 \cdot 78$ inches, the length being thus $31 \cdot 78$ times the diameter.

(166.) Mr. Hodgkinson, adopting 49 tons per square inch as the crushing strain of the particular iron used in his experiments, gives the length at 15 times the diameter for pillars with both ends pointed, and 30 times the diameter in those with both ends flat.

These ratios are, however, not constant for all diameters, as is

shown by Table 39, which has been calculated for cast-iron pillars by the rule:—

$$(167.)\quad L_D = \sqrt[1.7]{(M_P \times D^{3.6} \div \tfrac{1}{4} C_P)} \times 12 \div D.$$

In which L_D = the length in *terms of the diameter* with which correction for incipient crushing is nil; C_P = the crushing strain due to the area of section and the specific crushing strength of cast iron, or 43 tons; M_P = the multiplier for pillars, given by Table 34; and D = the diameter in inches. This table shows that the ratio of length to the diameter is reduced as the diameter is increased, in the case of cast iron considerably, and still more so with wrought-iron pillars (202).

TABLE 39.—Of the LENGTH OF CYLINDRICAL PILLARS, *in terms of the Diameter* with which correction for "Incipient Crushing" becomes *nil*.

Diameter.	2 Ends Pointed.	1 Flat. 1 Pointed.	2 Ends Flat.
	CAST IRON.		
1	16·64	25·04	31·78
2	16·08	24·01	30·47
3	15·60	23·48	29·78
6	14·98	22·55	28·60
	WROUGHT IRON.		
1	40·64	58·37	71·85
2	35·35	50·75	62·50
3	32·62	46·87	57·68
6	28·40	40·79	50·22
	STEEL.		
1	26·14	36·96	45·26
2	22·73	32·15	38·87
3	20·98	29·68	36·34
6	18·27	27·04	33·12

(168.) As an example of the application of the rule (164) we may take the pillar 6 inches diameter, $\frac{1}{2}$ inch thick, and 14 feet long, which we found in (160) to break by flexure with 151 tons. By a table of areas, 6 inches = 28·3, and 5 inch = 19·6, hence the area of the annulus = 28·3 − 19·6 = 8·7 square inches, and the mean crushing strength of cast iron being 43 tons per square inch (132), we obtain 8·7 × 43 = 374 tons for the value of C_P, and 374 × $\frac{3}{4}$ = 280 for $\frac{3}{4}$ C_P. Then the rule (164) becomes $P_C = 151 \times 374 \div (151 + 280) = 131$ tons, the reduced breaking weight.

(169.) The fact (164) that the correction for incipient crushing is necessary for those cases only where F is greater than $\frac{1}{4}$ C_P, supplies an easy method of finding beforehand where it is required, so as to save the labour of going through the whole calculation. Thus, taking No. 34 in Table 38, col. 10 gives 148,000 lbs. for the value of C_P, then $\frac{1}{4}$ C_P becomes 148000 ÷ 4 = 37,000; by col. 9, F = 19750 lbs., which is *less* than $\frac{1}{4}$ C, therefore the correction for incipient crushing is not necessary, and the breaking load of the pillar is simply that due by flexure. Again, in No. 40, col. 10 gives C_P = 38690 lbs., $\frac{1}{4}$ C_P becomes 38690 ÷ 4 = 9672 lbs., but F by col. 9 is 141,060 lbs., which being greater than $\frac{1}{4}$ C_P, requires correction by the rule in (164) by which we obtain 32,100 lbs. for the reduced breaking load, as in col. 7.

The effect of the application of this rule is in some cases remarkably great: for instance in Table 55, No. 9, col. 9 gives 10,750,000 lbs. for the value of F, but P_C = 79380 lbs. only by col. 7, or $\frac{1}{135}$th of F.

The rule for incipient crushing applies not only to cast-iron pillars, but equally to all other materials. It is used for wrought iron, steel, and timber in Tables 44, 57, and its correctness is proved by the general agreement of the calculations with the experimental results as shown by (959) and Table 150, that agreement being to a great extent due to the use of the rule.

(170.) "*Square Pillars of Cast Iron.*"—It is shown in (359) that the theoretical ratio of the strengths of square to round bars, either as pillars or beams, is 1·7 to 1·0, but the experimental

ratio for beams of cast iron is 1·5 to 1·0 (361), and we might suppose that the same ratio would apply to pillars also, but the experiments we have seem to show that the theoretical ratio is more correct. Admitting the theoretical ratio 1·7 to 1·0 we obtain for square and rectangular pillars the modified values of M_P in Table 34. Putting S for the side of the square and the rest as in (151), we have the following general rules for the resistance of square pillars of cast iron to flexure, irrespective of incipient crushing (163). For solid square pillars:—

(171.) $$F = M_P \times S^{3\cdot 6} \div L^{1\cdot 7}.$$

(172.) $$S = \sqrt[3\cdot 6]{(F \times L^{1\cdot 7} \div M_P)}.$$

(173.) $$L = \sqrt[1\cdot 7]{(M_P \times S^{3\cdot 6} \div F)}.$$

In which S = the side of the square pillar, and the rest as in (155). For hollow pillars these rules become

(174.) $$F = M_P \times (S^{3\cdot 6} - s^{3\cdot 6}) \div L^{1\cdot 7}.$$

(175.) $$(S^{3\cdot 6} - s^{3\cdot 6}) = F \times L^{1\cdot 7} \div M_P.$$

(176.) $$L = \sqrt[1\cdot 7]{\left\{M_P \times (S^{3\cdot 6} - s^{3\cdot 6}) \div F\right\}}.$$

In which S = the side of the square externally, s = the side of the square internally, and the rest as before, in (155).

(177.) "*Rectangular Pillars of Cast Iron.*"—A rectangular pillar, other than square, will fail by bending in the direction of its least dimension, and in that case, the strength will be simply proportional to the breadth or greatest dimension. For long pillars failing simply by flexure, the rules become:—

(178.) $$F = M_P \times t^{2\cdot 6} \times b \div L^{1\cdot 7}.$$

(179.) $$t = \sqrt[2\cdot 6]{\left\{F \times L^{1\cdot 7} \div (M_P \times b\right\}}.$$

(180.) $$b = F \times L^{1\cdot 7} \div (M_P \times t^{2\cdot 6}).$$

(181.) $$L = \sqrt[1\cdot 7]{(M_P \times t^{2\cdot 6} \times b \div F)}.$$

In which t = the thickness or least dimension of a rectangular pillar.

b = the breadth or greatest dimension of a rectangular pillar.

And the rest as in (155). The value of $t^{2\cdot6}$ may be found from col. 3 of Table 35; of $L^{1\cdot7}$ from Table 36; and of M_P from Table 34.

For hollow rectangular pillars the rules are modified, and become:—

(182.) $F = M_P \times \{t^{2\cdot6} \times b) - (t_0^{2\cdot6} \times b_0\} \div L^{1\cdot7}$.

(183.) $L = \sqrt[1\cdot7]{\{M_P \times (t^{2\cdot6} \times b] - [t_0^{2\cdot6} \times b_0) \div F\}}$.

In which t is the external, and t_0 the internal least dimension of the hollow rectangular pillar, or rather the dimensions measured in the direction in which the pillar will fail by bending. It is necessary to make this distinction in hollow pillars because it is possible that the least internal dimension may not coincide in direction with the least outside dimension:— for instance, in Fig. 40, flexure would take place in the direction of the arrow A rather than in that of B; then $t_0 = 3$, and $b_0 = 2$, and in that case t_0 is the greater, not the lesser dimension. We can easily determine in such a case, in which direction the pillar will bend, which will be the one where $(t^{2\cdot6} \times b) - (t_0^{2\cdot6} \times b_0)$ is the least: thus in Fig. 40, we obtain from Table 35, $4^{2\cdot6} = 36\cdot7$, and $3^{2\cdot6} = 17\cdot4$ and in the direction A, we have $(36\cdot7 \times 5) - (17\cdot4 \times 2) = 148\cdot7$. In the other direction B, $5^{2\cdot6} = 65\cdot7$, and $2^{2\cdot6} = 6\cdot06$, and we obtain $(65\cdot7 \times 4) - (6\cdot06 \times 3) = 244\cdot62$, or pretty nearly double that in the other direction A; the pillar will therefore certainly fail in the direction A.

(184.) As an example of the application of the rules in (177) say we take a pillar $1\frac{1}{2} \times 7$ inches, 14 feet long, with both ends flat: the value of M_P from Table 34 is 75 tons; of $1\frac{1}{2}^{2\cdot6}$ from Table 35 is 2·87, and of $14^{1\cdot7}$ from Table 36 is 88·8. Then $F = 75 \times 2\cdot87 \times 7 \div 88\cdot8 = 16\cdot96$ tons $= F$. This will not require correction for incipient crushing, for the area being $1\cdot5 \times 7 = 10\cdot5$ square inches, C_P becomes $10\cdot5 \times 19 = 199\cdot5$ tons, and $\frac{1}{4}$ $C_P = 49\cdot875$ tons. F being less than that, the correction is not required.

Again, say we require the thickness t, for a pillar 9 inches wide, 18 feet long, both ends flat, and 15 tons breaking weight.

Then M_P being as before = 75, and $18^{1\cdot7} = 131\cdot1$ by Table 36, we have $15 \times 131\cdot1 \div (75 \times 9) = 2\cdot913$, the nearest number to which in col. 3 of Table 35 is $2\cdot87$, which is opposite $1\frac{1}{2}$ inch, the thickness required, &c.

(185.) "*Cast-iron Pillars of + Section.*"—This form of pillar is commonly used for the connecting-rods of large steam-engines, and not unfrequently for carrying the floors of warehouses. The strength of such pillars may be found by a modification of the rules for rectangular pillars in (177). Let Fig. 38 be the section of such a pillar 8 × 8 inches, $\frac{7}{8}$ inch thick, 15 feet long, with both ends flat; assuming that the pillar will fail by flexure in the direction of the arrow B, we have virtually *two* pillars, one *a*, *a*, *forced* to bend in the direction of its larger dimension, flexure in the contrary direction being prevented by the ribs *c*, *c*. We have in that case $t = 8$ inches, and $b = \frac{7}{8}$ inch; the other pillar *c*, *c*, will bend in the normal direction, or that of its smaller dimension, t being $\frac{7}{8}$ inch, and $b = 8 - \frac{7}{8} = 7\frac{1}{8}$ inches.

(186.) By col. 3 of Table 35, $8^{2\cdot6} = 223$, and $\frac{7}{8}^{2\cdot6} = \cdot707$; by Table 36, $15^{1\cdot7} = 99\cdot8$, and by Table 34, $M_P = 75$. Then for *a*, *a*, we have $223 \times \frac{7}{8} = 195$, and for *c*, *c*, $\cdot707 \times 7\cdot125 = 5$:—the sum of the two is $195 + 5 = 200$, and it will be observed that *c*, *c*, adds only $2\frac{1}{2}$ per cent. to the strength, being 5 on a total of 200 (243). Having thus found the combined value of $t^{2\cdot6} \times b = 200$, the rule in (178) becomes $F = 75 \times 200 \div 99\cdot8 = 150$ tons, the breaking weight by flexure. In order to ascertain whether correction for incipient crushing (163) is necessary, we find the area of the section to be $15\cdot23$ square inches, and the crushing strength of cast iron being 43 tons per square inch (132), C_P becomes $15\cdot23 \times 43 = 655$ tons, hence $\frac{1}{4}\,C_P = 164$ tons; F, or 150 tons, being *less* than $\frac{1}{4}\,C_P$, the correction for incipient crushing is not necessary in this case (169), the correct breaking weight of the whole pillar is that due by flexure simply, or 150 tons.

(187.) We found in (186) that the cross ribs *c*, *c*, contributed only $2\frac{1}{2}$ per cent. to the strength of the pillar Fig. 38, hence there would be no appreciable error if we omit them altogether in calculating the strength, which then becomes simply that due

to the pillar *a*, *a*, forced to bend in the direction of its larger dimension. Table 40 has been calculated on that principle. This, however, is true for those cases only where the length of the pillar is so great that it fails by flexure only; with short pillars requiring correction for incipient crushing, the cross ribs *c*, *c*, yield their full share of resistance to the load.

It should be observed, that the breaking weight of a rectangular pillar like *a*, *a*, breaking by flexure in the direction of its *larger* dimension, will be simply proportional to its thickness, other things being the same; for instance, in Fig. 38, the breaking weight when $\frac{7}{8}$ inch thick being 150 tons, with $\frac{7}{16}$ inch thick, it would be 75 tons, and with $1\frac{3}{4}$ inch thick = 300 tons, &c.

TABLE 40.—Of the STRENGTH of CAST-IRON PILLARS of + SECTION, the ends being Flat and well supported.

Sizes of Pillar.		Length of the Pillar in Feet.						
Breadth.	Thickness of Metal.	6	7	8	9	10	12	15
		Reduced Breaking Weight in Tons.						
5 × 5	$\frac{5}{8}$	110	93·8	80·8	70·2	..	..	..
"	$\frac{3}{4}$	132	112	96·8	84·1	..	..	..
"	$\frac{7}{8}$	154	131	129	98·2	..	..	..
6 × 6	$\frac{3}{4}$	185	161	140	124	110	..	..
"	$\frac{7}{8}$	216	188	163	145	128	..	..
"	1	246	215	186	165	146	..	..
7 × 7	$\frac{7}{8}$	..	248	221	197	176	143	..
"	1	..	284	252	225	201	164	..
"	$1\frac{1}{8}$	..	320	283	253	227	184	..
8 × 8	1	..	..	323	291	261	216	..
"	$1\frac{1}{8}$	..	..	363	327	294	243	..
"	$1\frac{1}{4}$	..	..	404	364	326	270	..
9 × 9	$1\frac{1}{8}$	..	..	447	407	370	309	240
"	$1\frac{1}{4}$	..	..	497	452	411	343	267
"	$1\frac{3}{8}$	..	..	546	497	452	378	293
10 × 10	$1\frac{1}{4}$	..	..	592	543	500	422	332
"	$1\frac{1}{2}$	..	..	710	651	600	506	398
"	$1\frac{3}{4}$	..	..	829	760	700	591	465
12 × 12	$1\frac{1}{2}$	..	..	956	889	826	715	578
"	$1\frac{3}{4}$	..	..	1115	1037	964	834	674
"	2	..	..	1275	1185	1101	953	771

(188.) "*Relative Strength.*"—The fact that nearly half the section of a + pillar goes for nothing, will prepare us to find that this form of section is very uneconomical as compared with a cylindrical pillar of the same diameter, and area of section. Thus a pillar 8 inches external and $6\frac{5}{8}$ inches internal diameter will have an area of 15·8 square inches, or practically the same as that of Fig. 38, which we found (186) to be 15·23 square inches. Then from Table 35, $8^{3\cdot6} = 1783$, and $6\frac{5}{8}^{3\cdot6} = 904$, and M_P being 44·19, the rule in (157) gives $44\cdot19 \times (1783 - 904) \div 99\cdot8 = 380$ tons breaking weight by flexure. This, however, will require correction for incipient crushing by the rule (164); $C_P = 15\cdot8 \times 43 = 680$ tons, hence $\frac{3}{4}\,C_P = 510$ tons, and the rule $P_C = F \times C_P \div (F + \frac{3}{4}\,C_P)$ becomes $380 \times 680 \div (380 + 510) = 290$ tons, which is nearly double 150 tons, the breaking weight of a + pillar of the same weight of metal.

This ratio, however, is not constant; with a great length, such that both pillars would fail by flexure simply, the ratio in our case would be as we have seen, 380 to 150, or about $2\frac{1}{2}$ to 1; with a length of 15 feet, 2 to 1; as the length is reduced the ratio approaches equality, and with *very* short pillars in which the strength is governed almost exclusively by resistance to crushing, the two kinds of pillar become practically equal.

(189.) Mr. Hodgkinson made an experiment on a pillar of + section $3 \times 3 \times \cdot48$ inch thick, Fig. 36, the length being 7·562 feet, and both ends pointed; the breaking weight was 17,578 lbs. Calculating as in (186), and taking M_P at 56,100 lbs. from Table 34, we have for *a*, *a*, $3^{2\cdot6} = 17\cdot4$ by Table 35, then to find $L^{1\cdot7}$, we must use logarithms. The log. of $7\cdot562 = 0\cdot878637 \times 1\cdot7 = 1\cdot4936829$, the natural number due to which, or 31·17, is the 1·7 power of 7·562:—then $56100 \times 17\cdot4 \times \cdot48 \div 31\cdot17 = 15035$ lbs. For *c*, *c*, we have to find $\cdot48^{2\cdot6}$; the log. of $\cdot48 = \bar{1}\cdot681241 \times 2\cdot6 = \bar{1}\cdot1712266$, the natural number due to which, or ·1483, is the 2·6 power of ·48; then $b = 3 - \cdot48 = 2\cdot52$, and the rule becomes $56100 \times \cdot1483 \times 2\cdot52 \div 31\cdot17 = 673$ lbs. The sum of the two, or $15035 + 673 = 15708$ lbs., is the breaking load of the entire pillar, which is 10·6 per cent. less than 17,578 lbs., the experimental breaking weight.

(190.) "*Connecting-rods of Steam-engines.*"—The strength of cast-iron connecting-rods of the ordinary + sectional form cannot be calculated satisfactorily by the ordinary rules; in practice it is necessary to provide for extraordinary strains arising from forces in motion, &c., which the ordinary rule does not contemplate. The safer course is to use a theoretical formula with a constant multiplier derived from experience: the rule may then take the following form:—

(191.) $$B = \sqrt[4]{(H \times L^2 \div \cdot 42)}.$$

In which H = the reputed or nominal horse-power of the engine; L = the length of the connecting-rod between centres in feet; and B = the breadth of the rod at the centre in inches. Thus, for a 60-horse engine with a rod 14 feet $6\frac{1}{4}$ inches, or 14·52 feet between centres, we have $14 \cdot 52^2 = 210 \cdot 8$; then $60 \times 210 \cdot 8 \div \cdot 42 = 30114$, the log of which, or $4 \cdot 478773 \div 4 = 1 \cdot 119693$, the natural number due to which or 13·17, say $13\frac{1}{8}$ inches, is the breadth at the centre. We should obtain the same result by finding the square-root of the square-root of 30114: thus, the square-root of 30114 = 173·5, and the square-root of 173·5 = 13·17 inches as before. Table 41 gives the proportions of connecting-rods from cases in practice, col. 4 being calculated

TABLE 41.—Of the PROPORTIONS of CAST-IRON CONNECTING-RODS of + SECTION for STEAM-ENGINES: Cases in Practice.

Nominal Horse-power.	Length of Rod between Centres of Bearings.		Breadth at the Centre in Inches.		Sizes of Bearings.				Thickness of Ribs.
					Lower end, or Crank-pin.		Two, at the upper end.		
			Actual.	By Rule.	Diam.	Length.	Diam.	Length.	
	ft.	in.							
100	19	5	$17\frac{1}{4}$	17·3	$6\frac{3}{4}$	8	$5\frac{1}{4}$	$3\frac{1}{4}$	..
60	14	$6\frac{1}{4}$	13	$13\frac{1}{8}$	$4\frac{3}{4}$	$6\frac{1}{4}$	4	$5\frac{1}{2}$	..
42	13	$5\frac{1}{2}$	11	$11\frac{5}{8}$	$4\frac{1}{4}$	5	$3\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{1}{8}$
35	13	$3\frac{3}{8}$	11	11	4	5	$3\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{1}{8}$
30	11	$6\frac{1}{4}$	$8\frac{1}{2}$	$9\frac{1}{8}$	$3\frac{1}{2}$	$4\frac{3}{4}$	3	$3\frac{1}{2}$	$\frac{13}{16}$
22	9	$11\frac{1}{4}$	$8\frac{1}{2}$	$8\frac{1}{2}$	$3\frac{1}{16}$	$4\frac{1}{2}$	3	$3\frac{1}{2}$	$\frac{13}{16}$
12	8	$5\frac{1}{2}$	$7\frac{1}{2}$	$6\frac{3}{4}$	$2\frac{3}{4}$	$4\frac{1}{2}$	$2\frac{3}{8}$	$2\frac{1}{2}$	$\frac{3}{4}$
(1)	(2)		(3)	(4)	(5)	(6)	(7)	(8)	(9)

by the rule: the particulars of the bearings at the two ends are added as useful memoranda.

It should be observed that the rule supposes that the thickness of the ribs and the heavy mouldings with which the corners are filled in, are of the proportions usually adopted in practice and as shown in Fig. 37.

(192.) "*Cast-iron Pillars of* I *Section.*"—This form of pillar is sometimes used for stanchions: their strength may be calculated on the same principles as those of + section. Mr. Hodgkinson made an experiment on the pillar of the section shown by Fig. 39, the length was 7·562 feet, both ends pointed, and the breaking weight 29,571 lbs., the pillar breaking by flexure in the direction of the arrow C. Neglecting the middle web, as having very little influence on the result (187), we have simply to calculate for a rectangular pillar $3 \times \cdot 7$ inches forced to fail by flexure in the direction of its larger dimension, hence, using the rule in (178) $t = 3$ inches and $b = \cdot 7$ inch, and $3^{2 \cdot 6}$ being 17·4, we obtain $56100 \times 17 \cdot 4 \times \cdot 7 \div 31 \cdot 17 = 21925$ lbs., which is 25·8 per cent. less than 29,571 lbs., the experimental breaking weight.

It should be observed that the thickness of the metal in Fig. 39 has been *calculated*. Mr. Hodgkinson did not give that dimension unfortunately, but he states that the *area* of the cross-section was the same as that of the + pillar in (189); if we assume the thickness to be uniform all over, we obtain of necessity ·35 inch, as in the figure.

It is possible that the thinness of the metal in these cases (931) may be the reason for the excess of strength shown by the experiments; this appears to be the more probable from the fact that the casting ·35 inch thick gave a greater excess than the one ·48 inch thick. The difference although considerable is not of practical importance, being covered by the "factor" of safety (880); moreover the error is on the side of safety, calculation giving in both cases, the breaking weight *less* than by experiment.

(193.) "*Cast-iron Steam-engine Columns.*"—A common arrangement for beam engines is shown by Fig. 41, in which a cross-entablature A is built into the side walls, and is supported by

two columns. The strain on these columns is comparatively small, and the proper sizes cannot be calculated by the ordinary method, but may be found by the following empirical rules:—

(194.) $D = \sqrt[4]{(H \times L^2 \times 2 \cdot 2)}$.

(195.) $d = D \times 5 \div 6$.

In which H = the reputed, or nominal horse-power of the engine; L = the length of the column in feet; D = the diameter at the base, and d = the diameter at the top, both in inches. Thus for an engine of 100 nominal horse-power, and columns 16 feet long, we have $16^2 = 256$, giving by the rule $100 \times 256 \times 2 \cdot 2 = 56320$, which is the fourth power of D: then the log. of 56320 or $4 \cdot 750663 \div 4 = 1 \cdot 187666$, the natural number due to which is 15·4, or say $15\frac{3}{8}$ inches, the diameter of the column at the base, from which we obtain $15 \cdot 4 \times 5 \div 6 = 12 \cdot 83$, or say $12\frac{7}{8}$ inches, the diameter at the top. The actual diameters were $15\frac{1}{2}$ and 13 inches respectively, as shown by Table 42, which gives the sizes of engine columns from cases in practice with the corresponding sizes calculated by the rule. It should be observed that we might have found the 4th root of 56320 without the use of logarithms, for the square-root of the square-root of a number is the 4th root of that number; thus, the square-root of $56320 = 237 \cdot 3$, and the square-root of $237 \cdot 3 = 15 \cdot 4$, or the same as found direct by logarithms.

TABLE 42.—Of the DIAMETER of COLUMNS to BEAM ENGINES with CROSS-ENTABLATURE between the WALLS—Two Columns to each Engine (Fig. 41). From Cases in Practice.

Nominal Horse-power of Engine.	Length of the Columns.		Diameter of the Column at Base.		Diameter of the Column at Top.	
			Actual.	Calculated.	Actual.	Calculated.
	ft.	in.				
100	16	0	$15\frac{1}{2}$	$15\frac{3}{8}$	13	$12\frac{7}{8}$
60	12	3	$11\frac{3}{4}$	$11\frac{7}{8}$	10	10
40	10	3	10	$9\frac{13}{16}$	8	$8\frac{1}{8}$
30	9	$1\frac{5}{8}$	$7\frac{1}{4}$	$8\frac{5}{8}$	6	$7\frac{1}{8}$
20	7	9	$6\frac{3}{4}$	$7\frac{3}{16}$	$5\frac{1}{2}$	6
12	7	6	$6\frac{1}{8}$	$6\frac{3}{16}$	5	$5\frac{1}{8}$

Table 43 has been calculated by the rule, and the approximate depth of the cross-entablature is given as calculated by the rule in (953).

TABLE 43.—Of the APPROXIMATE SIZES of COLUMNS and DEPTH of CROSS-ENTABLATURE for BEAM ENGINES: two columns to each Engine, as in Fig. 41.

Nominal Horse-power.	Depth of Entablature in Inches.	Length of Column in Feet.								
		6	8	10	12	14	16	18	20	22
		Diameter of Columns at the Base, in Inches.								
10	6·3	5·3	6·1	6·8	..	..	..	..	..	..
20	9·0	6·3	7·3	8·1	8·4	..	..	..	..	..
30	11·0	7·0	8·1	9·0	9·3	10·6	..	..	..	..
40	12·6	..	8·7	9·7	10·0	11·4	12·2	..	..	..
50	14·1	..	..	10·2	10·6	12·0	12·9	..	..	..
60	15·5	..	..	10·7	11·7	12·6	13·5	14·3	..	..
80	18·0	..	..	11·5	12·6	13·6	14·6	15·4	16·3	..
100	20·0	..	..	..	13·3	14·3	15·4	16·3	17·2	..
120	22·0	..	..	..	..	15·0	16·1	17·1	18·0	..
140	23·6	..	..	..	..	15·6	16·7	17·7	18·7	19·6
160	25·2	..	..	..	..	..	17·3	18·3	19·3	20·3
180	26·8	..	..	..	..	..	17·8	18·9	19·9	20·9
200	28·2	..	..	..	..	..	..	19·4	20·5	21·4

NOTE.—The diameter at the top should be $\frac{3}{4}$ths of the diameter at the base.

(196.) "*Wrought-iron Cylindrical Pillars.*"—The strength of pillars of wrought iron is directly proportional to the 3·6 power of the diameter or side of square pillars, and inversely as the square of the length, this latter being the theoretical ratio as shown by (145). For solid cylindrical sections we have the following general rules for long pillars failing simply by flexure:—short pillars require correction for incipient crushing by the rules in (163).

(197.) $$F = M_P \times D^{3\cdot6} \div L^2.$$

(198.) $$D = \sqrt[3\cdot6]{(F \times L^2 \div M_P)}.$$

(199.) $$L = \sqrt[2]{(M_P \times D^{3\cdot6} \div F)}.$$

(200.) $$M_P = F \times L^2 \div D^{3\cdot6}.$$

In which F = the breaking weight on the pillar in lbs., tons, &c., by flexure, dependent on M_P.

D = the diameter of the pillar at the centre, in inches.

L = the length in feet.

M_P = constant multiplier, the value of which is given in Table 34.

Table 44 gives the result of 27 experiments on solid cylindrical pillars of wrought iron by Mr. Hodgkinson; col. 9 has been calculated by the rules, the value of M_P taken from Table 34 was 95,848 lbs. for pillars with both ends pointed; 197,700 for those with one end pointed and the other flat; and 299,620 lbs. for those with both ends flat.

(201.) Comparing cols. 9 and 7, it will be seen that many of these pillars require correction for incipient crushing by the rule (164), namely, $P_C = F \times C_P \div (F + \frac{3}{4} C_P)$. It is a matter of considerable difficulty to determine the crushing strain for wrought iron, or indeed for any very malleable metal (133). The ordinary method of crushing a small specimen is quite inapplicable in such a case:—by experiments on the transverse strength in (520) we found it to be 24 tons per square inch. But the only satisfactory course is to find by trial the resistance to crushing, or value of C, which when used in the rule for incipient crushing (164) will bring the calculated strength into agreement with the experimental strength. The result of a laborious application of that tentative method is that the value of C in wrought-iron pillars is 19 tons, or 42,560 lbs. per square inch, which, multiplied by the area of the pillar, will give the value of C_P in the rule for incipient crushing.

This value of C has been used for solid cylindrical pillars in Table 44; for solid rectangular pillars in Table 53; for hollow cylindrical pillars of thin plate-iron in Table 52; and for rectangular pillars of thin iron in Table 55; and its correctness is proved by the general agreement of the calculations with the experiments as shown by (958) and Table 150. The mean average error of those 4 tables is only 0·293, 0·0, 0·461, and 2·25 per cent. respectively:—of 99 experiments, 85 were reduced by the rule for incipient crushing, and the near agreement with

TABLE 44.—Of the STRENGTH of SOLID CYLINDRICAL PILLARS of WROUGHT IRON and CAST STEEL.

Number of Experiment.	Form of Ends.	Length. Feet.	Diameter. Inches.	Breaking Weight. By Experiment. Lbs.	Breaking Weight. By Experiment. Mean. Lbs.	Breaking Weight. By Calculation. Lbs.	Breaking Weight. By Calculation. Error. Per Cent.	Calculated. Breaking Weight by Flexure. F.	Calculated. Crushing Strain due to Area. C_p.
1	2 ends round ..	7·56	1·02	..	1,825	1,801	− 1·3	1,801	34,770
2	" ..	"	1·015	..	1,792	1,765	− 1·5	1,765	34,440
3	" ..	5·04	1·015	3,812	3,938	3,970	+ 0·8	3,970	34,440
4	" ..	"	"	4,064					
5	" ..	2·52	1·015	14,869					
6	" ..	"	"	16,213	15,480	13,110	−15·3	15,880	34,440
7	" ..	"	"	15,317					
8	" ..	2·52	0·52	1,330					
9	" ..	"	"	1,288	1,260	1,433	+13·8	1,433	17,380
10	" ..	"	"	1,162					
11	" ..	1·67	1·015	23,312	20,834	19,630	− 5·8	36,160	34,440
12	" ..	"	"	18,357					
13	" ..	1·26	1·005	23,395	23,535	23,910	+ 1·6	61,470	33,760
14	" ..	"	"	23,675					

[redacted]

15	1 round, 1 flat ..	7·56	1·02	..	3,355	3,715	+10·7	3,715	34,770
16	" " ..	5·04	1·03	..	8,137	8,657	+ 6·4	8,657	35,460
17	" " ..	2·52	1·015	21,355					
18	" " ..	"	"	22,699	21,243	19,260	− 9·1	32,760	34,440
19	" " ..	"	"	19,675					
20	" " ..	1·26	1·005	28,075	26,227	28,140	+ 7·3	126,800	33,760
21	" " ..	"	"	24,379					
22	2 ends flat ..	7·56	1·02	..	5,280	5,630	+ 6·6	5,630	34,770
23	" ..	5·04	1·02	..	12,990	11,370	−12·5	12,670	34,770
24	" ..	2·52	1·015	23,371	24,379	22,660	− 7·0	49,640	34,440
25	" ..	"	"	25,387					
26	" ..	1·26	1·005	27,099	27,099	29,720	+ 9·7	192,200	33,760
27	" ..	"	"	27,099					
	Cast Steel—not hardened.								
28	2 ends round ..	2·5	0·87	..	10,516	10,515	0·0	10,515	69,240
29	1 round, 1 flat ..	"	"	..	20,315	19,960	− 1·75	21,030	"
30	2 ends flat ..	"	"	..	26,059	26,070	0·0	31,550	"
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

the experiments is to a great extent due to the correct value of $C = 19$ tons.

As an illustration of the rules in (197) and (164), we may take Nos. 13, 14, in Table 44: then $95848 \times 1\cdot005^{3\cdot6} \div 1\cdot26$ $= 61470$ lbs., the value of F, or the breaking weight by flexure as in col. 9. This requires correction for incipient crushing (163):—to find C_P we have $1\cdot005^2 \times \cdot7854 \times 42560 = 3376$ lbs. as in col. 10, hence $\frac{3}{4} C_P = 25320$ lbs., and the rule $P_C = F \times C_P \div (F + \frac{3}{4} C_P)$ becomes $61470 \times 33760 \div (61470 + 2532$ $= 23910$ lbs., the reduced breaking weight or value of P_C, as in col. 7.

Table 44 has been calculated in this way throughout: th sum of all the + errors in col. 8 = 56·9 and of all the − erro 52·5, giving on 15 experiments an average of (56·9 − 52·5) ÷ 15 = 0·293 per cent. only. The greatest + erro = 13·8 per cent., and the greatest − error = 15·3 pe cent. (959).

(202.) Searching as in (165) for the length of wrought-iro pillars with which the correction for incipient crushing becom nil, beyond which length the rule must not be applied fo reasons given in (164); we find that a pillar 1 inch diamet crushes with $\cdot7854 \times 19 = 14\cdot92$ tons, the correction wil therefore be nil when $F = 14\cdot92 \div 4 = 3\cdot73$ tons (169), whic in a pillar with both ends pointed is due by the rule (199) to length of $\sqrt{42\cdot79 \div 3\cdot73} \times 12 = 40\cdot64$ inches, or 40·6 times the diameter. With one end pointed and one fla $\sqrt{88\cdot26 \div 3\cdot73} \times 12 = 58\cdot37$ inches:—with both ends fla $\sqrt{133\cdot8 \div 3\cdot73} \times 12 = 71\cdot86$ inches, &c. With cast-iro pillars (165) we obtained for the same diameter 16·64, 25·0 and 31·78 times the diameter respectively, which differ remarkably from 40·64, 58·37, and 71·86 as found for wrought-iron pillars.

These ratios vary with the diameter as we found to be th case with cast iron (166), and as shown by Table 39, which ha been calculated by the rule:—

$$(203.) \qquad L_D = \sqrt[2]{(M_P \times D^{3\cdot6} \div \tfrac{1}{4} C_P)} \times 12 \div D.$$

In which the letters have the same significance as in (16

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C =
As
take
= 61
as in
(163)
lbs. a
× C
= 23
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Ta
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52·5
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The table shows that the diameter is more influential on the length with wrought-iron pillars than with cast-iron ones, which is due in part to the lower value of C: this, as we have seen (132), is 43 tons per square inch with cast iron, and 19 tons with wrought iron in the form of a pillar (133).

(204.) "*Wrought-iron Connecting-rods.*"—Wrought-iron rods are commonly used for steam-engines, pumps, and many other purposes:—being jointed at both ends, they are assimilated to pillars with both ends pointed (149). The connecting-rods for double-acting pumps are subjected to heavy shocks from a mass of water in motion throughout the system of suction and delivery pipes, which are only partially obviated by air-vessels, &c.:—they therefore require special rules, which are given in (207).

Table 45 has been calculated by the rule (197); for a pillar with both ends pointed this becomes $F = 95848 \times D^{3 \cdot 6} \div L^2$, which has been corrected where necessary for incipient crushing by the rule (164), taking the crushing strain C at the reduced value of 33,600 lbs., or 15 tons per square inch, from which we have obtained C_P in col. 2.

(205.) It should be observed that the rod of a steam-engine is subjected to an *alternating* tensile and compressive strain which is exceedingly destructive to any material, necessitating the adoption of a high "factor of safety" (915). If we admit that for a statical or dead load the "factor" should be 3, that is to say the working or safe load should be $\frac{1}{3}$rd of the dead breaking load, then by col. 5 of Table 141 the equivalent alternating dynamic strain would be $\frac{1}{6}$th of the dead load, or $\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$th of the statical breaking weight, the "factor" being = 18.

Table 46 gives the sizes of a series of connecting-rods from cases in practice:—the breaking weight as calculated by the rules is given by col. 6, and the factor of safety by col. 7, its mean value being 15·52. Thus, taking No. 2 as an example: the 3·6 power of $3\frac{3}{4}$ is by Table 35 = 116; 7 feet 2 inches being = 7·167 feet, and $7 \cdot 167^2 = 51 \cdot 36$, and the rule becomes $95848 \times 116 \div 51 \cdot 36 = 216500$ lbs. = F, or the breaking weight by flexure, as in col. 4. This requires correction for incipient crushing (163):—the area due to $3\frac{3}{4}$ inches diameter

$= 11 \cdot 04$ square inches, hence $11 \cdot 04 \times 42560 = 469800$ lbs., the value of C_P, as in col. 5, and $\frac{3}{4} C_P = 352400$ lbs., from which we obtain $(216500 \times 469800) \div (216500 + 352400) = 178800$ lbs., the reduced breaking weight P_O, as in col. 6. The value of the factor of safety is $178800 \div 11000 = 16 \cdot 25$, as in col. 7.

The diameter of a connecting-rod might be calculated with sufficient precision for practice by an empirical rule as follows:—

(206.) $$D = \sqrt[3 \cdot 6]{(w \times L^2 \div 4640)}.$$

In which w = the strain on the rod in lbs., as found from the diameter of the cylinder and pressure of steam, &c.; L = the length between centres in feet, and D = the diameter at the centre in inches. Thus, for No. 3 in Table 46 we have $6411 \times 25 \div 4640 = 34 \cdot 54$ the nearest number to which in col. 4 of Table 35 is $32 \cdot 3$ opposite $2\frac{5}{8}$ inches diameter, &c.: col. 8 in Table 46 has been calculated by this rule.

TABLE 46.—Of the STRENGTH of WROUGHT-IRON CONNECTING-RODS to STEAM-ENGINES. Cases in Practice.

Strain on Rod. Lbs.	Length in Feet.	Actual Diameter at Centre in Inches.	F.	C_P.	Calculated Breaking Weight.	Factor of Safety, $\frac{\text{Col. 6.}}{\text{Col. 1.}}$	Calculated Diameter Rule (206).
15,350	7·83	$4\frac{3}{8}$	317,100	639,700	254,600	16·59	$4\frac{3}{8}$
11,000	7·17	$3\frac{3}{4}$	216,500	469,800	178,800	16·25	$3\frac{3}{4}$
6,411	5·04	$2\frac{7}{16}$	94,830	198,600	77,240	12·05	$2\frac{5}{8}$
4,564	4·75	$2\frac{5}{16}$	85,212	178,700	69,440	15·21	$2\frac{3}{8}$
3,150	3·50	$1\frac{7}{8}$	75,190	117,500	54,110	17·17	$1\frac{3}{4}$
1,772	3·50	$1\frac{1}{2}$	33,650	75,200	28,100	15·85	$1\frac{1}{2}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

(207.) "*Double-acting Pump-rods.*"—The ordinary rules for wrought-iron pillars do not apply satisfactorily to the rods of double-acting pumps, partly, perhaps, because those rules are adapted only to a statical load or dead weight, whereas pump-rods have to sustain heavy shocks at every stroke from the alternate inertia and momentum of the water in the pump and

mains, the amount of which depends on the frequency of the alternations or the speed of the pump. The following empirical Rule is derived from long and varied experience:—

(208.) $$D = \sqrt[4]{(w \times L^2 \times R \div 200000)}.$$

In which w = the weight or strain on the rod in lbs.; L = the length between centres of joints in feet; R = the revolutions per minute of the engine, or of the crank working the pump; and D = the diameter of the rod at the centre in inches.

(209.) Table 47 gives the proportions of double-acting pump-rods from cases in practice; col. 6 has been calculated by the Rule. Thus, with No. 9 we have $17 \cdot 5^2 = 306$, then $25125 \times 306 \times 24 \cdot 5 \div 200000 = 942$, which is the 4th power of the diameter. The logarithm of 942 or $2 \cdot 974051 \div 4 = 0 \cdot 743513$, the natural number due to which, or 5·54 inches, is the diameter at the centre, as in col. 9. We should have obtained the same result without the use of logarithms by finding the square-root of the square-root of 942; thus $\sqrt{942} = 30 \cdot 69$,

TABLE 47.—Of the STRENGTH of WROUGHT-IRON CONNECTING-RODS for DOUBLE-ACTING PUMPS, to resist a COMPRESSIVE STRAIN. From Cases in Practice.

Diameter of Pump.	Head of Water.	Pressure on the Rod.	Revolutions of Engine per Minute.	Length of Rod.		Calculated Breaking Weight in Lbs.	Ratio of Breaking Weight to Working Load.	Diameter of Rod at Centre in Inches.		Place, &c.
								Actual.	Calculated.	
in.	ft.	lbs.		ft.	in.					
13	50	2,926	16	23	0	11,683	4·00	$3\frac{1}{4}$	$3\frac{3}{16}$	Trafalgar Square, W.W.
$7\frac{1}{4}$	180	3,221	25	20	0	21,805	6·80	$3\frac{1}{2}$	$3\frac{1}{2}$	Ashridge, W.W.
13	64	3,724	28	13	0	29,492	7·93	3	$3\frac{1}{16}$	Colosseum.
9	160	4,450	24	14	0	31,790	7·14	$3\frac{3}{16}$	$3\frac{3}{16}$	Lord Carlisle.
$7\frac{1}{4}$	250	4,490	34	13	0	45,372	10·10	$3\frac{3}{8}$	$3\frac{3}{8}$	Tunbridge Wells, W.W.
9	203	5,533	22	16	9	35,259	6·37	$3\frac{5}{8}$	$3\frac{5}{8}$	Ipswich, W.W.
$12\frac{1}{2}$	203	9,000	22	17	1	53,286	5·92	$4\frac{1}{8}$	$4\frac{1}{8}$	Ipswich, W.W.
10	279	9,503	36	12	0	77,210	8·13	$3\frac{3}{4}$	4	Lewes, W.W.
$13\frac{5}{16}$	400	25,125	$24\frac{1}{2}$	17	6	145,028	5·78	$5\frac{1}{2}$	$5\frac{1}{2}$	South Essex, W.W.
20	180	24,500	20	22	0	218,240	8·90	7	$5\frac{7}{8}$	Portsmouth, W.W.
(1)	(2)	(3)	(4)	(5)		(6)	(7)	(8)	(9)	

and $\sqrt{30 \cdot 69} = 5 \cdot 54$ inches, as before. Table 85 would have given nearly the same result with less trouble.

The near agreement of the calculated sizes in col. 9 with the actual ones in col. 8 is due to the fact that the latter were in many cases fixed originally by the Rule (208); still, they all stand well in practice, and have done so for many years without one failure, confirming so far the accuracy of the Rule.

(210.) "*Wrought-iron Piston-rods.*" — The piston-rods of Steam-engines are subjected alternately to a tensile and compressive strain, and in calculating their strength both of those strains must be considered.

First, for the tensile strain :—a common mode of connecting the piston-rod to the cross-head is shown by Fig. 42, in which we have a $2\frac{1}{2}$-inch rod with a conical end, terminated by a $2\frac{1}{8}$-inch square-thread screw, $1\frac{3}{4}$ inch diameter at the bottom of the thread. The area of $2\frac{1}{2} = 4 \cdot 9$, and of $1\frac{3}{4} = 2 \cdot 4$ square inches; the area at the screw is therefore only *half* the area of the rod.

In large piston-rods a key is commonly used, as in Fig. 43; we have shown in (123) that the shearing strain is equal to the tensile, and as the key is subjected to a *double* shear, shearing at two places, its double area must be equal to the area of the rod at the key-way, supposing of course that the key is of the same material as the rod. With the sizes shown in the figure, the key will have a shearing area of $4 \cdot 75 \times 1 \cdot 25 \times 2 = 11 \cdot 87$ square inches. The area of $4\frac{3}{4}$ diameter being $17 \cdot 72$, and the area cut away by the key-way, $4 \cdot 75 \times 1 \cdot 25 = 5 \cdot 93$, we have $17 \cdot 72 - 5 \cdot 93 = 11 \cdot 79$ square inches, being practically the same as the area of the key :—this, again, is only *half* the area of the $5\frac{1}{2}$-inch rod, which $= 23 \cdot 75$ square inches.

(211.) We may therefore admit that the minimum area at the screw or key-way is half the area of the rod, and this of course limits the strength of the piston-rod; for obviously, whatever may be the strength as a pillar during the up-stroke, the rod cannot bear more than is due to its tensile strength during the down-stroke.

(212.) The mean strength of British wrought iron may be

taken at 57,500 lbs. per square inch from (4) and Table 1, hence the maximum strain admissible on the *body* of the rod is $57500 \div 2 = 28750$ lbs. per square inch, and the second column of Table 48 has been thus calculated. When a rod is so short that we are certain the strain will be limited by the tensile strength, we may find the area direct by the Rule :—

(213.) $$\text{Area} = W \times M_F \div 28750.$$

In which W = the strain on the rod due to the area of the piston, pressure of steam, &c., and M_F = the Factor of Safety (880). (216).

For the strength to resist the compressive strain at the up-stroke we may consider the rod as a pillar with one end flat and the other pointed, namely, flat at the piston, and pointed at the cross-head. Taking the value of M_P at 197,700 lbs. from col. 2 of Table 34, the Rule (197) then becomes :—

(214.) $$F = 197700 \times D^{3 \cdot 6} \div L^2.$$

In which F = the breaking weight by flexure in lbs., L = the greatest length unsupported in feet, or in most cases, the distance from the gland of the stuffing-box, to the centre of the cross-head at the top of the stroke, D = the diameter in inches.

(215.) This rule gives the breaking weight by flexure only :— as shown in (202), rods of a length less than 58·37 times the diameter will require correction for incipient crushing. Taking C, or the crushing strength of wrought iron at 19 tons, or 42,560 lbs. (201), and the areas due to the respective diameters, we obtain the value of C_P in the third column of Table 48, which gives the sizes of Piston-rods based on a combination of the rules for flexure (214), tensile (212), and crushing strength (164).

Thus, for a $3\frac{1}{4}$-inch rod, 8 feet long, $3\frac{1}{4}^{3 \cdot 6}$ being 69·6 by Table 35, the breaking weight by flexure becomes $197700 \times 69 \cdot 6 \div 64 = 215000$ lbs. = F. This requires correction for incipient crushing (163), the area of $3\frac{1}{4} = 8 \cdot 296$ square inches, and we obtain $8 \cdot 296 \times 42560 = 353100$ lbs., the value of C_P; hence $\frac{3}{4} C_P = 264800$ lbs., and the rule (164) gives P_C = (215000

$\times$ 353100) $\div$ (215000 + 264800) = 158200 lbs., as in the Table, which has been calculated throughout in this way. With four exceptions, the whole of the numbers required correction for incipient crushing. When the strength as a pillar is greater than the *tensile* strength in the second column, then the latter limits the strength of the whole:—for instance, a 3-inch rod, say 3 feet long, would bear as a pillar 251,500 lbs., but the tensile strength at the key-way, &c., is 203,200 lbs. only, and as the strain during the up and down strokes is usually equal, the strength both ways is limited by the lesser. It will be observed that in the Table the strengths of say a 5-inch rod 5, 6, and 7 feet long are all alike, being, in fact, equal to the *tensile* strength, or 564,600 lbs., &c.

(216.) Table 49 gives the particulars of Piston-rods from cases in practice:—the strain in col. 5 was obtained by multiplying the area of the piston by the pressure of steam, plus the vacuum in condensing engines, where the *total* has been taken at 20 lbs. per square inch; with the High-pressure Engines, the pressure has been taken at 45 lbs. above atmosphere. In col. 6, we have given the calculated breaking weight as found by Tables 48, 56, &c., and in col. 7, the "Factor of Safety," (880) or the ratio of the breaking to the working strain, the mean value of which is about 12 for ordinary Condensing-engines, and 14 for Marine Engines; it will be observed that the whole of the latter are dominated by the tensile strength, in consequence of the shortness of the stroke, and are marked by a *.

In col. 8 we have given the calculated diameters as found by Tables 48, 56; thus in the 80-Horse rod, No. 6, with Factor 12 we have 32520 $\times$ 12 = 390240 lbs. breaking strain, the nearest number to which in the column for 8 feet length in Table 48 is 391,100 lbs., opposite $4\frac{1}{2}$ inches diameter; the actual diameter was $4\frac{1}{4}$ inches. Again, in the 60-Horse Marine Engine, No. 9, with Factor 14 we obtain 29040 $\times$ 14 = 406560 lbs. breaking strain, the nearest number to which in the column for 5 feet long is 408,000 lbs., opposite $4\frac{1}{4}$ inches diameter, which was also the actual size; being governed by the Tensile strain in col. 2.

Wrought iron is not often used in modern times for first-class Engines; the superior character of steel, combined with its

TABLE 48.—Of the STRENGTH of WROUGHT-IRON PISTON-RODS to resist TENSILE and COMPRESSIVE STRAINS.

Diameter.	Tensile Breaking Weight at the Screw or Key-way.	Crushing Strain Cr.	Maximum Length of Rod unsupported, in Feet.								
			3	4	5	6	7	8	9	10	12
			Compressive Breaking Strain, in Lbs.								
inches.	lbs.	lbs.									
1	22,580	33,430	15,620	11,030	7,910†	5,490†	..	..	..	..	..
1¼	35,270	52,220	28,840	18,600	16,070	12,100	..	..	..	..	..
1½	50,800	75,200	47,090	36,500	28,280	22,180	17,400†	..	..	..	..
1¾	69,150	102,400	69,150*	56,010	44,610	35,760	28,970	..	..	..	..
2	90,330	133,700	90,330*	80,120	65,280	53,250	43,760	36,300	..	..	..
2¼	114,300	169,200	114,300*	108,700	90,520	74,980	62,640	52,510	..	..	..
2½	141,100	209,000	141,100*	141,100*	120,600	101,900	85,740	72,770	62,010	..	..
2¾	170,800	252,800	170,800*	170,800*	155,300	132,850	113,300	96,980	83,370	..	..
3	203,200	300,900	203,200*	203,200*	194,600	168,400	145,400	125,300	109,100	..	..
3¼	238,500	353,100	238,500*	238,500*	238,300	208,500	181,800	158,200	138,100	120,700	..
3½	276,600	409,500	276,600*	276,600*	261,500	253,500	223,600	195,700	171,800	151,300	..
3¾	317,400	469,900	317,400*	317,400*	317,400*	302,500	268,100	236,800	209,300	185,100	..
4	361,400	535,000	..	361,400*	361,400*	357,300	319,100	284,000	252,600	224,700	179,100
4¼	408,000	603,900	..	408,000*	408,000*	408,000*	374,200	335,200	300,000	268,200	215,500
4½	457,100	676,700	..	..	457,100*	457,100*	434,400	391,100	351,600	316,100	256,000
5	564,600	835,900	..	..	564,600*	564,600*	564,600*	515,900	468,700	425,000	349,300
5½	683,100	1,011,000	..	..	..	683,100*	683,100*	660,700	605,000	552,900	460,800
6	812,700	1,203,000	..	..	..	812,700*	812,700*	812,700*	760,400	698,900	590,200

NOTE.—The strains marked * are limited by the tensile strength in col. 2; those marked † are due to flexure simply. The rest have been reduced for incipient crushing by the rule in (164), with the value of Cr given by col. 3.

reduced cost, should commend it for universal adoption; the strength of Steel Piston-rods is considered in (271).

TABLE 49.—Of the STRENGTH and PROPORTIONS of PISTON-RODS to STEAM-ENGINES. From Cases in Practice.

Nominal Horse-power.	Diameter in Inches.		Maximum Length.	Actual Strain.	Calculated Breaking Weight.	"Factor," Col. 6 / Col. 5.	Calculated Diameter.	Kind of Engine.
	Cylinder.	Piston-rod.						
			ft.	lbs.	lbs.			
4	$12\frac{1}{2}$	$1\frac{3}{8}$ Iron	3·5	2,460	32,710	13·30	$1\frac{3}{8}$	Condensing.
12	$19\frac{1}{2}$	2 ,,	5·0	5,980	65,280	10·92	$2\frac{1}{16}$	,,
25	28	3 ,,	6·5	12,320	156,300	12·69	3	,,
30	30	$3\frac{1}{8}$,,	7·0	14,140	169,900	12·02	$3\frac{1}{8}$	,,
36	31	3 ,,	7·0	15,094	145,400	9·63	$3\frac{1}{4}$	,,
80	$45\frac{1}{2}$	$4\frac{1}{4}$,,	8·0	32,520	335,200	10·31	$4\frac{1}{2}$	,,
10	20	2 ,,	3·0	6,280	90,330*	14·39	2	Marine.
30	32	$3\frac{1}{2}$,,	4·0	16,080	238,500*	14·83	$3\frac{3}{16}$	,,
60	43	$4\frac{1}{4}$,,	5·0	29,040	408,000*	14·05	$4\frac{1}{4}$	,,
90	50	$4\frac{7}{8}$,,	6·0	39,270	536,500*	13·66	$4\frac{15}{16}$	,,
120	57	$5\frac{1}{2}$,,	7·0	51,040	683,100*	13·38	$5\frac{5}{8}$	,,
6	$9\frac{1}{2}$	$1\frac{3}{8}$ Steel	2·17	3,195	71,270*	22·31	$1\frac{1}{8}$	High-pressure.
10	$11\frac{1}{2}$	$1\frac{3}{4}$,,	2·5	4,580	115,400*	25·20	$1\frac{5}{16}$	,,
12	$10\frac{3}{4}$	$1\frac{1}{2}$,,	3·3	4,085	73,410	17·96	$1\frac{3}{8}$	Do. Woolf.
18	$14\frac{1}{2}$	$2\frac{1}{16}$ Iron	3·5	8,250	96,050*	11·64	$2\frac{1}{4}$	High-pressure.
20	$12\frac{1}{2}$	$1\frac{3}{4}$ Steel	3·3	5,521	115,400	20·85	$1\frac{1}{2}$	Do. Woolf.
22	13	$1\frac{3}{4}$,,	4·5	5,985	77,530	12·95	$1\frac{7}{8}$	,, ,,
25	$13\frac{1}{2}$	$1\frac{3}{4}$,,	4·5	6,435	77,530	12·05	$1\frac{7}{8}$	,, ,,
30	$14\frac{1}{2}$	2 ,,	4·8	7,425	107,300	14·46	2	,, ,,
35	$16\frac{7}{8}$	$2\frac{1}{2}$,,	5·5	10,080	178,400	17·70	$2\frac{3}{8}$	,, ,,
42	$17\frac{1}{8}$	$2\frac{1}{2}$,,	5·5	10,400	178,400	17·15	$2\frac{3}{8}$	,, ,,
60	22	$2\frac{3}{4}$,,	6·67	17,100	182,900	10·70	3	,, ,,
100	28	$3\frac{3}{8}$,,	9·5	27,710	205,400	7·41	$4\frac{1}{8}$	,, ,,
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

(217.) "*Radius-rods of Steam-engines.*"—In ordinary Beam Engines, the Radius-rods and "parallel bars" of the motion are subjected alternately to equal tensile and compressive strains; the latter, tending to cripple the rod as a pillar, is the most influential, and the diameter necessary to resist that strain will usually be sufficient for the tensile strain also. For Radius-rods we may give the following Empirical Rule:—

(218.) $$d = \sqrt[4]{(H \times L^2 \div 150)}.$$

In which H = the reputed or Nominal Horse-power of the Engine; L = the length of the radius-rod between centres, in feet; and d = the diameter of the rod in inches. Table 50 has been calculated by this rule. The "parallel bars" are usually made of the same diameter as the radius-rods; Table 51 gives the particulars of both from cases in practice, col. 4 having been calculated by the rule. Thus, for the 30-horse Engine with a

TABLE 50.—Of the SIZES of RADIUS-RODS to STEAM-ENGINES.

Diameter of Rod.	Length of Radius-rod, in Feet.											
	2	2¼	2½	3	3½	4	4½	5	5½	6	6½	7
	Nominal Horse-power of Engine.											
¾	12	9	7	..	..	..	..	..	..	..	..	..
⅞	22	17	14	10	..	..	..	..	..	..	..	..
1	37	30	24	17	12	..	..	..	..	..	..	..
1⅛	..	48	39	27	20	15	..	..	..	..	..	..
1¼	..	..	61	42	31	24	19	15	..	..	..	..
1½	..	..	..	84	62	47	37	30	25	..	..	..
1¾	..	..	..	156	100	88	70	56	47	..	..	..
2	..	..	..	..	200	150	120	96	80	67	..	..
2¼	..	..	..	..	..	240	190	155	125	105	91	..
2½	..	..	..	..	..	..	290	230	195	165	135	120
2¾	..	..	..	..	..	..	..	340	285	240	205	175
3	..	..	..	..	..	..	..	..	400	340	290	250

NOTE.—The parallel bars may be of the same diameter as the Radius-rods.

TABLE 51.—Of RADIUS-RODS, &c., to STEAM-ENGINES. Cases in Practice.

Nominal Horse-power.	Length. Feet.	Diameter of Radius-rod.			Diameter of Parallel Bars.
		Actual.	By Rule.	Bearings.	
60	4·57	1⅝	1·688	2	1⅝
42	3·43	1⅜	1·361	1⅝	1⅜
30	3·50	1⅛	1·251	1¼	1¼
22	3·27	1⅛	1·117	1¼	1⅛
12	2·25	⅞	0·798	1½	⅞
(1)	(2)	(3)	(4)	(5)	(6)

rod $3\frac{1}{2}$ feet long, we have $30 \times 12 \cdot 25 \div 150 = 2 \cdot 45$; then the logarithm of $2 \cdot 45$, or $0 \cdot 389166 \div 4 = 0 \cdot 097291$, the natural number due to which, or $1 \cdot 251$ inch, is the diameter required. Or we might have obtained the same result without logarithms by finding the square-root of the square-root of $2 \cdot 45$, thus $\sqrt{2 \cdot 45} = 1 \cdot 565$, and $\sqrt{1 \cdot 565} = 1 \cdot 251$, as before: see also, Table 85.

(219.) "*Hollow Cylindrical Pillars of Wrought Iron.*"—For hollow cylindrical pillars, the rules in (196) require a simple modification and become:—

(220.) $$F = M_P \times (D^{3 \cdot 6} - d^{3 \cdot 6}) \div L^2.$$

(221.) $$D^{3 \cdot 6} - d^{3 \cdot 6} = F \times L^2 \div M_P.$$

(222.) $$L = \sqrt[2]{\left\{M_P \times (D^{3 \cdot 6} - d^{3 \cdot 6}) \div F\right\}}.$$

In which F = the breaking weight on the pillar in lbs. or tons by flexure, dependent on M_P.
D = the external diameter in inches.
d = the internal " "
L = the length in feet.
M_P = Constant Multiplier, the value of which is given in Table 34.

Table 52 gives the result of thirty-six experiments on hollow pillars, made of thin plate-iron, by Mr. Hodgkinson; col. 10 has been calculated by the rule, the value of M_P for pillars with both ends flat being taken at 299,600 lbs. from Table 34.

(223.) Thus, with No. 3, the logarithm of $6 \cdot 187$ or $0 \cdot 791480 \times 3 \cdot 6 = 2 \cdot 849328$, the natural number due to which, or $706 \cdot 9$, is the $3 \cdot 6$ power of D:—then the logarithm of 6, or $0 \cdot 778151 \times 3 \cdot 6 = 2 \cdot 801344$, the natural number due to which, or $622 \cdot 9$, is the $3 \cdot 6$ power of d. Hence $706 \cdot 9 - 622 \cdot 9 = 74$, and the rule (220) becomes $F = 299600 \times 74 \div 100 = 221700$ lbs., the breaking weight by flexure, or F, as in col. 10. This requires correction for incipient crushing by the rule (164); taking the crushing strength at 19 tons, or 42,560 lbs. per square inch (201), and the area by col. 5 being $1 \cdot 799$ square inches, C_P becomes $1 \cdot 799 \times 42560 = 76570$ lbs., as in col. 11, and $\frac{3}{4} C_P = 57420$ lbs.; hence $(221700 \times 76570) \div (221700 + 57420) = 60810$ lbs.,

the reduced breaking weight, as in col. 8, showing an error of $60810 \div 60075 = 1 \cdot 012$, or $+ 1 \cdot 2$ per cent., as in col. 9, &c.

(224.) The sum of all the + errors in col. 9 is $225 \cdot 6$, and of all the − errors, $243 \cdot 2$, giving on the thirty-six experiments an average of $(243 \cdot 2 - 225 \cdot 6) \div 36 = 0 \cdot 461$, or $- 0 \cdot 461$ per cent. only. It will also be observed that the *range* of the error plus and minus is nearly equalized, the greatest + error being $+ 30 \cdot 4$, and the greatest − error is $- 33 \cdot 0$ (959).

(225.) "*Square Pillars of Wrought Iron.*"—Admitting that for wrought iron, the ratio of the strength of round and square pillars is $1 \cdot 0$ to $1 \cdot 7$, as shown by theory, we obtain the values of M_P given for square pillars by Table 34. The rules in (196) then require a simple modification, and become:—

(226.) $$F = M_P \times S^{3 \cdot 6} \div L^2.$$

(227.) $$S = \sqrt[3 \cdot 6]{(F \times L^2 \div M_P)}.$$

(228.) $$L = \sqrt[2]{(M_P \times S^{3 \cdot 6} \div F)}.$$

(229.) $$M_P = F \times L^2 \div S^{3 \cdot 6}.$$

In which S = the side of the square, and the rest as in (196). Say that we have a pillar $1\frac{1}{4}$ inch square, 5 feet long, with both ends pointed; then taking M_P from Table 34 at $72 \cdot 74$ tons, and $1\frac{1}{4}^{3 \cdot 6}$ from Table 35 $= 2 \cdot 2$, the rule becomes $72 \cdot 74 \times 2 \cdot 2 \div 25 = 6 \cdot 703$ tons = F, or the breaking weight by flexure. This result will not require correction for incipient crushing: thus the area is $1 \cdot 25 \times 1 \cdot 25 = 1 \cdot 5625$ square inches, and the crushing strength being 19 tons per square inch, C_P becomes $1 \cdot 5625 \times 19 = 29 \cdot 69$ tons, and $\frac{1}{4} C_P = 7 \cdot 425$ tons, and as F is less than that (169) the correction is not required.

For hollow square pillars of wrought iron the rules become:—

(230.) $$F = M_P \times (S^{3 \cdot 6} - s^{3 \cdot 6}) \div L^2.$$

(231.) $$(S^{3 \cdot 6} - s^{3 \cdot 6}) = F \times L^2 \div M_P.$$

(232.) $$L = \sqrt[2]{\left\{M_P \times (S^{3 \cdot 6} - s^{3 \cdot 6}) \div F\right\}}.$$

In which S = the side of square externally, and *s*, internally; the rest being as in (222). Thus for a pillar in which S =

TABLE 52.—Of the STRENGTH of CYLINDRICAL PILLARS,

Number of Experiment.	Diameter in Inches.		Thickness.	Area of Section.	Length.
	Outside.	Inside.			
			inches.	square inches.	feet.
1	6·366	6·1064	·1298	2·547	10·00
2	"	"	"	"	7·50
3	6·187	6·000	·0939	1·799	10·00
4	6·175	5·973	·101	1·799	5·00
5	6·125	5·929	·098	1·799	2·50
6	4·060	3·750	·150	1·905	9·94
7	4·052	3·790	·136	1·613	7·50
8	4·050	3·772	·139	1·7078	9·94
9	4·000	3·511	·2435	2·879	7·44
10	4·000	3·505	·250	2·897	7·44
11	4·000	3·504	·2425	2·873	7·44
12	3·995	3·513	·241	2·848	5·00
13	"	"	"	"	2·36
14	3·995	3·504	·2455	2·895	5·00
15	3·035	2·717	·168	1·414	7·50
16	3·000	2·712	·153	1·414	2·354
17	2·995	2·693	·151	1·349	9·940
18	2·490	2·275	·1075	·8045	9·917
19	"	"	"	"	5·000
20	"	"	"	"	2·500
21	2·382	1·891	·246	1·651	2·492
22	2·373	1·911	·231	1·554	2·425
23	2·350	1·865	·2425	1·605	10·000
24	2·350	1·910	·220	1·4721	5·000
25	2·343	1·923	·210	1·407	2·492
26	2·343	1·939	·202	1·3587	2·425
27	2·340	1·910	·215	1·4353	10·000
28	2·335	1·925	·205	1·3718	5·000
29	"	"	"	1·4353	2·500
30	"	"	"	1·4353	2·500
31	1·964	1·755	·1045	·6104	9·917
32	"	"	"	"	5·000
33	"	"	"	"	2·500
34	1·495	1·292	·1015	·4443	9·917
35	"	"	"	"	5·000
36	"	"	"	"	2·500
(1)	(2)	(3)	(4)	(5)	(6)

made of THIN WROUGHT-IRON PLATES: both ends Flat.

Breaking Weight by Experiment.	Calculated Breaking Weight.				Ultimate Strength per Square Inch by Experiment.
	Reduced.	Error or Difference per Cent.	By Flexure F.	Crushing Strain.	
lbs.	lbs.		lbs.	lbs.	tons
91,402	86,800	− 5·0	326,600	108,400	16·021
106,122	95,090	−10·4	580,500	108,400	18·600
60,075	60,810	+ 1·2	221,700	76,570	14·908
69,002	72,200	+ 4·6	950,300	76,570	17·123
74,411	75,370	+ 1·3	3,610,000	76,570	18·464
49,900	53,310	+ 6·8	116,750	81,075	11·710
53,770	53,070	− 1·3	175,200	68,650	14·880
47,212	47,880	+ 1·4	105,200	72,690	12·340
74,988	93,660	+24·9	297,900	122,500	11·628
76,780	94,320	+22·8	301,000	123,300	11·832
79,916	93,760	+20·3	301,500	122,300	12·418
98,122	106,350	+ 8·4	650,200	121,200	15·381
137,322	117,550	−14·4	2,919,000	121,200	21·530
86,922	108,100	+24·4	660,500	123,200	13·404
42,122	54,910	+30·4	95,230	60,180	13·299
52,874	56,934	+ 7·7	791,000	60,180	16·693
37,356	30,850	−17·4	50,010	57,420	12·362
23,958	16,020	−33·0	22,575	34,240	13·294
28,244	26,560	− 6·0	88,800	34,240	15·670
29,364	31,930	+ 9·0	355,200	34,240	16·290
54,666	64,750	+18·4	619,500	70,270	14·780
57,354	61,230	+ 6·8	619,000	66,140	16·476
37,516	28,940	−22·8	37,660	68,310	9·600
43,180	46,620	+ 8·0	136,600	62,650	13·094
53,770	55,180	+ 2·6	525,800	59,880	17·060
53,770	58,950	+ 9·6	540,000	64,210	17·665
31,828	25,650	−19·4	33,160	61,090	9·901
41,164	31,660	−23·1	127,150	58,380	13·392
52,588	56,040	+ 6·5	508,600	61,090	16·357
50,796	56,040	+10·3	508,600	61,090	15·799
14,158	9,667	−31·8	11,530	26,040	10·350
20,332	18,200	−18·4	45,360	26,040	14·866
22,572	23,510	+ 4·1	181,440	26,040	16·509
6,514	5,137	−21·2	5,295	18,870	6·550
13,860	11,235	−19·0	20,830	18,870	13·920
15,204	16,130	+ 6·1	83,310	18,870	15·277
(7)	(8)	(9)	(10)	(11)	(12)

4 inches, $s = 3\frac{1}{2}$ inches, L = 11 feet, both ends flat, we have $223 \times (4^{3\cdot6} - 3\frac{1}{2}^{3\cdot6}) \div 11^2$, or $223 \times (147 - 90\cdot9) \div 121 = 103\cdot5$ tons = F. Correcting for incipient crushing (164), the area of the section being 4·75 square inches, C_P becomes $4\cdot75 \times 19 = 90\cdot25$ tons, $\frac{3}{4}\, C_P = 67\cdot69$ tons, hence $P_C = (103\cdot5 \times 90\cdot25) \div (103\cdot5 + 67\cdot69) = 54\cdot56$ tons breaking weight.

With *thin* plates of wrought iron correction is required for incipient "Wrinkling" (249) rather than incipient crushing: in our case, however, the wrinkling strain is $(\sqrt{\cdot25} \div \sqrt{4}) \times 80$, or $(\cdot5 \div 2) \times 80 = 20$ tons per square inch, which being in excess of 19 tons, the *crushing* strength of wrought iron in pillars (201), the strength is governed by the latter.

(233.) "*Rectangular Pillars of Wrought Iron.*"—For rectangular sections, other than square, the rules for square pillars are modified, and we have:—

(234.) $$F = M_P \times t^{2\cdot6} \times b \div L^2.$$

(235.) $$t = \sqrt[2\cdot6]{\left\{F \times L^2 \div (M_P \times b)\right\}}.$$

(236.) $$b = F \times L^2 \div (M_P \times t^{2\cdot6}).$$

(237.) $$L = \sqrt[2]{(M_P \times t^{2\cdot6} \times b \div F)}.$$

(238.) $$M_P = F \times L^2 \div (t^{2\cdot6} \times b).$$

In which the letters have the same signification as in (177), namely F = the breaking weight by flexure in lbs., tons, &c., dependent on the value of M_P as given by Table 34, t = the thickness or least dimension of the rectangular pillar, the 2·6 power of which is given by col. 3 of Table 35; b = the greatest dimension, and L = length in feet, &c.

(239.) Table 53 gives the results of twenty-one experiments on solid rectangular pillars of wrought iron by Mr. Hodgkinson; col. 9 has been calculated by the rule (234), the value of M_P for rectangular pillars with both ends flat being taken at 498,500, or say 500,000 lbs., from Table 34. Thus taking No. 9 as an example, to find $t^{2\cdot6}$, the logarithm of $\cdot995$ or $\bar{1}\cdot9978 \times 2\cdot6 = \bar{1}\cdot99428$, the natural number due to which, or $\cdot9869$, is the 2·6 power of t, and $7\cdot5^2$ being $= 56\cdot25$, the rule becomes 500000

$\times \cdot 9869 \times 5\cdot 86 \div 56\cdot 25 = 51410$ lbs. $=$ F, or the breaking weight by flexure, as in col. 9. We shall find that this does not require correction for incipient crushing; the area of the section is $5\cdot 86 \times \cdot 995 = 5\cdot 8307$ square inches, and the crushing strength being 19 tons or 42,560 lbs. per square inch (201), C_P becomes $42560 \times 5\cdot 8307 = 248100$ lbs., as in col. 8, therefore $\frac{1}{4} C_P = 62025$, and as F is less that, namely 51,410 lbs., the correction is not required (164). The experimental breaking weight was 54,114 lbs., as in col. 5, hence we have $51410 \div 54114 = \cdot 95$, showing a difference or error of -5 per cent., as in col. 7.

TABLE 53.—Of the STRENGTH of RECTANGULAR, SOLID PILLARS of WROUGHT IRON, both ends Flat.

No. of Experiment.	Length.	Dimensions in Inches.	Ultimate Strength. By Experiment. Per Square Inch.	By Experiment. Total.	By Calculation. Total.	By Calculation. Error Per Cent.	By Crushing Strain, C_P.	By Flexure, F.
	feet.		tons.	lbs.	lbs.		lbs.	lbs.
1	10	2·98 × ·497	·364	? 1,222	2,418	..	63,030	2,418
2	10	3·01 × ·766	1·508	7,793	7,526	− 3·4	98,140	7,526
3	10	2·99 × ·995	1·911	12,735	14,760	+ 15·0	126,600	14,760
4	10	3·00 × 1·51	4·538	46,050	43,800	− 4·9	192,800	43,800
5	7·5	1·024 × 1·025	4·354	10,236	9,691	− 5·3	44,670	9,691
6	7·5	2·983 × ·5023	1·076	3,614	4,424	+ 22·4	63,770	4,424
7	7·5	3·005 × ·9955	4·425	29,616	26,400	− 10·9	127,300	26,400
8	7·5	3·00 × 1·53	8·923	91,746	59,600	− 34·8	195,300	64,310
9	7·5	5·86 × ·995	4·143	54,114	51,410	− 5·0	248,100	51,410
10	5	1·024 × 1·024	7·709	18,106	17,590	− 2·8	44,600	21,780
11	5	2·98 × ·507	2·502	8,469	10,190	+ 20·4	64,310	10,190
12	5	3·01 × ·767	5·790	29,955	28,400	− 5·2	98,270	30,060
13	5	3·01 × ·995	8·066	54,114	48,860	− 9·7	127,400	59,420
14	5	5·84 × ·996	7·901	102,946	94,980	− 7·7	247,600	115,600
15	2·5	1·0235 × 1·0235	11·307	26,530	32,200	+ 21·4	44,580	86,980
16	2·5	2·9867 × ·5026	7·524	25,299	29,080	+ 15·0	63,890	40,030
17	2·5	3·01 × ·763	12·396	63,786	60,530	− 5·1	97,760	119,200
18	2·5	3·00 × ·996	13·239	88,610	90,730	+ 2·4	127,200	237,500
19	1·25	1·023 × 1·023	15·426	36,162	40,630	+ 12 3	44,540	347,400
20	0·625	1·023 × 1·023	21·733	50,946	43,500	− 14·6	44,540	1,389,000
21	0·3125	1·023 × 1·023	23·549	52,749	44,284	..	44,540	5,558,400
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

To vary the illustration we will take a pillar very similar in section to the last, but only 5 feet long, say No. 14 in the Table 53: the logarithm of $\cdot 996 = \bar{1}\cdot 99826 \times 2\cdot 6 = \bar{1}\cdot 995476$, the natural number due to which, or $\cdot 9896 = t^{2\cdot 6}$; then $500000 \times \cdot 9896 \times 5\cdot 84 \div 25 = 115600$ lbs. = F, as in col. 9.

Then the area $= \cdot 996 \times 5\cdot 84 = 5\cdot 8166$ square inches, hence C_P becomes $5\cdot 8166 \times 42560 = 247600$ lbs., as in col. 8, $\frac{3}{4}$ C_P $= 185700$ lbs., and the rule in (164), gives $(115600 \times 247600) \div (115600 + 185700) = 94980$ lbs., reduced breaking weight, as in col. 6. The experimental result was 102,946 lbs., the difference being $94980 \div 102946 = \cdot 923$, or $1\cdot 0 - \cdot 923 = \cdot 077$, showing an error of $- 7\cdot 7$ per cent.

(240.) Table 53 has been calculated in this way throughout:—the sum of all the + errors in col. 7 is 108·9, and of all the − errors, 109·4, they are therefore practically equalized. The greatest + error is 22·4 per cent., and the greatest − error = 34·8 per cent. (959).

(241.) Messrs. Kennard have made some experiments on wrought-iron pillars of cruciform, ⊥ iron, and other sections, the results of which are given by Table 54; unfortunately there is some doubt as to the form at the ends, some were cut away there, so as to approximate the case to a pillar with both ends pointed, and others approximate to the form of pillars with ends flat. All we can do is to calculate for *both* conditions, and it will be found that the experimental results are between those extremes which do not differ nearly so much as 1 to 3, (149) being governed by incipient crushing.

(242.) "*Wrought-iron Pillars of + Section.*"—The cruciform pillars were of the sizes shown by Fig. 44, and were 5 feet long; they would therefore fail by flexure in the direction of the least dimension ($2\frac{1}{2}$ inches), and the strength may be calculated as in (239) by the rule (234). Thus $2\frac{1}{2}^{2\cdot 6} = 10\cdot 84$ by col. 3 of Table 35; for a pillar with both ends pointed $M_P = 162900$ by Table 34, then, $162900 \times 10\cdot 84 \times \cdot 375 = 662200$ lbs. due to the rib *a* as a pillar one foot long. For the resistance of the rib *c*, $\frac{3}{8}^{2\cdot 6} = \cdot 0781$ by col. 3 of Table 35, and the breadth *b* in the rule being $3 - 0\cdot 375 = 2\cdot 625$, we obtain $162900 \times \cdot 0781 \times 2\cdot 625 = 33400$ lbs. The sum of the two is $662200 + 33400 =$

TABLE 54.—Of EXPERIMENTS on WROUGHT-IRON PILLARS of T, L, and other Sections.

Form.	Length.	Two Ends Flat.		Two Ends Pointed.		Fig.
		Calculated.	Maximum: Experiment.	Minimum: Experiment.	Calculated.	
	feet.					
+	5	47,000 lbs.	38,304 lbs.	34,944 lbs.	25,350 lbs.	44
"	"	21·0 tons	17·1 tons	15·6 tons	11·32 tons	"
⊥	"	59,670 lbs.	62,989 lbs.	42,000 lbs.	35,350 lbs.	46
"	"	26·65 tons	28·12 tons	18·75 tons	15·78 tons	"
L	"	21·79 "	18·75 "	..	..	47
"	4	24·99 "	23·43 "	..	..	"
"	3	28·21 "	28·12 "	..	..	"
"	$1\frac{1}{2}$	32·22 "	31·30 "	..	..	"
"	5	..	..	12·25 "	12·55 "	"
⊔	"	20·28 "	17·10 "	14·00 "	9·85 "	49
(1)	(2)	(3)	(4)	(5)	(6)	(7)

695600 lbs., which for a length of 5 feet becomes $695600 \div 5^2 = 27824$ lbs., the breaking weight by flexure = F. Reducing this for incipient crushing (164); the area of the whole section is 1·88 square inch, hence $1{\cdot}88 \times 42560 = 80013$ lbs., the value of C_P, and 60,010 lbs. = $\frac{3}{4}$ C_P; then the rule (164) becomes $P_c = (27824 \times 80013) \div (27824 + 60010) = 25350$ lbs., the reduced breaking weight with both ends pointed, as in col. 6 of Table 54:—experiment gave 34,944 lbs., col. 5, which seems to show that the form at the ends did not conform to that condition.

With both ends flat, F would be greater, in the ratio of the respective Multipliers M_P given by Table 34, and becomes $27824 \times 500000 \div 162900 = 85400$ lbs., which corrected for incipient crushing becomes $(85400 \times 80013) \div (85400 + 60010) = 47000$ lbs., col. 3; experiment gave 38,304 lbs., col. 4, which seems to show that the condition of both ends flat, so favourable to strength, was not fully attained. The small difference between the results of the two experiments, namely $38304 - 34944 = 3360$ lbs. only, or about 9 per cent., shows that the form at the ends were nearly alike, although intended to be very different. The calculated difference was $47000 - 25350 =$

21650 lbs., and the experimental difference would have been about the same if the conditions assumed had been fulfilled. But it will be observed that the two experimental results fall between the two calculated ones.

(243.) "*Wrought-iron ⊥ Pillars.*"—Let Fig. 45 be the section of a ⊥ iron pillar of wrought iron, say 10 feet long, both ends flat. Such a pillar may fail by flexure in any one of the three directions shown by the arrows A, B, C, and of course it will select the one in which the resistance is the least, or where $t^{2\cdot6} \times b$ has the lowest value.

In the direction A, we have $(4^{2\cdot6} \times 0\cdot5) + (0\cdot5^{2\cdot6} \times 3\cdot5)$ or $(36\cdot7 \times 0\cdot5) + (\cdot165 \times 3\cdot5) = 18\cdot9275$.

It should be observed that the rib R contributes only 3 per cent. to the strength: it might therefore be neglected with impunity: thus $(\cdot165 \times 3\cdot5) \div 18\cdot9275 = \cdot03$, or 3 per cent. (186). In the direction of B, reckoning for wrought iron from the line N, as we found it necessary to do in calculating the transverse strength (378), we have $(3\cdot5^{2\cdot6} \times 0\cdot5) + \left\{4^{2\cdot6} - 3\cdot5^{2\cdot6}) \times 4\right\} = 55\cdot8$, or nearly three times the resistance in the direction of A: the pillar will therefore not give way in that direction unless it is forced to do so by the mode of fixing.

In the direction of C we must calculate for wrought iron from the line P, and $t^{2\cdot6} \times b$ then becomes $(0\cdot5^{2\cdot6} \times 4) + \left\{4^{2\cdot6} - 0\cdot5^{2\cdot6}) \times 0\cdot5\right\} = 18\cdot9275$, or the same as in the direction A; the pillar may therefore fail in either direction indifferently. Then by the rule in (234) we have $223 \times 18\cdot9275 \div 100 = 42\cdot24$ tons $=$ F. Then for incipient crushing (164), the area of the section being $3\cdot75$ square inches, C_P becomes $3\cdot75 \times 19 = 71\cdot25$ tons, and $\frac{3}{4}\ C_P = 53\cdot54$ tons, hence $(42\cdot24 \times 71\cdot25) \div (42\cdot24 + 53\cdot44) = 31\cdot46$ tons breaking weight.

(244.) Experiments were made on pillars whose section is given by Fig. 46, the length being 5 feet, one of them having flat ends or approximately so, and the other pointed. Assuming that the pillar will bend in the direction of the arrow D, and calculating as in (243), we have $(3^{2\cdot6} \times \cdot375) +$

$(0 \cdot 375^{2 \cdot 6} \times 2 \cdot 625)$ or $(17 \cdot 4 \times \cdot 375) + (\cdot 0781 \times 2 \cdot 625) = 6 \cdot 73$, which is $t^{2 \cdot 6} \times b$ in the rule (234), hence $162900 \times 6 \cdot 73 \div 25 = 43850$ lbs. = F, or the breaking weight by flexure. The area of the section $= 2 \cdot 1$ square inches, hence $C_P = 2 \cdot 1 \times 42560 = 89380$ lbs., and $\frac{3}{4}$ $C_P = 67030$ lbs.:—then the rule in (164) becomes $(43850 \times 89380) \div (43850 + 67030) = 35350$ lbs. breaking weight of a pillar with both ends pointed, col. 6 of Table 54: experiment gave 42,000 lbs., col. 5.

The same pillar with both ends flat, gives $500000 \times 6 \cdot 73 \div 25 = 134600$ lbs. for the value of F, hence $(134600 \times 89380) \div (134600 + 67030) = 59670$ lbs. breaking weight, col. 3: experiment gave 62,989 lbs., col. 4.

(245.) "*Wrought-iron* L *Pillars.*"—This form of pillar frequently occurs in the struts of roofs and other structures, the determination of the strength is therefore a matter of considerable practical importance. We have first to find the direction in which such a pillar will fail by flexure, which of course will be the one in which it is the weakest.

Let Fig. 48 be the section of a 3-inch angle-iron, $\frac{3}{8}$ inch thick, which as a pillar may fail by flexure in one of three directions indicated by the arrows A, B, C. To find $t^{2 \cdot 6} \times b$ in the direction A we must calculate from the line N for wrought iron (378), and we have $(\frac{3}{8}^{2 \cdot 6} \times 3) + \left\{3^{2 \cdot 6} - \frac{3}{8}^{2 \cdot 6}) \times \frac{3}{8}\right\}$ or $(\cdot 0781 \times 3) + \left\{17 \cdot 4 - \cdot 0781) \times \cdot 375\right\} = 6 \cdot 73$. In the other direction B, we must calculate from the line P, and we obtain $\left\{3^{2 \cdot 6} - 2\frac{5}{8}^{2 \cdot 6}) \times 3\right\} + (2\frac{5}{8}^{2 \cdot 6} \times \frac{3}{8})$ or $\left\{17 \cdot 4 - 12 \cdot 28) \times 3\right\} + (12 \cdot 28 \times \cdot 375) = 19 \cdot 965$, or nearly three times the strength in the direction A. For the direction C, t must be measured angle-wise, and in our case becomes $2\frac{1}{4}$ inches as in the figure, the breadth b becoming the sum of the thicknesses of the two ribs, or $\frac{3}{4}$ inch. We thus obtain $2\frac{1}{4}^{2 \cdot 6} \times \frac{3}{4}$ or $8 \cdot 23 \times \cdot 75 = 6 \cdot 1725$, which being less than either of the others, shows that the pillar will fail in the direction C.

(246.) Experiments were made on pillars whose section is given by Fig. 47; there were four, flat at both ends, the lengths

being $1\frac{1}{2}$, 3, 4, and 5 feet respectively; one was 5 feet long with both ends pointed or approximately so. Admitting that the pillar will fail by flexure in the direction of the arrow as in (245), t will be $2\frac{1}{4}$ inches, and $b = \frac{5}{8}$ inch, and taking M_P from Table 34 at 223 tons, and $2\frac{1}{4}^{2\cdot6}$ at $8\cdot23$ from Table 35, we obtain for $1\frac{1}{2}$ feet long $223 \times 8\cdot23 \times \cdot625 \div 2\cdot25 = 510\cdot2$ tons for the value of F. This requires correction for incipient crushing (164): the area being $1\cdot78$ inch, and the crushing strength of wrought iron in pillars = 19 tons per square inch (201), C_P becomes $1\cdot78 \times 19 = 33\cdot82$ tons, and $\frac{3}{4}\ C_P = 25\cdot36$ tons; then $(510\cdot2 \times 33\cdot82) \div (510\cdot2 + 25\cdot36) = 32\cdot22$ tons, col. 3: experiment gave $31\cdot3$ tons, col. 4. Table 54 has been calculated in this way for the several lengths, from $1\frac{1}{2}$ to 5 feet.

For the pillar with both ends pointed, $M_P = 72\cdot7$ from Table 34, and the rule becomes $72\cdot7 \times 8\cdot23 \times \cdot625 \div 25 = 14\cdot96$ tons $=$ F: then $(14\cdot96 \times 33\cdot82) \div (14\cdot96 + 25\cdot36) = 12\cdot55$ tons, col. 6, the reduced breaking weight: experiment gave $12\cdot25$ tons, col. 5

(247.) "*Wrought-iron* $\sqcup$ *Pillars.*"—Experiments were made on two pillars whose section is given by Fig. 49. Such a pillar might fail by flexure in one of three directions as indicated by the arrows A, B, C. In the direction A, the thin ribs R, R would be subjected to compression, therefore (378) we must calculate the strength from the line N, and $t^{2\cdot6} \times b$ by which the strength is governed, becomes $(\frac{3}{8}^{2\cdot6} \times 3) + \left\{1\frac{3}{4}^{2\cdot6} - \frac{3}{8}^{2\cdot6}) \times \frac{3}{4}\right\}$ or $(\cdot0781 \times 3) + \left\{4\cdot28 - \cdot0781) \times \cdot75\right\} = 3\cdot3857$. In the direction B, we have $(1\frac{3}{4}^{2\cdot6} \times 3) - (1\frac{3}{8}^{2\cdot6} \times 2\frac{1}{4})$ or $(4\cdot28 \times 3) - (2\cdot28 \times 2\cdot25) = 7\cdot71$, or about double that in the direction A. In the third direction C, we obtain $(3^{2\cdot6} \times 1\frac{3}{4}) - (2\frac{1}{4}^{2\cdot6} \times 1\frac{3}{8})$ or $(17\cdot4 \times 1\cdot75) - (8\cdot23 \times 1\cdot375) = 19\cdot134$, or nearly six times the strength in the direction A. We find from this that the strength of the pillar must be calculated for A; then for both ends flat $M_P = 223$, and for a length of 5 feet we have $223 \times 3\cdot3857 \div 25 = 30\cdot23$ tons $=$ F. Reducing for incipient crushing, the area being $2\cdot15$ square inches, C_P becomes

$2 \cdot 15 \times 19 = 40 \cdot 85$ tons, and $\frac{3}{4}$ $C_P = 30 \cdot 64$ tons, hence $(30 \cdot 23 \times 40 \cdot 85) \div (30 \cdot 23 + 30 \cdot 64) = 20 \cdot 28$ tons, col. 3. Experiment gave $17 \cdot 1$ tons, col. 4.

For the same pillar with two pointed ends we obtain $72 \cdot 74 \times 3 \cdot 3857 \div 25 = 9 \cdot 85$ tons $=$ F, which being less than $\frac{1}{4}$ C_P, correction for incipient crushing is not required (169), and the breaking weight is $9 \cdot 85$ tons: experiment gave 14 tons, col. 5.

The experiments gave $17 \cdot 1$ and 14 tons respectively; the difference is only $3 \cdot 1$ tons, whereas the calculated difference is $20 \cdot 28 - 9 \cdot 85 = 10 \cdot 43$ tons, which shows that in the experiments the assumed form at the ends was not complied with perfectly in either case (241). Here again the experimental results fall between the calculated ones, as in (242).

Table 54 gives a collected statement of the results of experiment and calculation on these pillars of unusual sections (242) to (247).

"*Hollow Rectangular Pillars of Wrought Iron.*"—For rectangular pillars other than square the rules in (230) become:—

$$(248.) \qquad F = M_F \times \left\{ t^{2 \cdot 6} \times b\,) - (t_0^{2 \cdot 6} \times b_0 \right\} \div L^2.$$

In which t = the least dimension of the pillar externally, and t_0 internally; b = the breadth or largest dimension externally, and b_0, internally, all in inches, and the rest as in (233), namely L = length in feet, M_F = multiplier, whose value is given by Table 34, and F = the breaking weight by flexure, in lbs. or tons dependent on M_F.

Thus for a pillar 3×4 externally, and $2\frac{1}{2} \times 3\frac{1}{2}$ internally, in which $t = 3$; $b = 4$; $t_0 = 2\frac{1}{2}$; $b_0 = 3\frac{1}{2}$; $L = 9$ feet, both ends being flat, we have $223 \times \left\{ 3^{2 \cdot 6} \times 4) - (2\frac{1}{2}^{2 \cdot 6} \times 3\frac{1}{2} \right\} \div 81$, or $223 \times \left\{ 17 \cdot 4 \times 4) - (10 \cdot 84 \times 3 \cdot 5 \right\} \div 81 = 87 \cdot 24$ tons $=$ F. Correcting for incipient crushing, the area being $3 \cdot 25$ square inches, C_P becomes $3 \cdot 25 \times 19 = 61 \cdot 75$ tons, $\frac{3}{4}$ $C_P = 46 \cdot 31$ tons, therefore by the rule in (164), $(87 \cdot 24 \times 61 \cdot 75) \div (87 \cdot 24 + 46 \cdot 31) = 40 \cdot 34$ tons breaking weight. In this case, as in (232), correction for incipient "Wrinkling" is not required.

(249.) "*Incipient Wrinkling.*"—It is shown in (306) that thin wrought-iron plates, where the breadth is considerable, will fail by wrinkling or corrugating under a compressive load, with a strain very much less than the absolute crushing strength of wrought iron. For plates forming the sides of a pillar, we have the rule:—

(250.) $$W_W = (\sqrt{t} \div \sqrt{b}) \times 80.$$

In which t = the thickness of plate, and b = the breadth unsupported, both in inches, W_W = the compressive wrinkling strain in tons per square inch. Thus for a plate $\frac{1}{8}$ inch thick, and 9 inches wide, forming one side of a pillar 9 inches square, we have $(\sqrt{\frac{1}{8}} \div \sqrt{9}) \times 80$, or $(\cdot 3535 \div 3) \times 80 = 9 \cdot 427$ tons per square inch, being less than half the crushing strength of wrought iron in pillars (201), which is 19 tons per square inch. When, however, the plate is thick in proportion to the breadth, the wrinkling strain may become greater than the crushing strength, and in that case the strength of the pillar is governed by the latter. An example of this is given in (409). Table 63 gives the wrinkling strain for wrought-iron plates of various thicknesses and breadth when forming part of a pillar particularly, for, as shown in (321) and Table 62, the resistance of plate-iron in beams is greater than in pillars in the ratio of 104 to 80.

(251.) It should be observed that the strength of a pillar is governed by that of the weakest plate:—for instance, in a rectangular pillar whose sides are in the ratio of 2 to 1, with equal thickness all over, the narrow end plates are stronger than the sides in the ratio of $\sqrt{2}$ to $\sqrt{1}$, or $1 \cdot 414$ to $1 \cdot 0$, but when the wide plate fails by wrinkling under the strain, the whole of the load is thrown upon the end plates, and they fail under that increased load in spite of their superior strength, which in such a case goes for nothing. Evidently, the most judicious course is to make the wide plates thicker than the narrow ones, so as to produce *equality* of strength all over; and as the resistance to wrinkling is proportioned to $\sqrt{t} \div \sqrt{b}$ by the rule in (250), it follows that the thickness of plate should be simply propor-

tionate to the breadth unsupported, so that for breadths in the ratio of 1, 2, 3, the thicknesses should be in the ratio 1, 2, 3 also.

When it is thus found that the Wrinkling strain is less than the crushing, the correction of F must be made by the rule:—

(252.) $$P_W = F \times C_W \div (F + \tfrac{3}{4} C_W).$$

But when the crushing strain is less than the wrinkling, the rule becomes:—

(253.) $$P_C = F \times C_P \div (F + \tfrac{3}{4} C_P).$$

In which F = the breaking weight of the pillar due to flexure by the rule in (248), &c.; C = the specific crushing strength of wrought iron per square inch, namely 19 tons, or 42,560 lbs.; C_P = resistance of the pillar to *crushing*, due to the area of the section, and the value of C; W_W = the wrinkling strain in lbs., tons, &c., per square inch, which varies with the thickness and breadth of the plate, and may be calculated by the rule in (250) or (308). C_W = the wrinkling strain on the whole pillar, due to the area of the section, and the value of W_W; P_C = the breaking weight of the pillar reduced for "Incipient Crushing" in tons, lbs., &c., dependent on the terms of F and C; P_W = the breaking weight of the pillar reduced for "Incipient Wrinkling" in tons, lbs., &c.

Of course it will be understood that the whole must be taken in the same terms; for instance, if F be taken in Tons, all the rest must be in Tons also.

(254.) "*Experimental Results.*"—Table 55 gives the results of 29 experiments by Mr. Hodgkinson on square and rectangular pillars of thin wrought-iron plate; col. 9 gives the calculated breaking weights by flexure, or F; the square pillars by the rules in (230), and the rectangular ones by those in (248).

(255.) "*Square Pillars.*"—As an example of the former we may take No. 4, whose section is shown by Fig. 50; to find $S^{3\cdot6}$, we have the logarithm of $8\cdot1 = 0\cdot908485 \times 3\cdot6 = 3\cdot270546$, the natural number due to which = 1864. Then for $s^{3\cdot6}$, the logarithm of $7\cdot98 = 0\cdot902003 \times 3\cdot6 = 3\cdot247211$, the natural

TABLE 55.—Of the STRENGTH of SQUARE and RECTANGULAR

No. of Experiment.	Dimensions.				Breaking-
	External Dimensions.	Thickness.	Area of Section.	Length.	By Experiment.
	in. in.	in.	sq. in.	ft.	lbs.
1	8·5 × 8·4	·2415	8·4665	10	225,835‡
2	8·5 × 8·375	·2191	7·7367	10	198,955
3	8·37 × 8·37	·139	4·9262	10	100,395
4	8·1 × 8·1	·06	2·07	10	27,545
5	" "	"	"	$7\frac{5}{8}$	27,531
6†	8·1 × 8 1	·0637	3·551	10·1	70,070
7†	" "	"	"	9·84	82,027
8†	" "	"	"	5·0	82,411
9†	" "	"	"	$2\frac{1}{3}$	85,771
10	8·5 × 4·75	·264	7·326	10	197,163
11*	8·5 × 4·75	·25	8·3466	10	207,915‡
12	8·4 × 4·25	·26	6·89	10	206,571
13	8·175 × 4·1	·061	1·532	10	23,289
14	" "	"	"	$7\frac{2}{3}$	24,843
15	" "	"	"	$2\frac{1}{3}$	24,395
16*	8·1 × 4·1	·059	1·885	10	43,673
17*	" "	"	"	$7\frac{5}{8}$	45,451
18*	" "	·06	"	$3\frac{2}{3}$	41,259
19*	" "	"	"	$1\frac{5}{8}$	49,035
20	4·25 × 4·25	·134	2·3947	10	51,690
21	" "	"	"	7·5	55,562
22	4·44 × 4·44	·136	2·539	1·68	73,034
23	4·25 × 4·25	·083	1·484	10	37,354
24	" "	·085	1·52	5	35,850
25	" "	"	"	2·5	41,674
26	4·1 × 4·1	·06	1·02	10	19,646
27	4·1 × 4·1	·03	·504	10	5,534
28	" "	"	"	5	5,803
29	" "	"	"	2·5	6,251
(1)	(2)	(3)	(4)	(5)	(6)

* Nos. 11, 16, 17, 18, and 19 were two-celled, as in Fig. 52.

‡ Nos. 1 and 11 sustained the strain given in col. 6, but were

PILLARS, of THIN WROUGHT-IRON PLATES, both ends Flat.

-Weight by Experiment and Calculation.

By Calculation.					By Experiment per Square Inch in Tons.
Lbs.	Error per Cent.	By Flexure.	Wrinkling Strain.		
			Total.	Per Sq. Inch.	
		lbs.	lbs.	tons.	
235,150	..	2,054,000	257,300	13·57	..
204,400	+ 2·7	1,864,000	222,600	12·85	11·480
106,200	+ 5·8	1,196,000	113,700	10·31	9·098
24,390	− 11·4	483,600	25,350	5·468	5·926
24,780	− 10·0	822,700	"	"	5·938
72,280	+ 3·1	573,800	79,830	10·04	8·809
72,630	− 11·5	604,400	"	"	10·312
77,830	− 5·6	2,341,000	"	"	10·361
79,380	− 7·5	10,750,000	"	"	10·783
188,100	− 4·6	753,700	231,400	14·10	12·015
257,400	..	772,700	343,200	18·36	..
170,700	− 17·4	598,200	217,300	14·08	13·384
21,580	− 7·3	143,100	23,720	6·911	6·786
22,110	− 11·0	243,400	"	"	7·239
23,560	− 3·4	2,628,000	"	"	7·108
33,610	− 23·0	147,900	40,520	9·596	9·877
36,190	− 20·4	254,700	"	"	10·764
38,530	− 4·4	1,101,000	"	"	9·772
40,280	− 17·7	5,606,000	"	"	11·613
58,630	+ 13·4	190,400	76,231	14·21	9·636
65,230	+ 17·4	338,500	"	"	10·358
78,950	+ 8·1	7,701,000	79,640	14·00	13·620
30,250	− 19·0	121,600	37,170	11·18	11·237
36,340	+ 1·0	498,500	38,440	11·31	10·529
39,680	− 4·8	1,994,000	"	"	12·240
18,360	− 6·5	81,250	22,110	9·675	8·5986
6,767	+ 22·2	40,870	7,725	6·843	4·902
7,460	+ 28·5	163,480	"	"	5·140
7,655	+ 22·4	653,920	"	"	5·537
(7)	(8)	(9)	(10)	(11)	(12)

† Nos. 6, 7, 8, and 9 were four-celled, Fig. 54.
not broken, nor were they likely to break, as shown by col. 7.

number due to which = 1767: from these we obtain 1864 − 1767 = 97 for the value of $S^{3\cdot6} - s^{3\cdot6}$ in the rule in (230). Taking the value of M_P from Table 34, at 498,500 lbs., we obtain $498500 \times 97 \div 100 = 483600$ lbs. = F, as in col. 9.

(256.) This will require correction for "Incipient Wrinkling": we have first to find W_W by the rule (250), namely $W_W = (\sqrt{t} \div \sqrt{b}) \times 80$, which in our case becomes $(\sqrt{\cdot06} \div \sqrt{8\cdot1}) \times 80$, or $(\cdot2449 \div 2\cdot846) \times 80 = 5\cdot468$ tons per square inch, as in col. 11. This is a very low result, and is due to the extreme thinness of the plate. To find C_W in lbs., the area of the whole section being 2·07 square inches by col. 4, we have $5\cdot468 \times 2240 \times 2\cdot07 = 25350$ lbs., as in col 10, therefore $\frac{3}{4}\,C_W = 19010$ lbs., and the rule (252), or $P_W = F \times C_W \div (F + \frac{3}{4}\,C_W)$ becomes $483600 \times 25350 \div (483600 + 19010) = 24390$ lbs., the reduced breaking weight, as in col. 7. The experiment gave 27,545 lbs., hence we have $24390 \div 27545 = \cdot886$, showing an error of $1\cdot0 - \cdot886 = \cdot114$, or − 11·4 per cent., as in col. 8.

We may now give an illustration of the error that would have occurred in this case if we had neglected "Wrinkling" and calculated for incipient *crushing* in the ordinary method for solid pillars. Thus C, or the crushing strength of wrought iron in pillars, being 19 tons, or 42,560 per square inch, and the area 2·07 square inches, C_P becomes $42560 \times 2\cdot07 = 88100$ lbs., and $\frac{3}{4}\,C_P = 66070$ lbs.; then the rule $P_C = F \times C_P \div (F + \frac{3}{4}\,C_P)$, becomes $483600 \times 88100 \div (483600 + 66070) = 77510$ lbs., whereas experiment gave 27,545 lbs. only. The calculated strength is $77510 \div 27545 = 2\cdot814$ times the experimental; an error of 181·4 per cent.!

(257.) "*Rectangular Pillars.*"—As an example of the method of calculating the strength of rectangular pillars of thin plate-iron by the rules in (248) and (252), &c., we will take No. 15 in Table 55, whose section is shown by Fig. 51. For $t^{2\cdot6}$, the logarithm of 4·1, or $0\cdot612784 \times 2\cdot6 = 1\cdot593238$, the natural number due to which is 39·2, hence $t^{2\cdot6} \times b$, becomes $39\cdot2 \times 8\cdot175 = 320\cdot5$; then for $t_0^{2\cdot6}$, the logarithm of 3·978, or $0\cdot599665 \times 2\cdot6 = 1\cdot559129$, the natural number due to which is 36·235, and $t_0^{2\cdot6} \times b_0$, becomes $36\cdot235 \times 8\cdot053 = 291\cdot8$.

With these values, and a length of 2·333 feet, the rule in (248), namely $F = M_P \times \left\{ t^{2.6} \times b) - (t_0^{2.6} \times b_0 \right\} \div L^2$, becomes in our case $498500 \times (320 \cdot 5 - 291 \cdot 8) \div 5 \cdot 444 = 2,628,000$ lbs. breaking weight by flexure = F, as in col. 9.

(258.) Correcting this result for "Incipient Wrinkling" (249), we have to find W_W by the rule (250), or $W_W = (\sqrt{t} \div \sqrt{b}) \times 80$, which in our case becomes $(\sqrt{\cdot 061} \div \sqrt{8 \cdot 175}) \times 80$, or $(\cdot 247 \div 2 \cdot 859) \times 80 = 6 \cdot 911$ tons per square inch, and the area of the whole section being 1·532 square inch by col. 4, we obtain $6 \cdot 911 \times 2240 \times 1 \cdot 532 = 23720$ lbs. for the value of C_W, and 17790 lbs. for $\frac{3}{4}$ C_W. Then the rule in (252), namely $P_W = F \times C_W \div (F + \frac{3}{4} C_W)$ becomes $2628000 \times 23720 \div (2628000 + 17790) = 23560$ lbs., the reduced breaking weight, as in col. 7. Experiment gave 24395 lbs., as in col. 6; hence $23560 \div 24395 = \cdot 966$, showing an error of $1 \cdot 0 - 0 \cdot 966 = \cdot 034$, or $-$ 3·4 per cent., as in col. 8.

(259.) If in this case we had neglected "Wrinkling," and corrected F for incipient crushing, the error would have been enormous. Thus the area being 1·532 square inch, C_P becomes $42560 \times 1 \cdot 532 = 65200$ lbs., and $\frac{3}{4} C_P = 48900$ lbs.: then the rule (253) becomes $2628000 \times 65200 \div (2628000 + 48900) = 80585$ lbs., whereas experiment gave 24,395 lbs. only. The calculated strength by this erroneous method is $80585 \div 24395 = 3 \cdot 30$ times the experimental, showing an error of 230 per cent.!

(260.) "*Cellular Pillars.*"—Several of the rectangular pillars in Table 55 were cellular, having a plate in the centre by which they were divided into two compartments or cells: in calculating the strength of such pillars some modification in the method of applying the rules becomes necessary. We will take No. 17 as an illustration, the section of which is shown by Fig. 52. Obviously the pillar would fail by flexure in the direction of its least dimension, t will therefore be 4·1 inches, and b = 8·1 inches, as usual. To find b_0, we may imagine the central plate x to be divided into two equal portions, one of which is added to each of the two side plates z, z, and we thus

obtain the simple section Fig. 53: we can then proceed with the calculation in the usual way, as illustrated in (257).

Thus to find $t^{2\cdot6}$, the logarithm of 4·1, or 0·612784 × 2·6 = 1·593238, the natural number due to which is 39·2; then for $t_0^{2\cdot6}$, the logarithm of 3·982, or 0·600101 × 2·6 = 1·560263, the natural number due to which is 36·33: the value of M_P = 498500 lbs. as before, and the length or $7\cdot625^2$ being 58·14, the rule in (248), namely $F = M_P \times \left\{t_0^{2\cdot6} \times b) - (t_0^{2\cdot6} \times b_0\right\} \div L^2$, becomes $498500 \times \left\{39\cdot2 \times 8\cdot1) - (36\cdot33 \times 7\cdot923\right\} \div 58\cdot14 = 254700$ lbs. = F, as in col. 9.

(261.) In correcting for incipient wrinkling it must be observed that the effect of the central plate is to reduce the breadth of the wide plate, in our case to half or 4·05 inches; then the greatest breadth unsupported is the end plate or 4·1 inches, and the wrinkling strain must be calculated for that width. The rule in (250), namely $W_W = (\sqrt{t} \div \sqrt{b}) \times 80$, becomes in our case $(\sqrt{\cdot059} \div \sqrt{4\cdot1}) \times 80$, or $(\cdot2429 \div 2\cdot035) \times 80 = 9\cdot596$ tons per square inch, as in col. 11: then the area of the whole section being 1·885 square inches by col. 4, C_W becomes 9·596 × 2240 × 1·885 = 40520 lbs., as in col. 10, and $\frac{3}{4}\,C_W$ = 30390 lbs. The rule in (252), namely $P_W = F \times C_W \div (F + \frac{3}{4}\,C_W)$, becomes 254700 × 40520 ÷ (254700 + 30390) = 36190 lbs., as in col. 7. Experiment gave 45,451 lbs., as in col. 6; hence 36190 ÷ 45451 = ·796, showing an error of 1·0 − 0·796 = ·204, or − 20·4 per cent., as in col. 8.

(262.) "*Four-celled Pillars.*"—Four of the pillars in Table 55, Nos. 6 to 9, were divided by central plates into four compartments or cells, as shown by Fig. 54, the lengths varying from 10·1 feet to $2\frac{1}{3}$ feet. In calculating the strength of this pillar we may assume that it will bend, and fail by flexure, say in the direction of the arrow. In that case the cross-plate B will be in the neutral axis N, A, and will add nothing to the strength of the pillar in resisting *flexure.* The *effective* section therefore becomes as in Fig. 55, and the case is assimilated to (260), and may be calculated in the same way: adding the thickness of the plate *e* to *f* and *g* in equal portions, we finally reduce the

case to Fig. 56, with this exception, that the "Wrinkling" strain will be that due to the breadth H, or 4·05 inches, Fig. 55, not 8·1 inches as at D in Fig. 56; again, although the cross-plate B adds nothing to the strength in resisting flexure, it has its full effect in resisting wrinkling, and so finally affects the strength of the pillar.

(263.) Taking No. 6 as an example of the mode of calculating pillars of this form:—to find $t^{2\cdot 6}$, the logarithm of 8·1 or 0·908485 × 2·6 = 2·362061, the natural number due to which is 230·176. For $t_0^{2\cdot 6}$, the logarithm of 7·79726, or 0·901599 × 2·6 = 2·344157, the natural number due to which is 220·9. The rule (248), namely $F = M_P \times \left\{t^{2\cdot 6} \times b) - (t_0^{2\cdot 6} \times b_0\right\} \div L^2$, becomes $498500 \times \left\{230\cdot 176 \times 8\cdot 1) - (220\cdot 9 \times 7\cdot 9089\right\} \div 102 = 573800$ lbs. = F, as in col. 9.

Reducing for incipient wrinkling (249), we have to take H in Fig. 55, or half the side of the square for the breadth, as explained in (262), and the rule $W_w = (\sqrt{t} \div \sqrt{b}) \times 80$, becomes $(\sqrt{\cdot 0637} \div \sqrt{4\cdot 05}) \times 80$, or $(\cdot 25239 \div 2\cdot 012) \times 80 = 10\cdot 04$ tons per square inch, as in col. 11, and the area of the whole section being 3·551 square inches by col. 4, we obtain 10·04 × 2240 × 3·551 = 79830 lbs. for the value of C_w, therefore 59,870 lbs. for $\frac{3}{4}$ C_w, and the rule (252), or $P_w = F \times C_w \div (F + \frac{3}{4} C_w)$ becomes 573800 × 79830 ÷ (573800 + 59870) = 72280 lbs. breaking weight, as in col. 7. Experiment gave 70,070 lbs., as in col. 6; hence 72280 ÷ 70070 = 1·031, showing an error of + 3·1 per cent., as in col. 8.

(264.) Table 55 has been calculated throughout in the way we have thus explained and illustrated:—the mean average error on the 27 experiments in which the pillars *were broken*, is − 2¼ per cent., for the sum of all the minus errors in col. 8, is 185·5, and of all the plus errors 124·6, giving a difference of 185·5 − 124·6 = 60·9, which on 27 experiments gives an average of 60·9 ÷ 27 = 2·25 per cent. The range of the errors is pretty nearly equal, the greatest + error being 28·5 per cent., and the greatest − error, 23 per cent. (959). It will be observed that the highest wrinkling strain in col. 11, is 18·36 tons per

square inch, which being less than 19 tons, the *crushing* strength of wrought iron in pillars, correction had to be made for "Incipient Wrinkling" rather than "Incipient Crushing," as explained in (252). Comparing cols. 7 and 9, it will be seen that correction was required in every case, without exception. In two cases, Nos. 1 and 11, the pillars were not strained up to the breaking point, but it is satisfactory to observe that the calculation shows they were not likely to break with the experimental strain, the calculated breaking weight in col. 7, being in both cases in excess of that strain in col. 6.

(265.) "*Economic Value of Cells.*"—We are now in a position to estimate the value of division-plates in cellular beams as a matter of economy. Taking a plain rectangular pillar with sides in the ratio of 2 to 1, and so short that the case is practically governed by the resistance to wrinkling, irrespective of flexure; by the addition of a central plate, we increase W_w, or the wrinkling strain per square inch, in the ratio of $\sqrt{2}$ to $\sqrt{1}$ or $1\cdot414$ to $1\cdot0$, or $41\cdot4$ per cent. But the area of the whole section is also increased in the ratio of 7 to 6, or from $1\cdot0$ to $7 \div 6 = 1\cdot167$ or $16\cdot7$ per cent., so that the total increase in strength is $1\cdot414 \times 1\cdot167 = 1\cdot65$, or 65 per cent. As the strength is increased by the centre plate 65 per cent., and the weight $16\cdot7$ per cent., the net *economic* advantage from it is $65 - 16\cdot7 = 48\cdot3$ per cent.

But this will apply only to a pillar so short that the strength is governed exclusively by the resistance to wrinkling, irrespective of flexure, which in long pillars will so affect the case that no general ratio can be given, as it will vary with the length and general sizes of the pillar.

(266.) The fairest comparison may be made by taking a concrete case: say we take the four-celled pillar No. 6 in Table 55 and Fig. 54, whose *calculated* strength as given in col. 7 was 72,280 lbs. Then removing the *two* centre-plates and adding the material in them to the *four* side-plates we obtain a plain square pillar as in Fig. 57, whose total area or weight is precisely the same as that of the cellular one Fig. 54, thus the 4 sides in the plain pillar, or $\cdot09555 \times 4 = \cdot3822$, and the 4 sides

and 2 centre-plates in the cellular one give $\cdot 0637 \times 6 = \cdot 3822$ also.

Calculating as in (254); the logarithm of $8\cdot 1$ or $0\cdot 908485 \times 3\cdot 6 = 3\cdot 270546$, the natural number due to which is $1864\cdot 4$; then the logarithm of $7\cdot 9089$ or $0\cdot 898117 \times 3\cdot 6 = 3\cdot 233221$, the natural number due to which is 1711. The rule in (230), namely $F = M_P \times (S^{3\cdot 6} - s^{3\cdot 6}) \div L^2$, becomes in our case $498500 \times (1864\cdot 4 - 1711) \div 102 = 749700$ lbs. = F.

Reducing for incipient wrinkling, the rule in (250) namely $W_w = (\sqrt{t} \div \sqrt{b}) \times 80$, becomes $(\sqrt{\cdot 09555} \div \sqrt{8\cdot 1}) \times 80$, or $(\cdot 309 \div 2\cdot 846) \times 80 = 8\cdot 686$ tons per square inch, and the area being as before $3\cdot 551$ square inches, C_W becomes $8\cdot 686 \times 2240 \times 3\cdot 551 = 69090$ lbs., and $\frac{3}{4}\, C_W = 51820$ lbs. Then the rule in (252), namely $P_W = F \times C_W \div (F + \frac{3}{4}\, C_W)$ becomes $749700 \times 69090 \div (749700 + 51820) = 64620$ lbs.

Now the cellular pillar of the same external size, length, and area or weight of metal, gave 72280 lbs., or $72280 \div 64620 = 1\cdot 1185$ or $11\cdot 85$ per cent. in excess of the plain pillar. The advantage of the cellular form is thus shown to be the greatest in the case of short pillars; see (265), which gave $48\cdot 3$ per cent.

(267.) "*Steel Pillars.*"—The only experiments we have, are three by Mr. Hodgkinson, the results of which are given in Table 44, Nos. 28 to 30. Under these circumstances, all we can do is to assume that steel pillars follow the same laws as those of wrought iron in (196), (225), &c.: the values of M_P are given by Table 34, and are based on the experiments. It will be observed that they follow the ratio 1, 2, 3 exactly, which agrees almost precisely with the experimental ratios when correction is made for incipient crushing.

In Table 44, col. 9 has been calculated by the rule $M_P \times D^{3\cdot 6} \div L^2$:—thus with No. 30, having both ends flat for $D^{3\cdot 6}$ we have the logarithm of $\cdot 87 = \bar{1}\cdot 939519 \times 3\cdot 6 = \bar{1}\cdot 782268$, the natural number due to which is $\cdot 6058$; then $325500 \times \cdot 6058 \div 6\cdot 25 = 31550$ lbs. = F, as in col. 9.

(268.) Correcting for incipient crushing, we have the same difficulty as with wrought iron (503) in determining the crushing strength of Steel. From experiments on the transverse

strength (507) we found it to be 61·48 tons per square inch:—if we adopt that for steel pillars, only the one with two ends flat in the experiments requires correction for incipient crushing (164), and applying it to that one, we obtain a result too high by 6·7 per cent. For reasons indicated in (504), &c., the crushing strength of malleable metals appears to be less in pillars than in beams:—thus experiments on the transverse strength of wrought iron (520) seem to give C = 24 tons per square inch, but in pillars we found by the experiments (201) that C = 19 tons only. Following the same course with steel, we find the crushing strength in pillars to be 52 tons, or 116,480 lbs. per square inch.

With this value for C, we obtain for our pillar ·87 inch diameter, $\cdot 87^2 \times \cdot 7854 \times 116480 = 69240$ lbs. for the value of C_P, and 51,930 lbs. for $\frac{3}{4}\,C_P$; hence $(31550 \times 69240) \div (31550 + 51,930) = 26070$ lbs., col. 7, breaking weight, or practically the same as by experiment, which gave 26,059 lbs., col. 6.

(269.) To compare the relative strength of wrought iron and steel pillars we must observe that the ratio will not be constant, but will vary with the length. With long pillars breaking merely by flexure, the ratios are simply those of the respective multipliers in Table 34; thus for pillars with both ends pointed, we obtain $108500 \div 95848 = 1\cdot 132$, or 13·2 per cent. in favour of Steel:—by theory based on the transverse strength and deflection, we obtained 9 per cent. (300). But with very short pillars, where the strength is dominated almost exclusively by the crushing strain, the ratio will be 52 to 19, or nearly 3 to 1 in favour of steel. The ratio will vary between those extremes, dependent on the length of the pillar in proportion to the diameter.

(270.) The length of steel pillars with which the correction for incipient crushing becomes nil (169) may be found by the rule (203). Thus, for pillars 1 inch diameter, $C_P = \cdot 7854 \times 52 = 40\cdot 84$ tons, therefore $\frac{1}{4}\,C_P = 10\cdot 21$ tons, and the rule becomes for those with two pointed ends $(48\cdot 44 \times 1^{3\cdot 6} \div 10\cdot 21)\sqrt{\ } \times 12 = 26\cdot 1$ inches long:—for those with one end flat and one pointed $(96\cdot 88 \times 1^{3\cdot 6} \div 10\cdot 21)\sqrt{\ } \times 12 = 37$ inches; and for those with both ends flat $(145\cdot 3 \times 1^{3\cdot 6} \div 10\cdot 21)\sqrt{\ } \times 12 =$

45·26 inches, &c. But these ratios of length to diameter will vary with the diameter, as shown by (148) and Table 39, which has been calculated by the rule (203).

(271.) "*Steel Piston-rods.*"—With long rods, or rather where the length is great in proportion to the diameter, the advantage of a steel pillar over a wrought iron one is not very great, as shown by (300). But strength is not the only, nor indeed the principal quality in which steel is superior as a piston-rod:—the continued action of passing to-and-fro through the gland, has a tendency to "score" the surface and wear longitudinal furrows in it; in resisting this tendency, the superior hardness of steel gives it a great advantage over wrought iron. For these reasons steel is used almost exclusively for first-class work, and as it is now so much reduced in price, it is probable that its use will become still more general for engines of all classes.

(272.) It is shown in (210) that a piston-rod is subjected alternately to tensile and compressive strains, both of which must be considered in calculating the strength:—also that the area at the screw or key-way is *half* only of that due to the diameter of the body of the rod. The mean tensile strength of Steel may be taken at 96,000 lbs. per square inch, hence the maximum strain admissible on the body of the rod is 48,000 lbs. per square inch, and the second column of Table 56 has been thus calculated. In any case where we are certain from the shortness of the rod, &c., that its strength will be governed by the tensile strain, we may find the area necessary direct by the rule:—

(273.) $$\text{Area} = W \times M_F \div 48000.$$

In which W = the strain on the rod due to the area of piston, pressure of steam, &c., and M_F = the "Factor of Safety" (880).

For the strength to resist the compressive strain at the up-stroke, the rod may be taken as a pillar flat at one end and pointed at the other, namely, flat at the piston, and pointed at the cross-head. Then taking M_P from col. 2 of Table 34 at 217,000 lbs., the rule in (197) becomes:—

(274.) $$F = 217000 \times D^{3\cdot6} \div L^2.$$

In which F = the breaking weight by flexure in lbs., L = the maximum length unsupported in feet, or the distance from the

gland of the cylinder to the centre of the cross-head at the top of the stroke, and D = the diameter in inches.

(275.) In many cases the results of this rule require correction for incipient crushing by the rule (164); taking the absolute crushing strain, or C, in Steel at 52 tons or 116,480 lbs. per square inch, we obtain the value of C_P in the third column of Table 56.

Thus, for a steel rod $3\frac{1}{2}$ inches diameter, 7 feet long, taking $3\frac{1}{2}^{3\cdot6}$ at 90·9 from col. 3 of Table 35, the rule (274) becomes $217000 \times 90\cdot9 \div 49 = 402500$ lbs. = F, or the breaking weight by flexure. Then C_P being by the third column, 1,121,000 lbs., $\frac{3}{4}\, C_P = 840500$ lbs., and the rule (164), or $P_C = F \times C_P \div (F + \frac{3}{4}\, C_P)$, becomes $402500 \times 1121000 \div (402500 + 840500) = 363000$ lbs., as in Table 56.

(276.) When the strength as a pillar as thus calculated exceeds the tensile strength as given in the second column, the latter governs the case and limits the strength (211). Table 56 has been calculated by a combination of the rules for flexure (274), incipient crushing (164), and tensile strength (272): thus with the $3\frac{1}{2}$-inch rod, the lengths 3, 4, 5 feet give strengths all alike, being limited by the tensile strength, or 461,800 lbs., as given by the second column. For the lengths 9 and 10 feet, the breaking weights are those due with flexure simply by the rule in (274), while the strains for the intermediate lengths of 6, 7, and 8 feet have been reduced for incipient crushing by the rule (164), &c.

(277.) Table 49 gives the particulars of steel piston-rods in practice; col. 6 has been calculated by the rules, and col. 7 gives the "Factor of Safety" (880), or the ratio of the breaking to the working strain:—it will be observed that it is very variable, which is due to the fact, that the sizes of piston-rods are usually fixed by "rule of thumb." The mean value of the Factor may be taken at 14, and we have thus obtained the sizes given by col. 8, with the help of Table 56:—as in most cases the actual lengths were intermediate between those in the Table, the nearest diameter had to be *estimated*, and will not be exact, but will usually be sufficiently so for practical purposes.

(278.) "*Timber Pillars.*"—By the experiments of Mr.

TABLE 56.—Of the STRENGTH of STEEL PISTON-RODS to resist TENSILE and COMPRESSIVE STRAINS.

Diameter.	Tensile Breaking Weight at the Screw or Key-way.	Crushing Strain. C_F.	Maximum Length of Rod unsupported, in Feet.								
			3	4	5	6	7	8	9	10	12
			Compressive Breaking Strain, in Lbs.								
in.	lbs.	lbs.									
1	37,700	91,480	23,775	13,560†	8,680†	..	..	..	..	..	..
1¼	58,910	142,900	47,310	29,840†	19,100†	13,260†	..	..	..	..	..
1½	84,820	205,800	82,690	56,420	37,320†	25,920†	19,050†	..	..	..	..
1¾	115,400	280,200	115,400*	91,330	65,080†	45,210†	33,220†	..	..	..	..
2	150,800	365,900	150,800*	136,900	101,300	72,950†	53,600†	41,030†	..	..	..
2¼	190,860	463,200	190,860*	190,860*	149,800	111,500†	81,940†	62,740†	..	..	..
2½	235,600	571,800	235,600*	235,600*	202,550	157,800	120,000†	91,890†	72,610†	..	..
2¾	285,100	691,800	285,100*	285,100*	241,650	190,550	169,200†	129,500†	102,200†	..	..
3	339,300	823,400	339,300*	339,300*	339,300*	278,000	224,300	177,000†	139,900†	..	..
3¼	398,200	966,300	398,200*	398,200*	398,200*	354,300	288,400	236,000†	186,500†	151,000†	..
3½	461,800	1,121,000	461,800*	461,800*	461,800*	442,400	363,000	300,800	243,500†	197,200†	..
3¾	530,150	1,286,000	530,150*	530,150*	530,150*	530,150*	446,900	396,900	310,800†	251,700†	..
4	603,200	1,464,000	..	603,200*	603,200*	603,200*	545,600	457,500	386,500	319,000†	..
4¼	680,960	1,652,000	..	680,960*	680,960*	680,960*	653,200	551,300	468,400	397,100†	..
4½	763,400	1,852,000	..	..	763,400*	763,400*	763,400*	656,600	560,600	481,700	339,100†
5	942,500	2,287,000	..	..	942,500*	942,500*	942,500*	899,600	774,900	670,800	494,300†
5½	1,140,400	2,767,000	..	..	..	1,140,400*	1,140,400*	1,140,400*	1,035,000	902,600	696,200†
6	1,357,200	3,293,000	..	..	..	1,357,200*	1,357,200*	1,357,200*	1,340,500	1,176,500	917,400
6½	1,517,400	3,865,000	..	..	..	..	1,517,400*	1,517,400*	1,517,400*	1,496,000	1,133,500
7	1,847,300	4,483,000	..	..	..	..	1,847,300*	1,847,300*	1,847,300*	1,847,300*	1,483,000

NOTE.—The strains marked * are limited by the tensile strength in col. 2; those marked † are due to flexure simply. The rest have been reduced for incipient crushing by the rule in (164), with the value of C_F given by col. 3.

Hodgkinson, we find that pillars of timber follow precisely the theoretical law (147), the strength of long pillars failing by flexure, being directly proportional to the fourth power of the diameter or side of square, and inversely as the square of the length :—hence we have for cylindrical pillars the rules :—

(279.) $$F = M_P \times D^4 \div L^2.$$

(280.) $$D = \sqrt[4]{(F \times L^2 \div M_P)}.$$

(281.) $$L = \sqrt[2]{(M_P \times D^4 \div F)}.$$

For square pillars these rules are modified and become :—

(282.) $$F = M_P \times S^4 \div L^2.$$

(283.) $$S = \sqrt[4]{(F \times L^2 \div M_P)}.$$

(284.) $$L = \sqrt[2]{(M_P \times S^4 \div F)}.$$

And for rectangular pillars other than square, into :—

(285.) $$F = M_P \times t^3 \times b \div L^2.$$

(286.) $$t = \sqrt[3]{\left\{F \times L^2 \div (M_P \times b\right\}}.$$

(287.) $$b = F \times L^2 \div (M_P \times t^3).$$

(288.) $$L = \sqrt[2]{(M_P \times t^3 \times b \div F)}.$$

In which D = the diameter of a solid cylindrical pillar in inches.
S = the side of a square pillar in inches.
t = the thickness or least dimension of a rectangular pillar in inches.
b = the breadth or greatest dimension of a rectangular pillar in inches.
F = the breaking weight by flexure in lbs., &c., dependent on M_P.
L = the length in feet.
M_P = Multiplier, whose value is given in lbs. and tons, by Table 34.

(289.) Mr Hodgkinson made some experiments on square and rectangular pillars of Dantzic Oak and Red Deal, the results of

which are given by Table 57: col. 8 has been calculated by the rules. Thus, taking No. 5 as an example, the value of M_P for a pillar of Dantzic Oak is given by Table 34 at 27,000 lbs., S^4 or $1\frac{3}{4}^4 = 9\cdot379$, and $2\cdot52^2 = 6\cdot35$: then rule (279) becomes $27000 \times 9\cdot379 \div 6\cdot35 = 39880$ lbs., as in col. 11, this being due by flexure.

This will require correction for incipient Crushing (163): by Table 32, C, or the crushing strength of Dantzic Oak = 6840 lbs. per square inch, and the area being $1\frac{3}{4} \times 1\frac{3}{4} = 3\cdot0625$ square inches, C_P becomes $6840 \times 3\cdot0625 = 20950$ lbs., as in col. 10, hence $\frac{3}{4}$ $C_P = 15724$ lbs., and the rule (164) becomes $39880 \times 20950 \div (39880 + 15724) = 15020$ lbs., the breaking weight, as in col. 8: experiment gave 14,305 lbs., hence 15020 $\div$ 14305 = $1\cdot05$, or + 5 per cent. error, as in col. 9.

(290.) As an example of rectangular pillars, we may take No. 10:—the value of M_P for a pillar of Red Deal with both ends flat = 24000 lbs. by Table 34; then rule (285) becomes $24000 \times 1\cdot41^3 \times 2\cdot82 \div 4\cdot833^2$, or $24000 \times 2\cdot803 \times 2\cdot82 \div 23\cdot36 = 8121$ lbs., the breaking weight by flexure as in col. 11. Correcting for incipient crushing: Table 32 gives 6167 lbs. for the value of C, or the specific crushing strength of Red Deal, and the area being $1\cdot41 \times 2\cdot82 = 4$ square inches, we obtain $6167 \times 4 = 24668$ lbs. for the value of C_P, hence $\frac{3}{4}$ C_P = 18500 lbs. Then the rule (164) becomes $8121 \times 24668 \div (8121 + 18500) = 7525$ lbs., as in col. 8: experiment gave 7681 lbs., hence $7525 \div 7681 = \cdot98$, or $-$ 2 per cent. error.

It should be stated that the actual dimensions of the rectangular pillars Nos. 10 and 11 were not given by Mr. Hodgkinson, but the areas were 4 square inches, and the ratios of the sides 2 to 1 in No. 10, and 3 to 1 in No. 11, and the dimensions in the Table were obtained from those data.

(291.) Table 57 has been calculated in this way throughout: the sum of the minus errors in col. 9 = $44\cdot3$, and of the plus errors $39\cdot6$, which on the 11 experiments gives a mean of $(44\cdot3 - 39\cdot6) \div 11 = -0\cdot427$, or less than $\frac{1}{2}$ per cent. The maximum minus and plus errors = $-15\cdot5$ and $+15\cdot9$ per cent. respectively, thus showing equality of range (959).

With the exception of Dantzic Oak and Red Deal, we have no

TABLE 57.—OF EXPERIMENTS on the STRENGTH of SQUARE and RECTANGULAR PILLARS of TIMBER.

No. of Experiment.	Kind of Timber.	Form of Ends.	Dimensions in Inches.	Length. Feet.	Breaking Weight in Lbs.					Calculated.	
					By Experiment.			By Calculation.		Crushing Strain. C_p.	By Flexure. F.
					Max.	Min.	Mean.	Lbs.	Error per Cent.		
1	Dantzic Oak	2 ends pointed	1·75 × 1·75	5·04	3,645	2,749	3,197	3,323	+ 3·0	20,950	3,323
2	"	1 flat, 1 pointed	" "	"	7,229	4,989	6,109	6,221	+ 1·9	"	6,646
3	"	2 ends flat ..	" "	"	11,179	8,323	9,625	8,129	− 15·5	"	9,970
4	"	"	" "	2·48	..	..	13,083	15,160	+ 15·9	"	41,180
5	"	"	" "	2·52	..	..	14,305	15,020	+ 5·0	"	39,880
6	"	"	" "	4·00	..	..	9,229	10,510	+ 13·8	"	15,830
7	"	"	1·02 × 1·02	3·84	1,791	1,679	1,754	1,928	− 10·0	7,116	1,983
8	"	"	1·5 × 1·5	3·84	8,069	7,545	7,888	6,858	− 13·1	15,390	9,273
9	Red Deal ..	"	2·0 × 2·0	4·833	12,385	11,602	11,993	11,605	− 3·2	24,668	16,440
10	"	"	1·41 × 2·82	"	..	..	7,681	7,525	− 2·0	"	8,121
11	"	"	1·153 × 3·46	"	..	..	4,349	4,329	− 0·5	"	4,329
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)

experimental data for Timber pillars, and are compelled to resort to theory, in order to obtain the values of M_P for other kinds of Timber: this we have done in (303), and we have thus obtained most of the numbers in Table 34. This method is of course not so satisfactory as direct experiment, but it is shown in (139) that as applied to Timber, the theoretical and experimental results practically agree with one another.

(292.) Table 58 gives the strength of square pillars of Red Deal calculated by the rule (282): we have selected the case of flat at one end and round at the other as approximating to ordinary conditions more nearly than any other. In most cases timber pillars are *nominally* flat at both ends, but this supposes that the surfaces between which the pillar is strained are perfectly parallel and unyielding, conditions which are seldom realised in practice: for example, when a soft-wood Bressummer is supported by a pillar, the effect of flexure in the latter is to compress the fibres of the former unequally, the soft wood yielding, so that the result is little if any better than it would have been with a round end. If we suppose that the foot of the pillar is well bedded on a large stone or cast-iron plate, and the upper end loaded by the Bressummer in the usual way, we should have in effect a pillar flat at one end and round at the other, being the conditions assumed in Table 58.

(293.) Say we take the case of a pillar 7 inches square; then by col. 5 of Table 34, the value of M_P for a pillar of Riga Fir, with one flat and one round end $= 7 \cdot 14$ tons, and with a length of say 12 feet, rule (282) becomes $7 \cdot 14 \times 7^4 \div 12^2$, or $7 \cdot 14 \times 2401 \div 144 = 119$ tons, the breaking weight by flexure. This requires correction for incipient crushing, being greater than $\frac{1}{4}$th C_P given by col. 4 of Table 58 (169). By col. 4 of Table 32, the specific resistance of Red Deal to crushing, or $C = 2 \cdot 75$ tons per square inch, and as we have $7^2 = 49$ square inches area, C_P becomes $2 \cdot 75 \times 49 = 135$ tons, as in col. 2: $\frac{3}{4}\, C_P = 101$ tons, col. 3: and $\frac{1}{4}\, C_P = 33 \cdot 8$, col. 4. Then in our case, the rule (164) becomes $P_C = 119 \times 135 \div (119 + 101) = 73$ tons, as in Table 58. By the use of col. 4, we can easily determine when the correction for incipient crushing is necessary: thus, for a pillar 3 inches square 10, 12, and 14 feet long, the

TABLE 58.—Of the STRENGTH of SQUARE PILLARS of

Side of Square.	Crushing Strain, in Tons.			LENGTH OF							
				5		6		7		8	
				BREAKING							
	C_P.	$\frac{3}{4}C_P$.	$\frac{1}{4}C_P$.	By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.
inches.											
1½	6·2	4·6	1·55	1·44	1·44	1·00	1·0	·74	·74	·56	·56
2	11·0	8·2	2·75	4·56	3·9	3·17	3·0	2·3	2·3	1·8	1·8
2½	17·2	12·9	4·30	11·1	7·9	7·8	6·5	5·7	5·3	4·4	4·3
3	24·7	18·5	6·18	23·0	13·8	16	11·5	11·8	9·6	9·0	8·1
4	44·0	33·0	11·0	73	31	51	27	37	23	29	21
5	68·7	51·5	17·2	178	53	124	48	91	44	70	40
6	99·0	74·2	24·8	370	82	257	77	189	71	145	65
7	135	101	33·8	686	118	476	108	350	102	268	98
8	176	132	44·0	1170	158	812	151	597	144	457	136
9	223	167	55·8	1875	205	1300	198	956	190	732	182
10	275	206	68·8	2856	256	1984	249	1457	241	1116	233
12	396	297	99·0	5922	377	4112	369	3022	361	2313	351

breaking weights *by flexure* = 5·8, 4·0, and 2·9 tons respectively, which are all *less* than 6·18 tons or $\frac{1}{4}$ C_P by col. 4; the correction is therefore not required, as shown by the Table, but for the shorter lengths, 5, 6, 7, 8, and 9 feet, that correction is necessary.

(294.) Table 58 may be adopted for conditions of fixing other than that of flat at one end and round at the other, as in that Table. By (149) it is shown that the breaking weights *by flexure* are in the ratio 1, 2, 3, for the three cases—both ends pointed,—one flat, one pointed,—and both ends flat respectively. Thus a pillar 6 inches square, 16 feet long = 36 tons by flexure from Table 58: then with both ends pointed we have 36 ÷ 2 = 18 tons, which being less than 24·8 tons given by col. 4, correction for incipient crushing is not required (163). The same pillar with both ends flat = 36 × 3 ÷ 2 = 54 tons breaking weight by flexure, which being greater than 24·8 or $\frac{1}{4}$ C_P by col. 4, correction for crushing will be necessary. Taking

RED DEAL: one end flat, and the other rounded.

PILLAR, IN FEET.													
9		10		12		14		16		18		20	
WEIGHT, IN TONS.													
By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.
·45	·45	..	..	..	..	..	..	..	..	..	..	..	..
1·4	1·4	1·1	1·1	..	..	..	..	..	..	..	..	..	..
3·4	3·4	2·8	2·8	1·9	1·9	..	..	..	..	..	..	..	..
7·1	6·8	5·8	5·8	4·0	4·0	2·9	2·9	..	..	..	..	..	..
22	18	18	15	13	12	9·3	9·3	7·1	7·1	..	..	..	..
55	36	45	32	31	26	23	21	18	18	14	14	..	..
114	60	92	55	64	46	47	38	36	32	29	28	23	23
212	91	171	85	119	73	87	63	67	54	53	46	43	40
361	129	292	121	203	107	150	94	114	81	90	71	73	63
578	173	468	164	325	147	239	131	183	116	145	104	117	92
882	223	714	214	496	194	364	176	278	158	224	143	178	128
1828	341	1480	330	1028	307	755	285	578	262	457	240	370	220

from cols. 2 and 3 of Table 58, the values of C_P and $\frac{3}{4}\,C_P$, or 99 and 74·2 respectively, the Rule (164) becomes $54 \times 99 \div (54 + 74{\cdot}2) = 41{\cdot}8$ tons.

Thus the reduced strengths under the 3 different forms at the ends are 18, 32, and 41·8 tons, those due by flexure only, being 18, 36, and 54 tons respectively.

(295.) "*Rectangular Pillars.*"—Table 58 may easily be applied to rectangular pillars: these always fail by bending in the direction of the least dimension, so that a rectangular pillar may be regarded as a number of square ones; thus 2×6 is equivalent to three 2-inch pillars, the breaking weight of which, say 8 feet long, would be $= 1{\cdot}8 \times 3 = 5{\cdot}4$ tons. Again, a pillar 7 feet long, and say $3 \times 7\frac{1}{2}$ would give a breaking weight $= 9{\cdot}6 \times 7\frac{1}{2} \div 3 = 24$ tons, &c.: from this it will be seen that the strength of rectangular pillars of all kinds is simply proportional to their larger dimension. Table 59 has been thus calculated from Table 58.

TABLE 59.—Of the STRENGTH of RECTANGULAR PILLARS of RED DEAL: one end flat and the other rounded.

Sizes.	Length of Pillar, in Feet.										
	5	6	7	8	9	10	12	14	16	18	20
	Reduced Breaking Weight, in Tons.										
1½ × 1½	1·44	1·0	0·74	·56	·45	..	..	..	..	..	..
,, 5	4·8	3·3	2·5	1·9	1·5	..	..	..	..	..	..
,, 7	6·7	4·7	3·4	2·6	2·1	..	..	..	..	..	..
,, 9	8·6	6·0	4·4	3·3	2·7	..	..	..	..	..	..
,, 11	10·5	7·3	5·4	4·1	3·3	..	..	..	..	..	..
2 × 2	3·9	3·0	2·3	1·8	1·4	1·1	..	..	..	..	..
,, 7	13·7	10·5	8·0	6·3	4·9	3·8	..	..	..	..	..
,, 9	17·5	13·5	10·3	8·1	6·3	4·9	..	..	..	..	..
,, 11	21·4	16·5	12·6	9·9	7·7	6·0	..	..	..	..	..
2½ × 2½	7·9	6·5	5·3	4·3	3·4	2·8	1·9	..	..	..	..
,, 7	22	18	15	12	9·5	7·8	5·3	..	..	..	..
,, 9	28	23	19	15	12	10	6·8	..	..	..	..
,, 11	35	28	23	19	15	12	8·3	..	..	..	..
3 × 3	14	11·5	9·6	8·1	6·8	5·8	4·0	2·9	..	..	..
,, 7	32	27	22	19	16	13	9·3	6·8	..	..	..
,, 9	41	34	29	24	20	17	12	8·7	..	..	..
,, 11	51	42	35	30	25	21	15	10·6	..	..	..
4 × 4	31	27	23	21	18	15	12	9·3	7·1	..	..
,, 8	62	54	46	42	36	30	24	19	14	..	..
,, 10	77	67	57	52	45	37	30	23	18	..	..
,, 12	93	81	69	63	54	45	36	28	21	..	..
5 × 5	53	48	44	40	36	32	26	21	18	14	..
,, 8	85	77	70	64	57	51	41	33	29	22	..
,, 10	106	96	88	80	72	64	52	42	36	28	..
,, 12	127	115	105	96	86	77	62	50	43	33	..
6 × 6	82	77	71	65	60	55	46	38	32	28	23
,, 10	137	129	119	108	100	91	76	63	53	46	38
,, 12	164	154	142	130	120	110	92	76	64	56	46
7 × 7	118	108	102	98	91	85	73	63	54	46	40
,, 10	168	154	146	140	130	121	104	90	77	65	57
,, 12	202	185	175	168	156	146	125	108	92	79	68
,, 14	236	216	204	196	182	170	146	126	108	92	80
8 × 8	158	151	144	136	129	121	107	94	81	71	63
,, 12	217	226	216	204	193	182	160	141	122	106	94
,, 14	276	264	252	238	226	212	187	164	142	124	110
9 × 9	205	198	190	182	173	164	147	131	116	104	92
10 × 10	256	249	241	233	223	214	194	176	158	143	128
12 × 12	377	369	361	351	341	330	307	285	262	240	220

CHAPTER VIII.

ON THE CONNECTION BETWEEN THE STRENGTH OF PILLARS, AND THE TRANSVERSE STRENGTH AND DEFLECTION OF A BEAM OF THE SAME MATERIAL.

(296.) By the theory of the strength of pillars in (136), &c., it is shown that the transverse strain on a beam multiplied by the distance between supports in inches, and divided by four times the deflection produced by that strain, will give the equivalent longitudinal strain which tends to break the same beam as a pillar (137). It is also shown (147) that the theoretical strength of pillars is directly proportional to d^4, and inversely as L^2.

We now propose to test the accuracy of the theoretical laws which connect the transverse and longitudinal strains by comparing calculation with experiment. This comparison is rendered very difficult by the fact that the strengths of cast iron, wrought iron, and steel pillars do not follow precisely the theoretical law $d^4 \div L^2$; the experimental law (147) for cast iron being $d^{3 \cdot 6} \div L^{1 \cdot 7}$, and for wrought iron and steel $d^{3 \cdot 6} \div L^2$. Timber pillars, however, follow the theoretical law precisely.

TABLE 60.—CAST-IRON PILLARS: comparison of Theory with Experiment.

1 Inch Diameter.			2 Inches Diameter.			3 Inches Diameter.		
Length.	Calculated.	Experiment.	Length.	Calculated.	Experiment.	Length.	Calculated.	Experiment.
feet.	tons.	tons.	feet.	tons.	tons.	feet.	tons.	tons.
1	23·5	15	1	376	181·5	1	1903	783
2	5·88	4·61	10	3·76	3·62	10	19·03	15·63
3	2·61	2·32	11	3·11	3·09	15	8·46	7·85
4	1·47	1·42	11½	2·84	2·81	18	5·873	5·758
4¼	1·30	1·28	11¾*	2·725	2·753	18½	5·564	5·490
4½*	1·161	1·163	12	2·611	2·665	19	5·271	5·255
5	·94	·972	13	2·225	2·318	19½*	5·008	5·020
6	·653	·714	14	1·918	2·044	20	4·757	4·804
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

(297.) "*Cast Iron.*"—Taking first, cast iron; Table 67 shows that a bar 12 inches long and 1 inch square deflects 0·0785

inch, with 2063 lbs. in the centre; hence the rule in (138), namely $W = w \times l \div (\delta \times 4)$, becomes $2063 \times 12 \div (\cdot 0785 \times 4) = 78840$ lbs., or 35·2 tons breaking weight as a pillar 1 inch square, the strain being in the centre, or the pillar having pointed ends. By (361) the ratio of the strength of square to round is by experiment 1·5 to 1·0; hence we have $35 \cdot 2 \div 1 \cdot 5 = 23 \cdot 5$ tons for a round pillar 1 foot long: calculating in this way, we obtain col. 2 of Table 60, which gives the strength with various lengths.

Mr. Hodgkinson found by his experiments that the strength of cast-iron pillars is governed by $L^{1 \cdot 7}$, instead of L^2 as by theory:—the effect of this divergence is very great, for instance, with a length of 10 feet, $L^2 = 100$, but $L^{1 \cdot 7} = 50$ only, giving thus double strength to that due by theory for that particular length. By col. 1 of Table 34, the experimental strength of a pillar 1 inch diameter and 1 foot long, with both ends pointed, is 14·73, or say 15 tons, and admitting the strength to be inversely as $L^{1 \cdot 7}$, we obtain col. 3 of Table 60, which shows that the theoretical and experimental strengths agree when the length is about $4\frac{1}{2}$ feet, or 54 times the diameter.

The length with which theory and experiment agree will not, however, be the same for all diameters, because according to theory, the strength varies directly as D^4, whereas by experiment it is as $D^{3 \cdot 6}$. Thus, if the strength of a pillar 1-inch diameter $= 1 \cdot 0$, then another of the same length, but 6 inches diameter, would by theory have a strength of $6^4 = 1296$, whereas by the experimental ratio it would be $6^{3 \cdot 6} = 633$ only, or about half. The effect of this divergence of the laws is shown by Table 60 to be that with a pillar 2 inches diameter, the results coincide with a length of about $11\frac{3}{4}$ feet, or 70 times the diameter: with a 3-inch pillar they coincide with a length of $19\frac{1}{2}$ feet, or 78 times the diameter.

(298.) "*Wrought Iron.*"—By Table 67 a bar of wrought iron 1 inch square and 1 foot between supports, loaded transversely with 2000 lbs., or ·893 ton in the centre, deflects ·0313 inch; hence the equivalent load as a pillar will be $\cdot 893 \times 12 \div (\cdot 0313 \times 4) = 85 \cdot 6$ tons, breaking weight of a pillar 1 inch *square* and 1 foot long, with both ends pointed. From this we

obtain by the ratio given in (225) $85 \cdot 6 \div 1 \cdot 7 = 50 \cdot 4$ tons for a cylindrical pillar 1 inch diameter and 1 foot long. By col. 1 of Table 34, the experimental strength is 42·79 tons:—this ratio, 50·4 to 42·79 will prevail for all lengths, for, as we have seen in (296), the strength varies inversely as L^2, both theoretically and experimentally. But it will not be the same for all diameters, theory giving D^4, and experiment $D^{3 \cdot 6}$, the effect of which is shown by Table 61, where col. 2 is the theoretical, and col. 3 the experimental strength for different diameters. It will be observed that the two rules agree in their results with a diameter of ·67 inch:—with larger diameters the theoretical results are in excess, and with smaller diameters in defect.

TABLE 61.—PILLARS of WROUGHT IRON and STEEL: comparison of Theory and Experiment.

Wrought Iron.			Steel.		
Diameter. Inches.	Theory. d^4.	Experiment. $d^{3 \cdot 6}$.	Diameter. Inches.	Theory. d^4.	Experiment. $d^{3 \cdot 6}$.
1·0	50·4	42·79	1·0	55·0	48·44
·8	20·6	19·17	·9	36·1	33·90
·75	15·92	15·19	·87	31·5	29·35
·7	12·09	11·85	·85	28·7	26·98
·67*	10·15	10·13	·82	24·9	23·68
·66	9·56	9·59	·8	22·6	21·70
·65	9·00	9·08	·75	17·4	17·19
·64	8·46	8·58	·74*	16·5	16·38
·6	6·53	6·80	·7	13·2	13 42
(1)	(2)	(3)	(4)	(5)	(6)

(299.) "*Steel Pillars.*"—By Table 67, a bar 1 inch square and 1 foot long between bearings, loaded transversely with 5600 lbs. in the centre, deflects ·0802 inch: hence by Rule (138) $W = 5600 \times 12 \div (\cdot 0802 \times 4) = 209500$ lbs. is the equivalent strain as a pillar 1 inch square, or $209500 \div 1 \cdot 7 =$ 123300 lbs., or 55 tons, for a cylindrical pillar 1 inch diameter with both ends pointed. The experimental strength by col. 1 of Table $34 = 48 \cdot 44$ tons. With Steel, as we found with wrought iron, a comparison of the theoretical with the experi-

mental strength will not be affected by the length, both materials being governed by L^2; but the diameter affects the comparison considerably, as shown by Table 61, where col. 5 follows the theoretical law D^4, and col. 6, the experimental law $D^{3 \cdot 6}$, the two laws coinciding nearly in their results with the diameter of $0 \cdot 74$ inch.

(300.) The superiority of steel over wrought iron as a pillar is shown to be remarkably small: with 1 inch diameter and 1 foot long, theory gives $50 \cdot 4$ to $55 \cdot 0$, or $55 \cdot 0 \div 50 \cdot 4 = 1 \cdot 09$, an increase of 9 per cent. only. The experimental strengths are $42 \cdot 79$ and $48 \cdot 44$, or $48 \cdot 44 \div 42 \cdot 79 = 1 \cdot 13$, an increase of 13 per cent. This applies to long pillars only; with short pillars requiring correction for incipient crushing the superior crushing strength of steel will give it much greater advantage as a pillar. We have shown (133) that the value of C for wrought iron in pillars is 19 tons per square inch, whereas for Steel (268) it is as much as 52 tons; a *very* short steel pillar, where the strength depends almost exclusively on the resistance to crushing, will have $52 \div 19 = 2 \cdot 74$ times the strength of a similar one of wrought iron.

(301.) From the preceding investigation it will be evident that in pillars of cast iron, wrought iron, and steel, the divergence of the theoretical from the practical laws governing the strength, renders the former unreliable for those materials.

Fortunately, under these circumstances, we have practical rules whose general accuracy has been experimentally proved as shown by our various Tables, and more particularly by (959) and Table 150.

(302.) "*Timber Pillars.*"—The experiments on Timber pillars in Table 57 show that they follow precisely the theoretical law $D^4 \div L^2$, which simplifies comparisons very considerably. Unfortunately the experimental information available is very scanty, this however will only enhance the value of the theoretical investigation, as we shall obtain thereby a knowledge of the strength of pillars for many kinds of Timber of which nothing is known experimentally. As the few experiments we have agree well with the theoretical results, as shown for Dantzic Oak by (139), we may have the more confidence in the

theory as applied to other cases. Thus the mean error of all the calculated strengths in Table 57 is shown in (291) to be less than $\frac{1}{2}$ per cent.

(303.) We may now find the value of the constant M_P for Timber pillars from the transverse load, and corresponding deflection by the theoretical law (138), namely $W = w \times l \div (\delta \times 4)$. Taking the values of w and δ from cols. 3 and 4 of Table 67, we obtain for pillars 12 inches long, 1 inch square, both ends pointed, the values of M_P:—

	w	l	δ	M_P
Teak	145 ×	12 ÷	(·026 × 4) =	16,730 lbs.
Red Pine	98 ×	12 ÷	(·023 × 4) =	12,780 ,,
Canadian Oak ..	117 ×	12 ÷	(·028 × 4) =	12,540 ,,
Deal	123 ×	12 ÷	(·031 × 4) =	11,900 ,,
Ash	136 ×	12 ÷	(·035 × 4) =	11,660 ,,
Beech	112 ×	12 ÷	(·036 × 4) =	9,333 ,,
Pitch-pine	115 ×	12 ÷	(·041 × 4) =	8,403 ,,
Dantzic Oak ..	71 ×	12 ÷	(·026 × 4) =	8,192 ,,
English Oak ..	102 ×	12 ÷	(·040 × 4) =	8,160 ,,
Riga Fir	78 ×	12 ÷	(·030 × 4) =	7,800 ,,
Larch	76 ×	12 ÷	(·037 × 4) =	6,162 ,,
Willow	73 ×	12 ÷	(·056 × 4) =	3,902 ,,
Cedar	91 ×	12 ÷	(·081 × 4) =	3,370 ,,

We obtain from this col. 4 in Table 34 for pillars with both ends pointed; then adopting Mr. Hodgkinson's Ratios 1, 2, 3 for, both ends pointed, 1 pointed and 1 flat, and both ends flat respectively (149), we have obtained cols. 5 and 6.

The theoretical ratio of the strength of square and round pillars (519) is 1·7 to 1·0, but the experimental ratio (361) is 1·5 to 1·0: adopting the latter we obtain cols. 1, 2, 3, in Table 34. For Dantzic Oak and Red Deal we have taken the experimental values of M_P which agree the best with Table 57.

The strength of Timber Pillars may be found from the Modulus of Elasticity: thus in Table 34, the pillars are arranged in the order of their strength, and in col. 7 of Table 105 we have the Modulus of Elasticity. Taking as examples Teak, Pitch-pine, and Cedar, we have strong, medium, and weak pillars: Teak gives for a cylindrical pillar with

both ends pointed 2413410 × ·0046 = 11100 lbs., col. 1 of Table 34 gives 11150 lbs.: Pitch-pine = 1224840 × ·0046 = 5634 lbs., Table 34 gives 5600 lbs.: Cedar = 448237 × ·0046 = 2246 lbs., Table 34 gives 2247 lbs., &c.

CHAPTER IX.

ON THE WRINKLING STRAIN.

(304.) When a rectangular pillar is made of thin wrought-iron plates, and the sizes are such as to preclude yielding by flexure, it is necessary not only to have sufficient *area* to resist crushing, but also considerable *thickness* to prevent failure by Wrinkling or Corrugation.

Let Fig. 58 be a square pillar so short in proportion to the side of the square as to avoid the probability of failure by bending. We have seen in (201) that the absolute crushing strength of wrought iron in the form of pillars is 19 tons per square inch; but if the plates are very thin, and the breadth unsupported, or the distance, say from A to B, very considerable, the plate would fail by wrinkling or corrugation near the centre-line C, with a strain much less than that required to crush the material. At, and near the corners, wrinkling would be prevented by the support which those corners afford, but the centre is only imperfectly supported by them, and the more imperfectly as the breadth of the plate, or the distance from a corner is greater.

(305.) Fig. 59 is half the tubular pillar Fig. 58, and we may admit as self-evident, that the edges D and E being supported at *one* side only, or from F and G, will fail by wrinkling with a strain much less than the plate A, B, which was supported at *both* sides, although the distance from a support is the same in both cases. In Fig. 60, we have a pillar where the distance H, J, or the distance from a support on *one* side is $\frac{1}{4}$th of the width of the plate A, B, which was supported on both sides:— in the absence of experimental information we may assume that

this would probably give equality of strength, the edge J failing by wrinkling with the same strain per square inch as the centre C of the wide plate A, B. In Fig. 61, we have the application of the same principles to a pillar of I section, the analogy of which with Fig. 60 is obvious:—this will be useful when we come to consider the crushing strain on the top flange of a plate-iron (395) or lattice girder.

(306.) A rectangular pillar of thin plate-iron may fail in one of three ways. 1st, by Flexure; 2nd, by Crushing; 3rd, by Wrinkling; each being governed by laws peculiar to itself and differing from the other two. Of course it will actually fail from that particular strain to which its power of resistance is the least.

Taking No. 15, in Table 55, as an example; col. 9 shows that by flexure it would fail with 2,628,000 lbs. or 1173 tons, and the area by col. 4 being 1·532 square inch, this is equal to $1173 \div 1 \cdot 532 = 766$ tons per square inch. But we have seen (201) that the absolute crushing strength of wrought iron in pillars is only 19 tons per square inch, or $\frac{1}{40}$th of the theoretical breaking weight by flexure in this case. Col. 12 shows that even this reduced strain was not borne by the pillar, which really failed by wrinkling with 7·108 tons per square inch, or little more than $\frac{1}{3}$rd of the crushing strain, and $\frac{1}{108}$th of the strength due by flexure: thus by

	Wrinkling	Crushing	Flexure
the breaking strains per square inch, were			
	7·108	19·0	766 tons
the Ratios of which are:—			
	1·0	2·7	108

Here evidently the pillar failed by Wrinkling, the actual breaking load being $\frac{1}{108}$th only of the bending strength and $\frac{1}{2 \cdot 7}$th of the crushing strength.

By increasing the length of the pillar, the resistance to flexure might be reduced until it became even less than the wrinkling strain: thus, for the sake of illustration, with 10 times the length the resistance to flexure (being proportional to L^2) would become $766 \div 100 = 7 \cdot 66$, or nearly the wrinkling strain.

With intermediate lengths we should obtain a mixed result, part of the strength being employed in resisting flexure, and part in resisting "*incipient* wrinkling" (249), (253). In searching for the laws governing Wrinkling, it will therefore be necessary to take the experiments on *short* pillars, where the strength is dominated almost exclusively by that strain.

(307.) "*Laws of Wrinkling.*"—The laws governing the wrinkling strain may be obtained from Mr. Hodgkinson's experiments on square pillars in Table 55, and on tubular beams of thin plate-iron in (406), and Table 77. They may be expressed by the rules:—

(308.) $$W_w = \sqrt{t_w \div b_w} \times \mathsf{M}_w.$$

(309.) $$t_w = \left(W_w \times \sqrt{b_w} \div \mathsf{M}_w\right)^2$$

(310.) $$b_w = \left(\sqrt{t_w} \times \mathsf{M}_w \div W_w\right)^2$$

(311.) $$\mathsf{M}_w = W_w \times \sqrt{b_w \div t_w}.$$

In which W_w = the compressive strain in tons per square inch with which the plate will wrinkle; t_w = the thickness of the plate in inches; b_w = the breadth in inches of a plate supported at both edges, as in Fig. 58; where the corners are connected by angle-irons in the usual way, the breadth must be measured between their edges, as at C in Fig. 62. When the plate is supported at one edge only, as in Fig. 59, four times the distance projecting beyond the angle-iron must be taken for the value of b_w, as explained in (305). M_w = the Multiplier found from experiment, the mean value of which in rectangular pillars = 80, and in Beams = 104, as shown by Table 62.

Table 63 gives the Wrinkling strain for plates of different thickness and breadth calculated by the rule.

(312.) The value of M_w for pillars may be found from the experiments in Table 55; selecting the short pillars for reasons given in (306) we obtain Table 62, the mean being 79·45, say 80. Thus taking No. 15 as an example which failed with 7·108 tons per square inch by col. 12; then rule (311) becomes 7·108

$\times \sqrt{8 \cdot 175 \div \cdot 061} = 82 \cdot 29$, the value of M_w, as in Table 62: for the value of M_w for Beams, see (322).

Col. 11 of Table 55 has been calculated by the rule (308); it will be observed that the actual experimental strain in col. 12, is often less than the wrinkling strain in col. 11; this is due to the fact that the strains in col. 12 are complicated by flexure (306), and are affected by the length of the pillar, as shown for example by Nos. 27, 28, and 29.

TABLE 62.—Of the VALUE of M_w for the RESISTANCE of THIN WROUGHT-IRON PLATES to WRINKLING.

Number in Table 55.	For Rectangular Pillars.		
	W_w		M_w
9	$10 \cdot 783 \times \sqrt{4 \cdot 05 \div \cdot 0637}$	=	85·96
15	$7 \cdot 108 \times \sqrt{8 \cdot 175 \div \cdot 061}$	=	82·29
22	$13 \cdot 62 \times \sqrt{4 \cdot 44 \div \cdot 136}$	=	77·82
25	$12 \cdot 24 \times \sqrt{4 \cdot 25 \div \cdot 085}$	=	86·55
29	$5 \cdot 537 \times \sqrt{4 \cdot 1 \div \cdot 03}$	=	64·74
	Mean		79·45
Table 77.	For Tubular Beams.		
	f		
1	$17 \cdot 75 \times \sqrt{24 \div \cdot 75}$	=	100·4
2	$18 \cdot 52 \times \sqrt{24 \div \cdot 75}$	=	104·8
5	$19 \cdot 20 \times \sqrt{15 \cdot 5 \div \cdot 525}$	=	104·3
6	$18 \cdot 50 \times \sqrt{16 \div \cdot 5}$	=	104·6
11	$14 \cdot 42 \times \sqrt{15 \cdot 5 \div \cdot 272}$	=	108·8
12	$7 \cdot 74 \times \sqrt{15 \div \cdot 124}$	=	85·1
14	$15 \cdot 32 \times \sqrt{3 \cdot 8 \div \cdot 065}$	=	117·1
16	$13 \cdot 38 \times \sqrt{1 \cdot 9 \div \cdot 03}$	=	106·4
	Mean		104

(313.) It should be observed that the wrinkling strain is independent of the *length of the plate*; this is shown by the

same experiments, for although with lengths of 10 and $2\frac{1}{2}$ feet respectively, the actual compressive strains in col. 12 varied from 4·902 to 5·537 tons per square inch, when correction is made for flexure, and the effect of the length is thus eliminated, col. 8 shows the same error in both, or 22·2 and 22·4 per cent. respectively.

Another proof that the wrinkling strain is independent of length, is that even with long pillars, the plate often fails near *the end;* for instance, No. 2 was 10 feet long, but failed by wrinkling 14 inches from one end. No. 8 was 5 feet long, but gave way at 7 inches from one end. Now the crushing strain due to *flexure* is a maximum at the centre, and is reduced progressively towards the ends, where it becomes nil; but the crushing strain due to *direct pressure* is the same from end to end. These two facts, that the length has so little effect on the strength of the pillar; and that failure by wrinkling takes place indifferently in any part of the length, show that the wrinkling strain is independent of the length of the plate.

(314.) An obvious and economical method of increasing the strength of a plate in resisting wrinkling, is by adding vertical ribs as at A, B, &c., in Fig. 62, which in effect reduce the breadth, and thereby increase the strength in a much higher ratio than the weight:—thus one central rib reduces the width to half, and the wrinkling strain is increased in the ratio of $\sqrt{2}$ to $\sqrt{1}$, or from 1·0 to 1·41, or 41 per cent. Similarly two ribs give an increase of $\sqrt{3} = 1 \cdot 73$, or 73 per cent., &c.

(315.) It should be observed that the *total* wrinkling strain increases in a much higher ratio than the square-root of the thickness which governs the resistance *per square inch* only: thus for thicknesses in the ratio

1	2	3	4	5	6

the wrinkling strain per square inch follows the ratio $\sqrt{t_w}$ and becomes,—

1	1·41	1·73	2	2·24	2·45

But the areas are also increased in the simple proportion of the

thickness, and the *total* strains are therefore increased in the ratio, $\sqrt{t_W} \times t_W$, or $t_W^{1\cdot5}$, and become :—

1	2·82	5·2	8	11·2	14·7

See (396) and Tables 74 and 75.

The total strain due to flexure is practically simply proportional to the thickness of the plate :—thus with a pillar 12 inches square externally, and $\frac{1}{16}$ inch thick, therefore $11\frac{7}{8}$ inches internally, $D^{3\cdot6} - d^{3\cdot6}$ becomes 284 :—with $\frac{1}{8}$ inch thick, therefore $11\frac{3}{4}$ inches internally, $D^{3\cdot6} - d^{3\cdot6} = 559$, which is nearly proportional to the thickness.

(316.) Another result of the rules is that in order to obtain equality of strength in a rectangular pillar other than square, the thickness of the plates should be simply proportional to the breadths :—thus, if the ratio of the sides is 3 to 1, the thicknesses should be in the same ratio (472).

The absolute crushing strength of wrought iron in pillars is 19 tons per square inch, and in order to obtain the full value of the material, the wrinkling strain should not be less than that. For example, No. 4 in Table 55 failed with 5·926 tons per square inch, col. 12, being very nearly the calculated Wrinkling strain in col. 11, whereas by crushing it would not have failed with less than 19 tons, so that $5\cdot926 \div 19 = \cdot31$, or 31 per cent. only of the strength of the material is utilised and 69 per cent. is wasted. We can easily find the ratio of the breadth to the thickness which is necessary to secure that equality of strength :—say we take 1 inch thick, then the breadth of a plate supported at both edges to give $W_W = 19$ tons will be given by the rule (310), which becomes $b_W = (\sqrt{1} \times 80 \div 19)^2 = 17\cdot72$, or say 18 inches. The same thickness of plate supported at one edge only, would have a breadth of $18 \div 4 = 4\frac{1}{2}$ inches projecting beyond the angle-iron, as P in Fig. 93. These dimensions apply only to plates subjected to direct pressure as in a pillar, and they may be taken as ratios applicable to all thicknesses.

(317.) For plates 1 inch thick forming part of a plate-iron tubular beam or girder, and supported at both edges as in Fig. 58, the value of $M_W = 104$, and the rule (310) gives the breadth

for $W_W = 19$ tons, $b_W = (\sqrt{1} \times 104 \div 19)^2 = 29 \cdot 96$, say 30 inches:—for the top flange of a girder, supported at one edge only, and measured as in Fig. 60, the breadth would be $30 \div 4 = 7\frac{1}{2}$ inches.

Calculating in this way we may find the breadths of wrought-iron plates under different conditions, such that the Wrinkling Strain shall be equal to the Crushing strain, or 19 tons per square inch in all cases: for thicknesses of:—

$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ 1 inch,

the breadth of plate supported at both edges and forming part of a pillar =

$2\frac{1}{4}$ $4\frac{1}{2}$ $6\frac{3}{4}$ 9 $11\frac{1}{4}$ $13\frac{1}{2}$ $15\frac{3}{4}$ 18 inches.

The same plates forming the top of a Tubular beam would have breadths of:—

$3\frac{3}{4}$ $7\frac{1}{2}$ $11\frac{1}{4}$ 15 $18\frac{3}{4}$ $22\frac{1}{2}$ $26\frac{1}{4}$ 30 inches.

The breadths for plates supported at one edge only in pillars become:—

$\frac{9}{16}$ $1\frac{1}{8}$ $1\frac{5}{8}$ $2\frac{1}{4}$ $2\frac{3}{4}$ $3\frac{3}{8}$ $3\frac{7}{8}$ $4\frac{1}{2}$ inches,

and in plates forming the top flange of a plate-iron girder, Fig. 93, we have:—

1 $1\frac{7}{8}$ $2\frac{7}{8}$ $3\frac{3}{4}$ $4\frac{3}{4}$ $5\frac{5}{8}$ $6\frac{1}{2}$ $7\frac{1}{2}$ inches.

With breadths greater than those given above the Wrinkling strain would be less than 19 tons per square inch: with less breadths the wrinkling strain by calculation would come out *more* than 19 tons, but this would not be realised; in that case the strength of the plate would be limited by the *crushing* strength.

(318.) Let Fig. 62 be the section of a short pier for a bridge, &c., 6 feet square, of wrought-iron plate $\frac{5}{8}$ inch thick, strengthened with T ribs A, B, &c., giving $11\frac{1}{4}$ inches between their edges, we should then have 19 tons per square inch wrinkling strain, and should thus have obtained the utmost possible effect from the material. If in this case we dispense with the ribs,

TABLE 63.—*Of the* RESISTANCE *of* WROUGHT-IRON PLATES *to* WRINKLING *in Tons per Square Inch.*

Thickness of Plate.	Breadth of Plate, supported at Both Edges: in Inches.										
	4	6	9	12	15	18	24	30	36	48	60
	In Square and Rectangular Pillars.										
$\frac{1}{16}$	10·00	8·2	6·6	5·76	5·16	..	..	..	..	..	..
$\frac{1}{8}$	14·14	11·54	9·43	8·17	7·31	6·67	5·77	..	..	..	..
$\frac{1}{4}$	19·80	16·4	13·2	11·53	10·33	9·4	8·2	7·3	6·7	..	..
$\frac{3}{8}$	..	..	16·4	14·2	12·7	11·6	10·0	8·95	8·17	7·1	6·3
$\frac{1}{2}$	..	..	18·85	16·35	14·62	13·34	11·54	10·32	9·42	8·16	7·3
$\frac{5}{8}$	..	..	..	18·28	16·34	15 00	12·91	11·54	10·54	9·1	8·2
$\frac{3}{4}$	..	..	..	..	17·9	16·34	14·17	12·64	11·55	10·0	9·0
$\frac{7}{8}$	..	..	..	..	19·5	17·64	15 27	13·66	12·48	10·8	9·6
1	..	..	..	..	..	18·87	16·33	14·60	13·33	11·54	10·3
	In Tubular Beams.										
$\frac{1}{16}$	13·0	10·7	8·6	7·49	6·7	..	..	..	..	..	..
$\frac{1}{8}$	18·4	15·0	12·2	10·6	9·5	8·67	7·50	..	..	..	..
$\frac{1}{4}$	..	..	17·2	15 0	13·4	12·2	10·7	9·5	8·7	..	..
$\frac{3}{8}$	..	..	..	18·5	16·5	15·1	13·0	11·6	10·6	9·2	8·2
$\frac{1}{2}$	..	..	..	..	19·0	17·3	15·0	13·4	12·2	10·6	9·5
$\frac{5}{8}$	..	..	..	..	..	19·5	16·8	15·0	13·7	11·8	10·7
$\frac{3}{4}$	..	..	..	..	..	..	18·4	16·4	15·0	13·0	11·7
$\frac{7}{8}$	..	..	..	..	..	..	..	17·7	16·2	14·0	12·5
1	..	..	..	..	..	..	..	19·0	17·3	15·0	13·4

the effective width of the $\frac{5}{8}$-inch plate would be 5 feet $5\frac{3}{4}$ inches, and the wrinkling strain by the rule (308) becomes $\sqrt{\cdot 625 \div 65\cdot 75} \times 80 = 7\cdot 8$ tons per square inch, which is $7\cdot 8 \div 19 = \cdot 41$, or 41 per cent. only of the maximum strength, and thus 59 per cent. would be wasted in that case.

The vertical ribs would not only increase the strength of the thin plate, but would also yield their full quota of strength to the pier in the simple proportion to their own area or weight. The additional strength to the plates is thus a clear *net* advantage.

If the pillar is of considerable length it would be expedient to make it cellular as in Fig. 54, rather than a simple square as in Fig. 62. See (266).

(319.) "*Wrinkling Strain in Beams.*"—In an ordinary rectangular tubular beam, supported at both ends, a transverse load causes a compressive strain on the top plates. With very thick plates the limit to that strain is the crushing strength of the material, or 19 tons per square inch, but with ordinary thicknesses and breadths the plate will fail by Wrinkling with a much lower strain, as shown by col. 12 of Table 77.

We have first to ascertain the longitudinal compressive strain from the transverse load, &c., which will be given by rule (514).

Thus, taking No. 5 in Table 77 as an example, whose section is shown by Fig. 63, the rule (514) for finding *f* becomes

$$\frac{3 \times 58\cdot 66 \times 360 \times 24}{2 \times \left\{24^3 \times 15\cdot 5) - (22\cdot 95^3 \times 14\cdot 45\right\}} = 19\cdot 2 \text{ tons,}$$

as in col. 12.

(320.) It will be interesting and instructive to check this result by an analytical investigation. The beam being 30 feet long supported at each end, our first step will be to reduce it to the equivalent case of a cantilever of half the length, say built into a wall, and loaded at the end with half the central load. See (385) and Figs. 91, 92.

Assuming the value of *f*, or $19\cdot 2$ tons per square inch, as the maximum strain at A and B in Fig. 63, this is reduced at C and D, or the *centres* of the top and bottom plates to $19\cdot 2 \times 23\cdot 475 \div 24 = 18\cdot 78$ tons per square inch, this being simply propor-

tional to the relative distances from the neutral axis N. The area of say the top plate being $15 \cdot 5 \times \cdot 525 = 8 \cdot 137$ square inches, and the leverage by which the strain acts in supporting the load at W, 23·475 and 180 inches, we obtain $8 \cdot 137 \times 18 \cdot 78 \times 23 \cdot 475 \div 180 = 19 \cdot 93$ tons at W, and as we have taken for the leverage the whole depth C, D, not the distance from the neutral axis N to C or D, this will be the *sum* of the resistances of tension at C and compression at D, which in that case occupy the positions of fulcra and resistances to each other reciprocally.

Following the same course with the sides, we have below N the area of the two half sides $= 22 \cdot 95 \times \cdot 525 = 12 \cdot 04$ square inches. The mean resistance at o and p will be proportional to the distances from N: hence 19·2 tons at A is reduced to $19 \cdot 2 \times 11 \cdot 475 \div 24 = 9 \cdot 18$ tons per square inch at o and p. By (495) it is shown that the true mean is not found by multiplying the area by the mean tensile strain and the mean leverage simply, but by $\frac{4}{3}$ of that product. Then we obtain $(12 \cdot 04 \times 9 \cdot 18 \times 11 \cdot 475 \times \frac{4}{3}) \div 180 = 9 \cdot 4$ tons at W, as the resistance of the sides, making a total of $19 \cdot 93 + 9 \cdot 4 = 29 \cdot 33$ tons at the end of a cantilever 15 feet long, which is equivalent to $29 \cdot 33 \times 2 = 58 \cdot 66$ tons in the centre of the girder 30 feet long, agreeing precisely with the experiment; col. 8 of Table 77. See (412) and (414).

(321.) The value of f in col. 12 of Table 77 has been calculated by the rule (514); it represents the maximum strain at the edge of the section due to the transverse load, but does not determine whether failure takes place by wrinkling or by crushing. When, however, f is much less than 19 tons per square inch, the plate must have failed by wrinkling; in two cases, Nos. 13 and 15, f was greater than 19 tons, namely, 23·13 and 24·56 tons respectively, which must be regarded as exceptional and anomalous. They may be accounted for by the variableness common to all materials under all kinds of strain, as shown by Table 147, which gives for Boiler-plate under tensile strain 29 per cent. in excess of the mean strength, col. 1. In our two cases the excess was 22 and 29 per cent. respectively under wrinkling or crushing strains.

(322.) "*Value of* M_W *for Beams.*"—The experiments in

Table 77 will give the value of M_w for a plate forming part of a tubular beam and subjected to compression, which is usually the top plate. Selecting cases where the top and bottom plates were of one and the same thickness, and where the results are likely to be more correct than under other conditions, omitting also the anomalous cases Nos. 13 and 15, we obtain Table 62; the mean of the whole is 104, which is 30 per cent. higher than 80, the mean value of M_w for pillars where the plate is subjected to direct compression (312). We found the same remarkable difference to prevail in the *crushing* strength of Wrought-iron and Steel, the former giving 26 and the latter 18 per cent. greater resistance in Beams than in Pillars (133).

The variations in the value of M_w for beams in Table 62 are $117\cdot1 \div 104 = 1\cdot126$ or $+ 12\cdot6$ per cent., and $85\cdot1 \div 104 = \cdot818$ or $- 18\cdot2$ per cent., which are not greater than the variableness of plate-iron under tensile strains, namely $+ 29$ and $- 33$ per cent., as shown by Table 149. This is the more satisfactory when it is remembered that the thicknesses ranged from $\cdot03$ to $\cdot75$ inch, or 1 to 25, and the breadths from $1\cdot9$ to 24 inches, or 1 to $12\cdot6$. These relative numbers, however, fail to give an adequate idea of the great differences of the dimensions: Fig. 100, where the beams are drawn to the same scale, will convey a clearer conception. It should also be observed that the largest and the smallest beams give nearly the same value for M_w, namely, $100\cdot4$ and $106\cdot4$ respectively, and that both differ but little from 104, the mean value of the whole.

The application of the laws of Wrinkling to rectangular pillars is shown by (249), and to Tubular Beams by (406), &c.

CHAPTER X.

ON THE TRANSVERSE STRAIN.

(323.) The general investigation of the Transverse Strength of Materials is complicated very much by the variable conditions in the mode of fixing and loading. It will therefore be expedient to take first a standard case, say that of a horizontal

beam supported at both ends and loaded with a dead weight in the centre: the effect of other conditions may be considered afterwards.

For solid Rectangular sections we have the Rules:—

(324.) $$W = D^2 \times B \times M_T \div L.$$

(325.) $$D = \sqrt{W \times L) \div (M_T \times B)}.$$

(326.) $$B = (W \times L) \div (D^2 \times M_T).$$

(327.) $$M_T = (W \times L) \div (D^2 \times B).$$

(328.) For solid square sections:—

$$W = D^3 \times M_T \div L.$$

(329.) For hollow square sections:—

$$W = \frac{D^4 - d^4}{D} \times M_T \div L.$$

(330.) For hollow rectangular sections:—

$$W = \frac{(D^3 \times B) - (d^3 \times b)}{D} \times M_T \div L.$$

(331.) For solid cylindrical sections:—

$$W = D_0^3 \times M_T \div L.$$

(332.) For hollow cylindrical sections:—

$$W = \frac{D_0^4 - d_0^4}{D_0} \times M_T \div L.$$

(333.) For solid Elliptical sections:—

$$W = D_D^2 \times D_B \times M_T \div L.$$

(334.) For hollow Elliptical sections:—

$$W = \frac{(D_D^3 \times D_B) - (d_D^3 \times d_B)}{D_D} \times M_T \div L.$$

In which D = the external, and d = the internal depth, in inches.

" B = the external, and b = the internal breadth, in inches.

TABLE 64.—Of the TRANSVERSE STRENGTH, &c., of BRITISH CAST IRON, reduced to bars 1 inch square, 1 foot long between supports, and loaded in the centre.

No. in the Scale of Strength.	Names of Irons.	No. of Experiments on each.	Specific Gravity.	Modulus of Elasticity in Lbs. per Square Inch.	Mean Deflection per Lb. up to the Safe Weight.	Breaking Weight. Lbs.	Breaking Weight. Ultimate Deflection.	Breaking Weight. Resilience.	Safe Weight (⅓). Lbs.	Safe Weight (⅓). Deflection.	Safe Weight (⅓). Resilience.
					inches.		in.	inch-lbs.		in.	inch-lbs.
1	Ponkey .. .. 3 C	4	7·122	17,211,000	·00002510	2632	·08726	113·50*	877	·02201	9·653*
2	Devon .. .. 3 H	2	7·251	22,473,650	·00001923	2448	·05459	61·36	816	·01569	6·400
3	Oldberry .. 3 H	5	7·300	22,733,400	·00001900	2416	·05038	60·86	805	·01530	6·157
4	Carron .. .. 3 H	2	7·056	17,873,100	·00002422	2403	·06836	82·17	801	·01940	7·770
5	Coed-Talon .. 3 H	4	6·970	14,707,900	·00002937	2362	·07904	93·34	787	·02312*	9·097
6	Beaufort .. 3 H	5	7·069	16,802,000	·00002571	2358	·07998	94·29	786	·02021	7·948
7	Butterley ..	4	7·038	15,379,500	·00002809	2290	·09072	103·90	763	·02143	8·176
8	Bute .. .. 1 C	4	7·066	15,163,000	·00002849	2241	·08823	98·82	747	·02128	7·948
9	Windmill End 2 C	4	7·071	16,490,000	·00002620	2332	·07911	88·28	744	·01949	7·251
10	Old Park .. 2 C	5	7·049	14,607,000	·00002958	2214	·08121	82·00	738	·02183	8·055
11	Carron .. .. 2 C	3	7·066	17,270,500	·00002501	2173	·06583	71·56	724	·01811	6·556
12	Beaufort .. 2 H	4	7·108	16,301,000	·00002650	2164	·07571	81·92	721	·01911	6·889
13	Low Moor .. 2 C	4	7·055	14,509,500	·00002977	2155	·09263	99·85	718	·02138	7·673
14	Low Moor .. 3 C	2	7·052	13,918,740	·00003104	2133	·09722	103·70	711	·02207	7·688
15	Buffery .. .. 1 C	3	7·079	15,381,200	·00002809	2115	·07765	82·17	705	·01981	6·982
16	Carron .. .. 2 H	3	7·046	16,085,000	·00002686	2115	·06708	70·97	705	01893	6·675
17	Brymbo .. .. 2 C	5	7·017	14,911,666	·00002897	2097	·08746	90·70	699	·02025	7·078
18	Apedale .. .. 2 H	3	7·017	14,852,000	·00002909	2083	·08657	90·18	694	·02018	7·004
19	Oldberry .. 2 C	4	7·059	14,307,500	·00003020	2079	·09062	92·10	693	·02093	7·250
20	Pentwyn .. 2	4	7·038	15,193,000	·00002843	2079	·07441	77·39	693	·01970	6·828
21	Maesteg 2	5	7·038	13,959,500	·00003095	2074	·09787	101·60	691	·02138	7·388
22	Muirkirk .. 1 C	4	7·113	14,003,550	·00003085	2074	·08684	90·05	691	·02132	7·365
23	Adelphi .. .. 2 C	5	7·080	13,815,500	·00003127	2052	·08809	90·38	684	·02138	7·314
24	Blaina .. .. 3 C	5	7·159	14,281,466	·00003032	2047	·08642	88·50	682	·02068	7·051
25	Devon .. .. 3 C	2	7·295*	22,907,700*	·00001886*	2047	·03976*	40·71*	682	·01286*	4·386*

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
26	Gartsherrie .. 3 H	5	7·017	13,894,000	·00003109	2043	·07811	79·83	681	·02117	7·210
27	Frood .. .. 2 C	5	7·031	14,112,666	·00003061	2043	·09133	91·21	681	·02037	6·936
28	Carron .. .. 3 C	5	7·094	16,246,966	·00002659	2038	·06701	68·28	679	·01805	6·130
29	Lane End .. 2	3	7·028	15,787,666	·00002737	2029	·07091	71·97	676	·01850	6·254
30	Dundyvan .. 3 C	4	7·087	16,534,000	·00002613	2025	·07357	74·53	675	·01764	5·952
31	Maesteg	5	7·038	13,971,500	·00003092	2020	·09440	95·34	673	·02082	7·002
32	Corbyn's Hall 2	5	7·007	13,845,866	·00003120	2020	·08454	85·38	673	·02100	7·066
33	Blaenavon .. 2	4	6·920	13,790,000	·00003133	2012	·08928	89·82	671	·02102	6·843
34	Pontypool .. 2	5	7·080	13,136,500	·00003288	2011	·09300	93·56	670	·02203	7·381
35	Wallbrook .. 3	5	6·979	15,394,766	·00002806	2011	·07236	72·80	670	·01880	6·299
36	Milton .. .. 3 H	4	7·051	15,852,500	·00002725	2002	·06862	68·69	667	·01818	6·062
37	Buffery .. .. 1 H	3	6·998	13,730,500	·00003147	1983	·08223	81·94	664	·02089	6·936
38	Level .. .. 1 H	5	7·080	15,452,500	·00002795	1975	·07596	75·01	658	·01840	6·052
39	Pant 2	5	6·979	15,280,900	·00002827	1971	·06289	61·98	657	·01857	6·102
40	Level .. .. 2 H	6	7·031	15,241,000	·00002834	1962	·06818	66·89	654	·01854	6·061
41	Eagle Foundry 2 H	4	7·038	14,211,000	·00003040	1953	·07587	74·09	651	·01979	6·442
42	Elsicar .. .. 2 C	4	6·928	12,586,500	·00003432	1953	·11087*	108·20	651	·02234	7·272
43	Varteg .. .. 2 H	4	7·007	15,012,000	·00002878	1948	·07274	70·85	649	·01868	6·061
44	Coltham .. 1 H	5	7·128	15,510,066	·00002785	1939	·07676	74·42	646	·01800	5·812
45	Carroll .. .. 2 C	4	7·069	17,036,000	·00002536	1917	·06179	59·91	639	·01620	5·177
46	Muirkirk .. 1 H	4	6·953	13,294,400	·00003250	1917	·07881	76·41	639	·02076	6·634
47	Bierley 2	5	7·185	16,156,133	·00002674	1912	·06141	58·71	637	·01703	5·425
48	Coed-Talon .. 2 H	4	6·969	14,322,500	·00003016	1903	·09412	89·55	634	·01912	6·062
49	Coed-Talon .. 2 C	5	6·955	14,304,000	·00003020	1890	·07378	69·72	630	·01903	5·993
50	Monkland .. 2 H	3	6·916	12,259,500	·00003524	1890	·08840	83·54	630	·02220	6·993
51	Bowling 2	3	7·014	14,130,000	·00003059	1850	·07768	71·85	617	·01886	5·820
52	Ley's Works .. 1 H	3	6·957	11,539,333*	·00003744*	1795	·07561	67·86	598	·02239	6·694
53	Milton .. .. 1 H	4	6·976	11,974,500	·00003608	1692	·07673	64·91	561	·02035	5·738
54	Plaskynaston 2 H	5	6·916*	13,341,633	·00003238	1638	·06873	56·29	546	·01768	4·827
	Sum =	221	..	..	..	..	..	..	..	..	..
	Mean	..	7·049	15,242,320	·00002886	2063	·07850	80·63	688	·01971	6·774

In which D_0 = the external, and d_0 = the internal diameter, in inches.

„ D_D = the external, and d_D = the internal depth or vertical diameter in Elliptical sections.

„ D_B = the external, and d_B = the internal breadth or horizontal diameter in Elliptical sections.

„ L = the length of the beam in feet.

„ W = the load in lbs., tons, &c., dependent on the terms of M_T.

„ M_T = Multiplier which varies with the Material, mode of fixing, loading, &c.: the value for rectangular and cylindrical beams is given by Tables 64, 65, 66, and the Ratios in (359), (362).

(335.) The value of M_T may be found from direct experiment by rule (327). Its most useful value is when W = the ultimate or breaking weight in the centre of a rectangular beam supported at both ends:—in that case it is simply the breaking load of a beam 1 inch square and 1 foot long.

Table 64 gives in col. 7 the mean value of M_T for the breaking weight of 54 kinds of British cast iron at 2063 lbs., or 18·4 cwts., or ·92 ton, and of course W will come out in lbs., cwt., or tons according to the Multiplier used:—col. 10 gives M_T for the safe dead load, which is taken at $\frac{1}{3}$rd of the breaking weight. This Table is based on Fairbairn and Hodgkinson's experiments.

Table 65 gives the value of M_T for Timber, and Table 66 a reduced and condensed general summary. Table 67 gives the Transverse strength in connection with the Stiffness for the Safe working load as well as for the breaking weight: the ratio which these should bear to one another, or the "Factor of Safety," varies with the nature of the material and the character of the Strain, &c. See (880), &c.

(336.) The application of the rules and Tables may be illustrated by examples. Thus to find the breaking weight for a beam of English Oak, 12 inches deep, 6 inches wide, and 15 feet long;—we may take M_T from col. 6 of Table 66 at ·2272 ton; then the rule (324) becomes $12^2 \times 6 \times \cdot 2272 \div 15 = 13 \cdot 1$ tons breaking weight in the centre. Again: to find the depth of a beam of Riga Fir 10 feet long, 3 inches wide, to carry the

TABLE 65.—Of the TRANSVERSE STRENGTH of TIMBER, or the Value of M_T, being the central Breaking Weight of a Beam 1 inch square, 1 foot long between end bearings.

Kind of Wood.	Max.	Min.	Mean.		Authority and Number of Experiments.
	lbs.	lbs.	lbs.	tons.	
Acacia	..	..	747	·3334	P. W. Barlow.
" green	..	..	622	·2776	1 Ebbels.
Alder	..	..	530	·2366	1 "
Ash, common	682	665	675	·3014	6 Barlow.
" "	597	490	542	·2420	3 Peake and Barrallier.
" "	..	..	815	·3638	2 P. W. Barlow.
" "	..	..	635	·2830	1 Ebbels.
" "	..	..	785	·3504	1 Tredgold.
" young	..	..	810	·3616	1 "
" American ..	..	..	517	·2308	Nelson.
" " ..	..	..	680	·3054	Denison.
" " Swamp	..	..	388	·1732	"
" " Black	..	..	287	·1281	"
Beech, common ..	538	494	519	·2317	3 Barlow.
" " ..	..	..	677	·3022	1 Ebbels.
" American, White	..	..	460	·2054	Denison.
" " Red	..	..	573	·2560	Nelson.
" " "	..	..	586	·2610	Denison.
Birch, common	815	727	771	·3442	2 P. W. Barlow.
" "	..	..	518	·2312	1 Ebbels.
" American, Black	896	572	727	·3250	4 P. W. Barlow.
" " "	..	..	616	·2750	Nelson.
" " "	..	..	603	·2700	Barlow.
" " "	..	..	842	·3760	Denison.
" " Yellow	..	..	445	·2000	"
Cedar, Lebanon ..	..	..	498	·2223	"
" " ..	..	..	412	·1840	1 Tredgold.
" Bermuda ..	..	..	465	·2760	Nelson.
" Guadaloupe ..	..	..	681	·3040	"
" American, White	..	..	255	·1140	Denison.
Chestnut, green ..	..	..	450	·2009	1 Ebbels.
" Spanish, dry	592	382	487	·2176	2 Tredgold.
Deal, Christiania ..	657	587	622	·2780	2 P. W. Barlow.
" " ..	..	..	686	·3060	1 Tredgold.
" " ..	..	..	521	·2326	Barlow.
" Memel	..	..	692	·3090	P. W. Barlow.
" "	..	..	545	·2433	1 Tredgold.
Elm, English	348	334	338	·1500	3 Barlow.
" "	481	412	447	·2000	2 P. W. Barlow.
" "	..	..	540	·2410	1 Ebbels.

TABLE 65.—TRANSVERSE STRENGTH of TIMBER, &c.—*continued.*

Kind of Wood.	Max.	Min.	Mean.		Authority and Number of Experiments.
	lbs.	lbs.	lbs.	tons.	
Elm, Wych, green ..	..	..	480	·2140	1 Ebbels.
Fir, Riga	518	308	404	·1803	25 Beaufoy.
„ „	425	275	359	·1602	6 Barlow.
„ „ dry	550	280	382	·1705	27 Peake and Barrallier.
„ „ wet	..	..	437	·1951	37 „ „
„ Dantzic	480	350	390	·1741	4 „ „
„ Mar Forest ..	525	327	407	·1817	9 Barlow.
„ Scotch	..	..	392	·1750	1 Ebbels.
„ Spruce	..	..	465	·2076	1 „
Larch..	414	225	330	·1473	13 Barlow.
„	425	332	375	·1674	4 Peake and Barrallier.
„	632	322	504	·2250	3 Tredgold.
Lignum-Vitæ	..	..	671	·3000	Nelson.
Mahogany, Spanish..	..	..	425	·1897	1 Tredgold.
„ Honduras	..	..	637	·2844	1 „
„ Nassau ..	..	..	584	·2607	Nelson.
„ „ ..	..	..	501	·2237	Moore.
„ „ ..	..	..	635	·2840	Young.
Maple, Canadian ..	..	..	565	·2522	Denison.
Oak, English	568	368	475	·2120	6 Barlow.

[redacted]

TABLE 65.—TRANSVERSE STRENGTH of TIMBER, &c.—*continued.*

Kind of Wood.	Max.	Min.	Mean.		Authority and Number of Experiments.
	lbs.	lbs.	lbs.	tons.	
Oak, African	1035	904	998	·4455	4 P. W. Barlow.
„ „	..	..	831	·3715	Barlow.
„ „	..	..	828	·3696	Nelson.
„ „	..	..	841	·3755	Moore.
„ „	..	..	865	·3861	Denison.
„ Memel	..	..	555	·2480	Moore.
„ Italian	..	..	563	·2513	„
Pine, Red	464	413	447	·2000	3 Barlow.
„ „	..	..	648	·2900	Nelson.
„ „	..	..	600	·2680	Young.
„ „	..	..	430	·1920	Moore.
„ „	..	..	420	·1875	Denison.
„ Pitch	569	521	544	·2428	3 Barlow.
„ „	672	364	544	·2428	24 Beaufoy.
„ „	912	505	702	·3134	7 Peake and Barrallier.
„ „	..	..	607	·2710	Denison.
„ Yellow	627	332	472	·2108	6 Peake and Barrallier.
„ „	..	..	396	·1770	Moore.
„ „	..	..	367	·1640	Barlow.
„ „	..	..	422	·1900	Denison.
„ White	307	227	272	·1213	7 Peake and Barrallier.
„ „	..	..	485	·2165	Nelson.
„ „	..	..	358	·1600	Young.
„ „	..	..	387	·1728	Denison.
„ Dantzic	..	..	475	·2120	Moore.
„ Memel	..	..	449	·2004	„
„ Riga	..	..	562	·2509	„
„ „	..	..	359	·1603	Barlow.
„ Archangel ..	..	..	457	·2040	Moore.
Plane-tree	..	..	607	·2710	1 Ebbels.
Poplar, Lombardy ..	..	..	328	·1467	1 „
„ Abele	..	..	570	·2540	1 „
Spruce	..	..	449	·2000	Moore.
„ American ..	..	..	345	·1540	Denison.
Sycamore	..	..	535	·2388	1 Ebbels.
Teak	892	717	821	·3660	3 Barlow.
„	677	642	660	·2947	2 Peake and Barrallier.
„	..	..	633	·2826	Nelson.
„	..	..	655	·2924	Denison.
Willow	..	..	365	·1629	Tredgold.
Walnut (green) ..	..	..	487	·2174	Ebbels.

TABLE 66.—Of the TRANSVERSE STRENGTH of BARS 1 inch square, 1 foot long: value of M_r for Breaking load.

Materials.	Maximum.		Minimum.		Mean.		No. of Experiments.	No. of Authorities.
	Lbs.	Tons.	Lbs.	Tons.	Lbs.	Tons.		
Cast-Iron	2632	1·175	1638	·7308	2063	0·921	221	2
Wrought Iron: plain bars	..	..	..	..	4000	1·786	..	..
„ limit of Elasticity	..	..	..	..	2000	0·893	..	..
„ Working dead load	..	..	..	..	1330	0·594	..	..
Rolled T and I bars ..	..	..	..	..	3200	1·43	..	..
„ limit of Elasticity	..	..	..	..	1500	0·67	..	..
„ Working dead load	..	..	..	..	1120	0·50	..	..
Steel: ordinary bar ..	7432	3·318	5671	2·532	6720	3·000	4	1
„ limit of Elasticity	..	..	..	..	5600	2·500	..	..
„ Working dead load	..	..	..	..	3360	1·500	..	..
Slate: Bangor, split ..	543	·242	354	·158	421	·188	9	2
„ Valentia, sawn ..	275	·123	220	·098	241	·107	5	1
York Paving	78	·035	67	·03	73	·033	2	1
Ash	815	·3638	490	·2188	681	·304	14	5
Beech	677	·3022	494	·2205	559	·250	4	2
Birch	815	·3638	518	·2312	687	·3067	3	2
Cedar	681	·3040	412	·1840	514	·2295	4	4
Chestnut	592	·2643	382	·1708	475	·2120	3	2
Deal	692	·3090	521	·2236	615	·2746	5	3
Elm	540	·2410	334	·1500	408	·1820	6	3
Fir, Riga	550	·2455	275	·1288	389	·1737	58	3
Larch	632	·2821	225	·1005	380	·1700	21	5
Mahogany	637	·2844	425	·1900	556	·2482	5	4
Oak, English	964	·4303	364	·1625	509	·2272	38	8
„ Dantzic	526	·2322	224	·1000	357	·1600	29	3
„ Canadian	620	·2768	566	·2530	584	·2600	6	4
„ African	1035	·4620	828	·3696	920	·4110	8	5
Pine, Pitch	912	·4072	364	·1630	577	·2576	35	4
„ Red	648	·2900	413	·1844	491	·2190	7	5
„ Yellow	627	·2800	367	·1640	446	·1983	9	4
„ White	485	·2165	227	·1013	331	·1478	10	4
Teak	892	·3982	633	·2826	724	·3230	7	4
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

working load of 1900 lbs. in the centre:—we may take M_r = 78 lbs. from col. 3 of Table 67, and the rule (325) becomes $\sqrt{1900 \times 10) \div (78 \times 3)} = 9$ inches, the depth required.

Again: to find the breadth for a beam of Pitch-pine, 12 feet long, 10 inches deep, to break with 24,000 lbs. in the centre:—taking M_T from col. 5 of Table 66, at 577 lbs., rule (326) becomes $(24000 \times 12) \div (100 \times 577) = 5\cdot16$ inches, the breadth required, &c.

TABLE 67.—Of the TRANSVERSE STRENGTH and STIFFNESS of BEAMS, 1 inch square and 1 foot long.

Material.	Ultimate Strength.		Safe Working Dead Load.		Resilience in Inch-Pounds.		Ratios.		
	Weight in Lbs. W_B.	Deflection in Inches. δ_B.	Weight in Lbs. W_S.	Deflection in Inches. δ_S.	Ultimate. R.	Safe Working. r.	$\frac{W_B}{W_S}$	$\frac{\delta_B}{\delta_S}$	$\frac{R}{r}$
Steel, Bar	5600*	·0802	3360†	·0481	224·5	80·8	1·66	1·66	2·78
Wrought Iron ..	2000*	·0313	1330‡	·0208	31·3	13·9	1·5	1·5	2·25
Cast Iron	2063	·0785	688	·0197	81·0	6·78	3	4	12
Slate (Bangor, split)	421	·0153	105	·0038	3·2	0·20	4	4	16
York Paving ..	73	·0264	18	·0067	0·96	0·06	4	4	16
Ash	681	·375	136	·0354	127·6	2·41	5	10·6	53
Beech	558	·234	112	·036	65·3	2·02	5	6·5	32·3
Cedar	455	·440	91	·081	100·0	3·68	5	5·44	27·2
Deal	615	·184	123	·031	56·6	1·91	5	5·94	29·7
Elm	408	·283	82	·052	57·7	2·13	5	5·45	27·1
Fir, Riga	389	·288	78	·030	56·0	1·17	5	9·60	47·9
Larch	380	·300	76	·037	57·0	1·41	5	8·11	40·4
Oak, English ..	509	·308	102	·040	78·4	2·04	5	7·70	38·4
" Canadian ..	584	·245	117	·028	71·5	1·64	5	8·75	43·6
" Dantzic ..	357	·198	71	·026	35·3	0·92	5	7·62	38·3
Pine, Pitch	577	·245	115	·041	70·7	2·36	5	5·98	30·0
" Red	491	·239	98	·023	58·7	1·13	5	10·40	52·0
Teak	724	·176	145	·026	63·7	1·88	5	6·77	33·9
Willow	365	·480	73	·056	87·6	2·04	5	8·57	42·9
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

* "Limit of Elasticity": Wrought-iron = ½, and Steel = ⅗ the Breaking-down loads.
† ⅗ of Breaking-down load. ‡ ⅔ of Breaking-down load.

"*Old Rule.*"—The rule (329) shows that in hollow *square* sections the strength is proportional to $\frac{D^4 - d^4}{D}$. The old rule very commonly used by practical men is:—

$$(337.)\qquad W = (D^3 - d^3) \times M_T \div L.$$

In this rule a hollow beam is considered as composed of two solid beams, one having the external, and the other the internal dimensions; then the strength of the smaller one subtracted from that of the larger was supposed to give the strength of the hollow one. But it is overlooked that the *ultimate* deflection is inversely proportional to the depth (694), and that the full strength will not be realised if that deflection is not permitted. For instance, in Fig. 64, the hollow beam A is supposed to be composed of the two solid beams B, Fig. 65, and C, Fig. 66, but the ultimate deflection of the two latter will be in the ratio 1 to 2. Now, when combined as in A, and when breaking with the deflection due to B, the deflection of C is *half* only of that due with its own breaking weight, therefore *half* only of its strength is to be subtracted from the *full* strength of B in order to find the real strength of A. By the old rule the strength of A would be $4^3 - 2^3$, or $64 - 8 = 56$, but allowing half of C only, we obtain $64 - 4 = 60$ as the actual strength. We should obtain the same result by the rule (329), namely $\dfrac{D^4 - d^4}{D}$, or in our case $\dfrac{4^4 - 2^4}{4}$ or $\dfrac{256 - 16}{4}$ or $\dfrac{240}{4} = 60$, as before, which is $60 \div 56 = 1 \cdot 07$, or 7 per cent. in excess of the old rule.

(338.) Again: in Fig. 67, the internal dimensions are $\frac{3}{4}$ of the internal; for the former we have $4^3 = 64$, and $\frac{3}{4}$ of the latter becomes $3^3 \times \frac{3}{4} = 20 \cdot 25$; hence the strength of the hollow beam is $64 - 20 \cdot 25 = 43 \cdot 75$. The rule (329) gives $\dfrac{4^4 - 3^4}{4}$ or $\dfrac{256 - 81}{4}$ or $\dfrac{175}{4} = 43 \cdot 75$ also. The old rule gives $4^3 - 3\frac{1}{2}^3 = 37$, the correct rule being $43 \cdot 75 \div 37 = 1 \cdot 18$, or 18 per cent. in excess of the old one.

Again: in Fig. 68, the internal sizes are $\frac{7}{8}$ of the external; hence, instead of deducting $3\frac{1}{2}^3 = 42 \cdot 875$, we have to deduct $42 \cdot 875 \times 7 \div 8 = 37 \cdot 5$, and we obtain $64 - 37 \cdot 5 = 26 \cdot 5$ for the hollow beam. By rule (329) $\dfrac{4^4 - 3\frac{1}{2}^4}{4}$ or $\dfrac{256 - 150}{4}$ or

$\frac{106}{4} = 26 \cdot 5$ as before. By the old rule $4^3 - 3\frac{1}{2}^3 = 21 \cdot 125$; the correct rule being $26 \cdot 5 \div 21 \cdot 125 = 1 \cdot 254$, or $25 \cdot 4$ per cent. in excess of the old rule.

From all this we find that the old rule is entirely incorrect; and, further, we have the result that the error increases with the relative thinness of the metal. Thus with metal 1, $\frac{1}{2}$, and $\frac{1}{4}$ inch thick, the error in our case was 7, 18, and 25·4 per cent. respectively.

(339.) The same erroneous method of calculation (337) is very commonly applied to girders of the ordinary flanged type. Thus in Fig. 69, A is the section of a girder, which is frequently considered as composed of two plain sections B and C:—then for B by the rule (324) we have $9^2 \times 3 = 241$, and for C, $7^2 \times 2\frac{1}{4} = 110 \cdot 25$, from which A becomes $241 - 110 \cdot 25 = 130 \cdot 75$. But by the correct rule (330), we obtain $\frac{(9^3 \times 3) - (7^3 \times 2\frac{1}{4})}{9} = 157 \cdot 3$; a difference of $157 \cdot 3 \div 130 \cdot 75 = 1 \cdot 20$, or 20 per cent.

SPECIAL RULES FOR CAST IRON.

(340.) The Rules in (323) are perfectly correct for all materials so far as solid sections are concerned, but for hollow tubular, and ordinary ⊥ and I sections in cast and wrought iron they are not correct except for very light strains. The rules are absolutely correct for those cases only where the Tensile and Crushing strength of the material and the corresponding extensions and compressions are equal to one another, and this, as shown by Table 79, is not the case with any known material when strained nearly to the breaking point. With cast iron the ratio of those strains is 1 to 6, and special rules become necessary when we would calculate the breaking weight. With wrought iron there is much greater equality between T and C, and the ordinary rules would be nearly correct but for the fact that in thin plates of wrought iron there is a tendency to wrinkle or become undulated under a compressive load with a strain much less than is necessary to crush the material. The great strength of cast iron in resisting compression, and the great

weakness of wrought iron, necessitate special rules for both materials, differing from one another, and differing also from the ordinary rules for other materials whose tensile and crushing strengths are more nearly equal. The necessity for special rules will be made apparent by comparing calculation with experiment. It is probable that with the working loads commonly adopted in practice, say $\frac{1}{3}$rd of the breaking weight, the ordinary rules will be sufficiently correct for practice (353); the usual course, however, is to calculate the breaking weight, and then to find the working load by the use of the "Factor of Safety" (880).

(341.) The best rule we can give for Cast-iron beams is an Empirical one, and is based on the assumption that the resistance to compression is infinite, and as a result, that the neutral axis coincides with the edge of that part of the section subjected to compression. This assumption is manifestly not absolutely true; nevertheless, we shall obtain with cast iron more correct results on that hypothesis than with any other.

(342.) "*Beams of ⊥ and T Section.*"—The best evidence of the necessity for special rules for cast-iron beams is given by sections of this form. Let A, in Fig. 70, be such a section. By the old method of calculation (339) this would be regarded as two solid beams, B and C; for B we have $8^2 \times 6 = 384$; for C, $7^2 \times 5 = 245$; hence A becomes $384 - 245 = 139$, which is the reduced value of $D^2 \times B$. Now, if we reverse the position as at D, we should by this mode of calculation obtain precisely the same result, whereas it is well known by experiment that with cast iron there is a great difference of strength in the two positions, A being much stronger than D.

By the new method of calculation we must calculate in both cases from the line N, A; then with A we have for the vertical web $7^2 \times 1 = 49$, and for the bottom flange $(8^2 - 7^2) \times 6 = 90$; the sum of the two $= 139$, or the same as by the old method. But with D we have for the top flange $1^2 \times 6 = 6$, and for the vertical web $(8^2 - 1^2) \times 1 = 63$; the sum of the two is $6 + 63 = 69$, which is very nearly half the strength in the other position, namely 139.

(343.) Mr. Hodgkinson made experiments on beams E and F

in Fig. 71; E broke with 1008 lbs.: and F with 270 lbs.; the ratio being $1008 \div 270 = 3 \cdot 73$ to 1. The length between supports was 4·25 feet. Calculating from the line N, A, in both cases we have with E, for the vertical web $1 \cdot 1^2 \times \frac{1}{4} = \cdot 302$, and for the bottom flange $(1 \cdot 35^2 - 1 \cdot 1^2) \times 4 = 2 \cdot 45$: the sum of the two is $\cdot 302 + 2 \cdot 45 = 2 \cdot 752$, which is the reduced value of $D^2 \times B$ in rule (324), and taking $M_T = 2063$ lbs., we obtain $2 \cdot 752 \times 2063 \div 4 \cdot 25 = 1336$ lbs.

In the position F we have for the top flange $\cdot 25^2 \times 4 = \cdot 25$, and for the vertical web $(1 \cdot 35^2 - \cdot 25^2) \times \cdot 25 = \cdot 44$:—the sum of the two $= \cdot 25 + \cdot 44 = \cdot 69$, which is the reduced value of $D^2 \times B$, and rule (324) becomes $\cdot 69 \times 2063 \div 4 \cdot 25 = 335$ lbs. The ratio of the strength in the two positions is $1336 \div 335 = 3 \cdot 988$ to 1: experiment gave 3·73 to 1.

(344.) In Fig. 72 we have sections of similar beams experimented upon by Mr. Hodgkinson, the length between supports being $6\frac{1}{2}$ feet; G broke with 1120 lbs., and H with 364 lbs., the ratio being $1120 \div 364 = 3 \cdot 08$ to 1. Calculating from the line N, A as before, with G we have for the vertical web $1 \cdot 26^2 \times \cdot 365 \doteq \cdot 58$; for the bottom flange $(1 \cdot 56^2 - 1 \cdot 26^2) \times 5 = 4 \cdot 23$. The sum of the two $= \cdot 58 + 4 \cdot 23 = 4 \cdot 81$, with which rule (324) becomes $4 \cdot 81 \times 2063 \div 6 \cdot 5 = 1526$ lbs.

In the position H, we have for the top flange $\cdot 3^2 \times 5 = \cdot 45$; for the vertical web $(1 \cdot 55^2 - \cdot 3^2) \times \cdot 36 = \cdot 8325$:—the sum of the two $= \cdot 45 + \cdot 8325 = 1 \cdot 2825$, with which rule (324) becomes $1 \cdot 2825 \times 2063 \div 6 \cdot 5 = 407$ lbs., giving as the ratio of strengths in the two positions, $1526 \div 407 = 3 \cdot 75$ to 1: experiment gave 3·08 to 1.

Considering the extreme disproportion between the flange and web in these experimental beams, whose forms were not such as would usually be found in practice, but were designed for the special purposes of research, the calculated results are perhaps as correct as could be expected.

(345.) "*Hollow Rectangular Beams.*"—Let Fig. 73 be the section of a hollow beam 4 inches square externally, 3 inches internally, 6 feet long, $M_T = \cdot 92$ ton. Calculating from the neutral axis N, A, $D^2 \times B$ becomes for the top plate a, $\cdot 5^2 \times 4 = 1 \cdot 0$; for the two sides $b\ b$, $(3\frac{1}{2}^2 - \frac{1}{2}^2) \times 1 = 12 \cdot 0$; and for

the bottom $c\ (4^2 - 3\frac{1}{2}^2) \times 4 = 15 \cdot 0$. The sum of the whole is $1 + 12 + 15 = 28$, with which the rule (324), or $W = D^2 \times B \times M_T \div L$, becomes $28 \times \cdot 92 \div 6 = 4 \cdot 293$ tons breaking weight in the centre.

By the old rule (337) we should have had $(4^3 - 3^3) \times \cdot 92 \div 6 = 5 \cdot 67$ tons. By rule (329) we obtain $\frac{(4^4 - 3^4)}{4} \times \cdot 92 \div 6 = 6 \cdot 71$ tons.

(346.) We have thus obtained three very different results, and in the absence of experiment should not know which was correct; fortunately, we have Mr. E. Clark's experiments on hollow beams of various sections by which the various rules may be tested.

Fig. 74 is the section of a hollow square beam, the *mean* breaking weight of which with a length of 6 feet was $2 \cdot 152$ tons. There were three experiments which gave $2 \cdot 0$, $2 \cdot 05$, and $2 \cdot 405$ tons respectively. Calculating from the line N, A, as in the last example, we have for the top $\frac{3}{8}^2 \times 3\frac{1}{8} = \cdot 439$; for the two sides $(2\frac{3}{4}^2 - \frac{3}{8}^2) \times \frac{3}{4} = 5 \cdot 666$; and for the bottom $(3\frac{1}{8}^2 - 2\frac{3}{4}^2) \times 3\frac{1}{8} = 6 \cdot 885$. The sum of the three is $12 \cdot 99$: then rule (324) becomes $W = 12 \cdot 99 \times \cdot 92 \div 6 = 2$ tons nearly, agreeing precisely with one of the experiments, but differing from the mean of the three $2 \cdot 0 \div 2 \cdot 152 = \cdot 929$, giving an error by Special Rule of $1 \cdot 0 - \cdot 929 = \cdot 071$ or $- 7 \cdot 1$ per cent.

If we calculate the same beam by rule (329), we obtain $W = \frac{3\frac{1}{8}^4 - 2\frac{3}{8}^4}{3\frac{1}{8}} \times \cdot 92 \div 6 = 3 \cdot 119$ tons: hence $3 \cdot 119 \div 2 \cdot 152 = 1 \cdot 45$, or an error of $+ 45$ per cent. by the ordinary rule.

(347.) Fig. 75 gives the section of hollow rectangular beams, the mean breaking weight of which by Mr. Clark's experiments was $2 \cdot 3$ tons, the length being 6 feet. There were four experiments, the maximum $= 2 \cdot 45$, and the minimum $= 2 \cdot 2$ tons. Calculating from the line N, A, we have for the top, $\frac{3}{8}^2 \times 2 \cdot 21 = \cdot 3107$; for the two sides $(3 \cdot 665^2 - \frac{3}{8}^2) \times \frac{3}{4} = 9 \cdot 975$; and for the bottom $(4 \cdot 04^2 - 3 \cdot 665^2) \times 2 \cdot 21 = 6 \cdot 033$. The sum of the whole is $16 \cdot 319$; then the rule becomes $16 \cdot 319 \times \cdot 92 \div 6 = 2 \cdot 502$ tons; hence $2 \cdot 502 \div 2 \cdot 3 = 1 \cdot 088$, or an error of $+ 8 \cdot 8$ per cent.

By rule (330) we obtain $\frac{(4 \cdot 04^3 \times 2 \cdot 21) - (3 \cdot 29^3 \times 1 \cdot 46)}{4 \cdot 04}$ $\times \cdot 92 \div 6 = 3 \cdot 558$ tons:—hence $3 \cdot 558 \div 2 \cdot 3 = 1 \cdot 547$, or an error of $+ 54 \cdot 7$ per cent. by the ordinary rule.

(348.) "*Circular Sections.*"—For circular sections, or cylindrical beams, we must still calculate from the top edge of the section, but shall have to resort to analysis in the manner explained and illustrated for rectangular sections in (494). Taking first a solid round bar 1 inch diameter, and 1 foot long, we know (335) that a square bar of those dimensions would break with ·92 ton, and admitting for the breaking weight 1·0 to 1·5 as the experimental ratio of the strength of round to square bars (361), a round bar would break with $\cdot 92 \div 1 \cdot 5 = \cdot 6133$ ton. Now by analysis, we have to reckon from the line N, A in Fig. 76; the maximum tension at $C = 7 \cdot 14$ tons per square inch (4), therefore at $B = 7 \cdot 14 \div 2 = 3 \cdot 57$ tons. The area $= \cdot 7854$ square inch, hence $\cdot 7854 \times 3 \cdot 57 = 2 \cdot 8$ tons:—but it is shown in (495) that to obtain the true mean, we must take $\frac{4}{3}$ of that product or $2 \cdot 8 \times 4 \div 3 = 3 \cdot 733$ tons. Reducing the case to a cantilever of half the length of the beam, as in Fig. 92, we find that the leverages by which the strain of 3·733 tons acts are $\frac{1}{2}$ inch and 6 inches, then the strain at the end of the cantilever becomes $3 \cdot 733 \times \frac{1}{2} \div 6 = \cdot 3111$ ton, which is equivalent to $\cdot 3111 \times 2 = \cdot 6222$ ton in the centre of the beam 1 foot long, differing little from ·6133 ton, as we found before.

(349.) Applying this method of calculation to hollow cylindrical beams; Fig. 77 gives the section of three which by Mr. Clark's experiments broke with 2·0875, 2·358, and 2·416 tons respectively, the mean being 2·287 tons, with a length of 6 feet. The area of the section = 4·12 square inches, hence $4 \cdot 12 \times 3 \cdot 57 \times \frac{4}{3} = 19 \cdot 6$ tons, which with leverages of 2 and 36 inches, gives $19 \cdot 6 \times 2 \div 36 = 1 \cdot 09$ ton at the end of a cantilever 3 feet long, equivalent to $1 \cdot 09 \times 2 = 2 \cdot 18$ tons in the centre of a beam 6 feet long. Experiment gave as the mean 2·287 tons; hence $2 \cdot 18 \div 2 \cdot 287 = \cdot 9532$, or $1 \cdot 0 - \cdot 9532 = \cdot 0468$, giving an error of $- 4 \cdot 68$ per cent.

By rule (332) taking the value of M_T for the *breaking* weight

(335) of circular sections at $\cdot 92 \div 1\cdot 5 = \cdot 6133$, we obtain $\dfrac{3\cdot 875^4 - 3\cdot 125^4}{3\cdot 875} \times \cdot 6133 \div 6 = 3\cdot 432$ tons: hence $3\cdot 432 \div 2\cdot 287 = 1\cdot 50$, or an error of $+ 50$ per cent. by the ordinary rule.

(350.) "*Girder-sections.*"—The same method of calculation will apply to girders of all sections with equal or unequal flanges. Fig. 78 is the section of one which by Mr. Hodgkinson's experiment broke with 6678 lbs. in the centre; the weight of the beam itself between supports $4\frac{1}{2}$ feet asunder was 32 lbs., equivalent to 16 lbs. in the centre, giving a total of $6678 + 16 = 6694$ lbs. Calculating from the line N, A, and taking $M_T = 2063$ lbs., we have for the top flange $\cdot 42^2 \times 1\cdot 76 = \cdot 31$; for the vertical web $(4\cdot 735^2 - \cdot 42^2) \times \cdot 29 = 6\cdot 45$; and for the bottom flange $(5\cdot 125^2 - 4\cdot 735^2) \times 1\cdot 76 = 6\cdot 758$: the sum of the whole is $\cdot 31 + 6\cdot 45 + 6\cdot 758 = 13\cdot 518$ which is the reduced value of $D^2 \times B$ in rule (324), which then becomes $13\cdot 518 \times 2063 \div 4\cdot 5 = 6197$ lbs.: experiment gave 6694 lbs., hence $6197 \div 6694 = \cdot 9258$, showing an error of $1\cdot 0 - \cdot 9258 = \cdot 0742$, or $- 7\cdot 42$ per cent.

By rule (330) we should have had

$$W = \frac{(5\cdot 125^3 \times 1\cdot 76) - (4\cdot 315^3 \times 1\cdot 47)}{5\cdot 125}$$

$\times 2063 \div 4\cdot 5 = 10630$ lbs.: hence $10630 \div 6694 = 1\cdot 588$, or an error of $+ 58\cdot 8$ per cent. by the ordinary rule.

(351.) As an example of the application of the special rules to girders with unequal flanges, we may take Fig. 79, which gives the section of girders of the proportions recommended by Mr. Hodgkinson, the ratio of the areas of flanges being exactly 1 to 6. Mr. Owen, the Government Inspector of Metals, made a series of 13 experiments on these girders, with very various kinds of British cast iron, some pure and others mixed; this fact, together with the large scale of the experiments, enhances their value very much. The general results are given by Table 68: the maximum $= 47\frac{1}{4}$ tons, the minimum $= 30$ tons, and the mean of the whole $= 38\cdot 3$ tons in the centre of the girder, 16 feet between supports.

Calculating as before from the line N, A, we have for the top flange $1^2 \times 3\frac{1}{2} = 3 \cdot 5$; for the vertical web $(12\frac{1}{4}^2 - 1^2) \times 1 = 149 \cdot 0$; and for the bottom flange $(14^2 - 12\frac{1}{4}^2) \times 12 = 552$. The sum of the whole $= 3 \cdot 5 + 149 \cdot 0 + 552 \cdot 0 = 704 \cdot 5$, which is the reduced value of $D^2 \times B$ in rule (324), which then becomes $704 \cdot 5 \times \cdot 92 \div 16 = 40 \cdot 5$ tons: experiment gave a mean $= 38 \cdot 3$ tons; hence $40 \cdot 5 \div 38 \cdot 3 = 1 \cdot 0575$, or an error of $+ 5 \cdot 75$ per cent. by Special Rule.

To calculate this girder in the ordinary way, we shall have to modify rule (330), which then becomes in our case

$$W = \left\{ \frac{14^3 \times 12}{14} - \left(\frac{12\frac{1}{4}^3 \times 8\frac{1}{2}}{14} \right] + \left[\frac{11\frac{1}{4}^3 \times 2\frac{1}{2}}{14} \right\} \times \cdot 92 \div 16$$

$= 56 \cdot 46$ tons, giving an error of $56 \cdot 46 \div 38 \cdot 3 = 1 \cdot 475$, or $+ 47 \cdot 5$ per cent.

(352.) "*General Results.*"—The whole of these experiments on square, rectangular, circular, and ordinary girder sections with equal and unequal flanges, show that the method of calculating from the top edge gives in all cases the most correct results, the errors in the five different kinds of section by the *special* rules being $- 7 \cdot 1$, $+ 8 \cdot 8$, $- 4 \cdot 68$, $- 7 \cdot 42$, and $+ 5 \cdot 75$ per cent. respectively; whereas by the ordinary rules they were $+ 45$, $+ 54 \cdot 7$, $+ 50 \cdot 0$, $+ 58 \cdot 8$, and $+ 47 \cdot 5$ per cent. respectively. It should also be observed that in the special rules the *sum* of the three $-$ errors is $19 \cdot 2$, and the sum of the two $+$ errors is $+ 14 \cdot 55$, so that we have as a general average result $(19 \cdot 2 - 14 \cdot 55) \div 5 = 0 \cdot 93$, or less than 1 per cent.

(353.) It is shown by all these experiments that the ordinary rules in (323), &c., give always much higher results than the special rules, the *difference* being $47 \cdot 5 - 5 \cdot 75 = 41 \cdot 75$ per cent. in the girder with unequal flanges (351), and $4 \cdot 68 + 50 \cdot 0 = 54 \cdot 68$ per cent. with the circular section in (349). Now it is admitted in (510) that for small strains, say up to $\frac{1}{3}$rd of the breaking weight, the ordinary rules in (512), &c., which are based on the supposed equality of the tensile and compressive strengths of the material, are *nearly* correct.

(354.) According to that, we might calculate the safe load by the ordinary rule, while the breaking weight must be found by the special rule. This method would conduct us to some curious

TABLE 68.—Of the COMPARATIVE STRENGTH of COMMON and STIRLING'S TOUGHENED CAST IRON, in Girders 16 feet long, &c. Fig. 79.

No.	Load in Centre, in Tons.									Ultimate.	
	7	14	21	28	35	42	45½	49	52½	Load.	Deflection.
	Deflection in Inches: Common Cast Iron.										
1	·16	·38	·62	1·12	..	..	..	..	..	30	1·20
2	·42	·85	1·14	1·55	..	..	..	..	..	33	1·87
3	·20	..	·92	1·53	..	..	..	..	..	33¼	1·82
4	..	·60	..	1·59	..	..	..	..	..	34	2·00
5	..	·42	·85	1·40	..	..	..	..	..	34½	1·87
6	·22	·61	·90	1·60	..	..	..	..	..	35	2·30
7	·27	·54	·78	1·10	1·56	..	..	..	..	36½	1·63
8	·12	·50	·84	1·17	1·58	..	..	..	..	38½	1·80
9	·14	·54	·89	1·27	1·90	..	..	..	..	38½	2·29
10	·22	·48	·68	·90	1·23	1·82	..	..	..	43¾	1·90
11	·33	·55	·75	1·02	1·30	1·84	2·14	..	..	46½	2·19
12	·27	·48	·75	1·01	1·40	1·84	2·10	..	..	47	2·18
13	·22	·35	·70	·95	1·42	1·84	2·06	..	..	47¼	2·14
	Stirling's Toughened Cast Iron.							Mean =		38·3	1·94
14	·30	·52	·76	·99	1·23	1·50	1·64	..	..	48	1·73
15	·18	·37	·53	·74	·99	1·20	1·38	..	..	48½	1·54
16	·20	·47	·65	·94	1·22	1·58	1·79	..	..	50½	1·99
17	·23	·45	·62	·80	·99	1·20	1·42	1·63	..	50½	1·68
18	·31	·53	·72	·99	1·25	1·54	1·75	1·98	..	52	2·13
19	·18	·37	·50	·67	·87	1·12	1·29	1·50	..	52	1·64
20	·14	·37	·57	·84	..	1·49	..	1·82	..	52½	1·98
21	·18	·37	·58	·78	·98	1·22	1·34	1·56	1·79	52½	1·79
22	·20	·38	·58	·79	1·02	1·28	1·42	1·57	1·76	52½	1·76
23	·16	·38	·56	·77	1·00	1·27	1·44	1·62	1·83	56	2·01
24	·24	·45	·62	·80	1·04	1·22	1·35	1·47	1·62	60½	1·95
								Mean =		52·3	1·84

1. ½ Madely Wood (Shropshire); ½ Colebrook Vale (Welsh), both No. 3, C. (cupola) slight flaw lower flange.

2. Calder (Scotch), No. 1, Hot-blast; air furnace. With 28 tons permanent set ·41 inch.

3. Russell's Hall (Staffordshire), No. 2, H.; cupola; sound fracture.

4. Same as No. 2; good sound fracture.

5. ⅓ Russell's Hall, No. 2, H.; ⅓ Madely Wood, No. 3, C.; ⅓ Colebrook Vale, No. 3, C.; cupola.

6. Same mixture as No. 1; cupola.

7. Ley's Works (Staffordshire), No. 2, H.; air furnace. With 14 tons, set $\frac{1}{16}$ inch.

8. Same mixture as No. 7.

9. Same mixture as No. 7, but from cupola; sound casting.

10. ½ Madely Wood, No. 3, C.; ½ Colebrook Vale, No. 3, C.; air furnace.

11. Same mixture as No. 10.

12. ⅓ Prior Field (Staffordshire), No. 1, H.; ⅓ Wednesbury Oak, No. 1, C.; ⅓ Lawley (Shropshire), No. 2, C.; air furnace.

13. Same mixture as No. 12.

14. ·8 Calder (Scotch), No. 1, H.; ·2 wrought iron scrap; air furnace.

15. ·208 Wednesbury Oak, No. 1, C.; ·417 Prior Field, No. 2, H.; ·209 Lawley, No. 2, C.; ·166 wrought-iron scrap; air furnace.

16. ·381 Russell's Hall, No. 2, H.; ·476 Prior Field, No. 1, H.; ·143 wrought-iron scrap; air furnace.

17. ·857 Ley's Works, No. 2, H.; ·143 wrought-iron scrap; air furnace; sound.

18. Same mixture as No. 14; defect in top flange.

19. Unknown.

20. Same mixture as No. 16; sound casting.

21. ·8 Calder (Scotch), No. 1, H.; ·2 wrought-iron scrap; slight defect; air furnace.

22. ·75 Calder (Scotch), No. 1, H.; ·25 wrought-iron scrap; defective; air furnace.

23. Same mixture as No. 17; air furnace.

24. Same mixture as No. 21; sound fracture; air furnace.

results: thus, taking for the values of $M_T = \cdot 92$ ton for the breaking weight, and $\frac{1}{3}$rd of $\cdot 92$, or $\cdot 3067$ for the safe weight, then calculating the former by the *special* rule and the latter by the ordinary rule, the safe load comes out considerably more than $\frac{1}{3}$rd of the breaking weight; in fact more than $\frac{1}{2}$ in some cases.

For instance: for hollow square beams in (346) the breaking weight by special rule = 2 tons, but by ordinary rule with $M_T = \cdot 92$ we obtained $3 \cdot 119$ tons: obviously with $M_T = \cdot 3067$, or $\frac{1}{3}$rd of $\cdot 92$, we should have $3 \cdot 119 \div 3 = 1 \cdot 0397$ ton, which is more than *half* the breaking weight as calculated by the special rule, namely, 2 tons.

Again: with the equal-flanged girder in (350), the calculated breaking weight by the special rule = 6197 lbs., and the safe load by ordinary rule would be $10630 \div 3 = 3543$ lbs., which again is considerably more than *half* the breaking weight by special rule and experiment, instead of $\frac{1}{3}$rd, as it would have been if both had been calculated by the *same* rule.

(355.) This reasoning is certainly correct for cast iron with very light strains up to $\frac{1}{7}$th of the breaking weight, as shown in (617); as the load is increased beyond that point, the ordinary rules give results more and more in excess of the true strength until the breaking weight is attained, when, as shown in (353), it becomes more than 50 per cent. On the other hand, the special rules may be taken as perfectly correct for the *breaking* weight; but as the load is reduced they give results in defect of the true value, the error increasing until with $\frac{1}{7}$th of the breaking weight it becomes $-$ 33 per cent.

It will be evident from this that with $\frac{1}{3}$ of the breaking weight, which is usually adopted for the safe load, the strength would be intermediate between those given by the two rules. It is therefore not quite correct to assume that the ordinary rules will give the working load with "Factor" 3.

(356.) In Fig. 81 we have a Diagram in which the combination of the two rules is represented graphically, the results by the special rule being shown by the line A, and by the ordinary rule by the line B, the latter being throughout 50 per cent. in excess of the former. Now, we have seen that the ordinary rule is correct with $\frac{1}{7}$th of the breaking weight, and the special

rule with the full breaking weight:—then the line C, which connects those two points, should give the correct load throughout. Thus, at $\frac{1}{4}$th the breaking weight, or at D, the line C is exactly intermediate between A and B, and the true strength is an arithmetical mean between the two rules. For example: the hollow square beams in (346) came out 2 tons breaking weight by the special rule, therefore at $\frac{1}{4}$th, or with Factor = 4, we have $2 \div 4 = 0 \cdot 5$ ton working load. By ordinary rule the breaking weight was $3 \cdot 119$ tons, and for the working load we have $3 \cdot 119 \div 4 = \cdot 78$ ton. Then, as by Diagram, the true load for $\frac{1}{4}$th the breaking weight is an arithmetical mean between those results, and becomes $(0 \cdot 5 + 0 \cdot 78) \div 2 = \cdot 64$ ton. Or we might have done it another way, by dividing the difference between the two rules into two equal parts; adding one of those parts to the calculated strength by the special rule, or, deducting it from that by the ordinary rule, we should obtain the same result. Thus, in our case $(\cdot 78 - \cdot 5) \div 2 = \cdot 14$, and $\cdot 5 + \cdot 14 = \cdot 64$; or $\cdot 78 - \cdot 14 = \cdot 64$ as before. In this case, the breaking weight is $2 \div \cdot 64 = 3 \cdot 13$ times the safe load, and yet the beam is strained to $\frac{1}{4}$th only of the breaking weight: the effect being to add $\cdot 64 \div \cdot 5 = 1 \cdot 28$, or 28 per cent. to the working load.

It seems paradoxical to say that a beam loaded with $\frac{1}{4}$th of the breaking load is not strained to $\frac{1}{4}$th of the breaking weight, but this is due to the peculiar character of the material. Thus, say we have a beam whose breaking weight = 100, then when loaded with 25 it seems to be self-evident that it will be strained to $\frac{1}{4}$th of the breaking weight, but we have shown that to produce that strain we shall require a load of $1 \cdot 28 \div 4 = 32$ instead of 25. See (133) and (504).

(357.) For $\frac{1}{3}$rd the breaking weight the diagram shows that the difference between the two rules must be divided into three equal parts, and the true strength will then be found by adding *one* of those parts to the special rule, or deducting *two* of the same from the ordinary rule. Thus, taking the same example as before, we have $2 \div 3 = \cdot 6667$ ton by special rule, and $3 \cdot 119 \div 3 = 1 \cdot 0397$ by ordinary rule: the difference $= 1 \cdot 0937 - \cdot 6667 = 0 \cdot 373$. Then $\cdot 373 \div 3 = \cdot 1243$ is to be added to

$\cdot 6667$, and we obtain $\cdot 6667 + \cdot 1243 = \cdot 791$ ton:—similarly *two* of those parts, or $\cdot 1243 \times 2 = \cdot 2486$, deducted from $1 \cdot 0397$ gives $1 \cdot 0397 - \cdot 2486 = \cdot 791$, as before; the ratio of which to the breaking weight, as found by special rule and experiment, is $2 \div \cdot 791 = 2 \cdot 528$ to $1 \cdot 0$ instead of 3 to 1, as by the ordinary method of procedure. The effect of this is to add $\cdot 791 \div \cdot 6667 = 1 \cdot 1865$, or $18 \cdot 65$ per cent. to the working load:—taking a beam whose breaking weight = 100 tons, then, when strained to $\frac{1}{3}$rd, the load will not be $100 \div 3 = 33 \cdot 33$ tons, but $118 \cdot 65 \div 3 = 39 \cdot 55$ tons.

(358.) The application of these principles to practice is very simple; for instance, the breaking weight of the girder, Fig. 79, was found in (351) to be $40 \cdot 5$ tons, as calculated by the special rule; then if we adopt 4 as the factor of safety, the working load would be $40 \cdot 5 \div 4 = 10 \cdot 12$ tons by the ordinary course, but by (356) we obtain $10 \cdot 12 \times 1 \cdot 28 = 13$ tons nearly. If we adopt 3 as the Factor, we have $40 \cdot 5 \div 3 = 13 \cdot 5$ tons in the usual way, which by (357) becomes $13 \cdot 5 \times 1 \cdot 1865 = 16$ tons, &c.

(359.) "*Ratio of Square to Round Bars.*"—It is shown in (519) that the theoretical ratio of the strength of square to round bars of the same dimensions is $1 \cdot 7$ to $1 \cdot 0$; but this is strictly true for those cases only where the tensile and compressive strengths and the corresponding extensions and compressions are equal to one another, and this, as we have seen, is not realised perfectly with any materials, except with very light strains (617). Under these circumstances, experiment alone can determine the real ratio for the Breaking weight, &c.

(360.) For cast iron we have the experiments of Mr. W. H. Barlow, which were made with direct reference to this question: the results are given in Table 69. It will be observed that the round bars were not made of the same linear dimensions as the square ones, but rather of the same sectional *area*. The object of this was possibly to avoid the complications due to the *size* of the casting, which, as shown by (932), is very influential on the transverse strength of cast iron. Thus, a bar 2 inches square would have the same area as another $2\frac{1}{4}$ inches diameter, and presumably there would be equality of strength so far as that is affected by the size of casting.

In the first set of experiments the bars were about one square inch in area: the mean value of M_T from five square bars = 2530 lbs., and from five round ones 1697 lbs., the ratio being $2530 \div 1697 = 1 \cdot 491$ to $1 \cdot 0$. In the second set the bars were about 4 square inches in area: four square bars gave M_T = 2173 lbs., and nine round ones = 1399 lbs., the ratio being $2173 \div 1399 = 1 \cdot 553$ to $1 \cdot 0$.

TABLE 69.—OF EXPERIMENTS on the RELATIVE TRANSVERSE STRENGTH of SQUARE and ROUND BARS of CAST IRON, all 5 feet long.

Depth.	Breadth.	Central Breaking Weight.	M_T	Diameter.	Central Breaking Weight.	M_T
Bars about One Square Inch Area.						
Square Bars.				Round Bars.		
in.	in.	lbs.		in.	lbs.	
1·01	1·02	505	2427	1·145	519	1729
1·01	1·025	505	2415	1·113	505	1831
1·01	1·02	561	2696	1·115	449	1620
1·02	1·025	533	2498	1·118	449	1606
1·00	1·02	533	2613	1·120	449	1598
		Mean	..2530		Mean	..1697
2530 ÷ 1697 = 1·491 to 1; Ratio of square to round.						
Bars about Four Square Inches Area.						
1·985	2·020	3303	2075	2·52	4283	1338
1·990	2·015	3303	2071	2·52	4283	1338
2·01	2·01	3443	2120	2·52	4003	1251
2·00	1·99	3863	2427	2·51	4003	1266
..	..	..	..	2·2	3068	1441
..	..	..	..	2·2	2988	1403
..	..	..	..	2·19	3388	1613
..	..	..	..	2·20	3228	1516
..	..	..	..	2·19	2988	1422
		Mean	..2173		Mean	..1399
2173 ÷ 1399 = 1·553 to 1; Ratio of square to round.						

(361.) The mean ratio from the whole of these experiments, 23 in number, is $1 \cdot 522$ to $1 \cdot 0$, or nearly $1 \cdot 5$ to $1 \cdot 0$, instead of $1 \cdot 7$ to $1 \cdot 0$, as due by theory. This may be taken, therefore, as the real ratio for cast iron.

For timber we have the experiments of Mr. P. Barlow on 2-inch round and square bars cut from the same plank of Christiania deal 4 feet long:—there were three round bars which broke with 740, 780, and 796 lbs. respectively, the mean being 772 lbs. Two square bars gave 1125 and 1110 lbs. respectively, the mean being 1117 lbs., and the ratio $1117 \div 772 = 1 \cdot 447$ to $1 \cdot 0$. Mr. Couch's experiments on round beams in the form of spars $3\frac{1}{4}$ inches diameter give as the value of M_T for

Riga Fir	Red Pine	Yellow Pine	English Oak
296	310	272	337 lbs.

By Table 66, the mean value of M_T for square bars of the same materials is—

389	491	446	509 lbs.

hence the ratio of the transverse strengths of square to round bars comes out—

1·314	1·584	1·64	1·51

The mean $= 1 \cdot 512$ to $1 \cdot 0$: Mr. Barlow's experiments, as we have seen, gave $1 \cdot 447$ to $1 \cdot 0$.

(362.) We may therefore take $1 \cdot 5$ to $1 \cdot 0$ as the ratio of square to round beams of Timber, being the same as found for Cast iron; both being for the breaking weights (354).

For wrought iron, and still more certainly for steel, whose elasticity is nearly perfect, especially for the moderate strains usually adopted as the working load in practice, we may admit the theoretical ratio $1 \cdot 7$ to $1 \cdot 0$.

(363.) "*Bottoms of Round Vessels: Cast Iron.*"—The bottoms of air-vessels and the sides of valve-boxes for pumps, &c., are frequently made flat for reasons of necessity or convenience, although that form is not well adapted to bear the heavy internal pressure to which they are usually subjected; the calculation of the strength is therefore a matter of some importance. We may find the strength of the whole bottom by taking a portion of it, say a strip in the direction of a diameter, 1 inch wide, and having found the strength of that by the ordinary rules for beams, then multiplying the result by the

number of such strips in the whole circumference will give the strength of the whole.

(364.) Let Fig. 82 be a vessel 3 feet internal diameter with a plain flat bottom 1 inch thick:—we have then a bar A, B, 1 inch deep, 1 inch wide, 3 feet long, *fixed* at both ends and loaded equally all over by the pressure of the water. By column 5 of Table 66, the value of M_T = 2063 lbs. in the centre when the bar is merely supported at the ends, but when fixed at the ends and loaded equally all over, as in our case, the ratio given by (431) is 3, hence M_T becomes $2063 \times 3 = 6189$ lbs., and rule (324) gives $1^2 \times 1 \times 6189 \div 3 = 2063$ lbs. spread equally all over. Now, a circle 36 inches diameter has a circumference of 113 inches, and as each of our imaginary beams occupies *two* inches, namely an inch at each end, we have $113 \div 2 = 56 \cdot 5$ such beams in the whole bottom, and the breaking weight of the whole number will be $2063 \times 56 \cdot 5 =$ 116560 lbs. This load being distributed all over an area of 1018 square inches gives $116560 \div 1018 = 114$ lbs. pressure per square inch, or $114 \times 2 \cdot 3 = 262$ feet, the height of the column of water which would burst the vessel. An air-vessel is exposed to more or less violent shocks from the alternating action of the pump, &c.; it is therefore expedient to adopt a high Factor of safety (924) for such cases, say 10, hence the safe working pressure would be $262 \div 10 = 26 \cdot 2$ feet of water.

(365.) It may be objected to this method of calculation, that we have reckoned on the strength of the material at the centre, or where the bars cross each other, twice over or more; but this is not the fact. Taking, for instance, the two bars A, B, and C, D, in Fig. 82, at right angles to each other, it will be observed that the longitudinal filaments which form the strength of A, B, are not at all strained by loading C, D, because the filaments of the latter form another series at right angles to those of the former; and the same reasoning applies to all the imaginary beams of which the bottom is composed.

(366.) "*Bottoms of Square Vessels.*"—In applying this method of calculation to the bottoms of square vessels some modifications are necessary. Let Fig. 83 be a square vessel with a bottom 1 inch thick, &c.:—we have as before an imaginary

bar 1 inch square and 3 feet long, with a breaking load of 2063 lbs. distributed all over, and if all the imaginary beams of which the bottom is composed were allowed to deflect alike, and therefore to be equally strained, we should have 2063 × 36 × 2 = 148536 lbs. as the total breaking load. But it will be observed that from the conditions of the case, this equality of strain is not realised, for while the beams E, F, and G, H, have equal and great deflections, the beam I, J, will deflect very little, being prevented from doing so by the beam E, F, the fibres of both beams being interlaced with one another. It will be thus seen that between the edges where the deflection is nothing, to the centre where it is a maximum, we have a series of beams with a progressively increasing deflection. The *mean* deflection of the whole series of beams is $\frac{2}{3}$rds of the maximum central deflection, hence the bottom will bear $\frac{2}{3}$rds only of the strain we calculated before, or 148536 × 2 ÷ 3 = 99024 lbs. distributed over an area of 36 × 36 = 1296 square inches, or 99024 ÷ 1296 = 76 lbs. per square inch. Hence the ratio of the strength of round to square bottoms is 114 ÷ 76 = 1·5 to 1·0.

For plain unribbed flat bottoms of round and square vessels, we have the rules:—

(367.) For Round Vessels $p = t^2 \times 148390 \div d^2$.

(368.) „ „ $h = t^2 \times 342780 \div d^2$.

(369.) For Square Vessels $p = t^2 \times 99010 \div S^2$.

(370.) „ „ $h = t^2 \times 228720 \div S^2$.

In which t = the thickness of the plate in inches; d = inside diameter in inches; S = side of square, inside, in inches; p = bursting pressure in pounds per square inch; and h = head of water, bursting pressure, in feet. Thus, for a vessel 40 inches diameter 1½ inch thick, rule (368) gives 2·25 × 342780 ÷ 1600 = 482 feet of water bursting pressure, or with Factor 10, = 48·2 feet working pressure, &c. See (961).

(371.) "*Ribbed Bottoms.*"—The same reasoning may be applied to a flat-bottomed vessel, with strengthening ribs inside or outside; with cast iron, and an internal pressure, the ribs

should always be inside for the same reason that a ⊥ section girder should be broken flange downwards (342), the metal of the bottom plate being then brought into tension. With external pressure, the ribs should be external for the same reason.

Say, we have a vessel 3 feet diameter with ribs cast inside as in Fig. 84; we may take the central part, or the rib *a*, and its proper share of the bottom plate, namely, from *b* to *c*, as the index of the strength of the whole bottom. We have then in effect, a girder of the section given by Fig. 85, 3 feet long, *fixed* at the ends, and loaded all over. The value of M_T for this case we found in (364) to be 6189 lbs., and by the method of calculation explained in (342), we have to calculate from the line N, A, and we have for the vertical web $4^2 \times 1 = 16 \cdot 0$; for the bottom flange $(5^2 - 4^2) \times 9 = 81$, the sum of the two $= 97$, and rule (324) gives $97 \times 6189 \div 3 = 200000$ lbs. spread all over. The circle 36 inches diameter has a circumference of 113 inches, and as each beam, like Fig. 85, occupies 18 inches, namely, 9 inches at each end, we have $113 \div 18 = 6 \cdot 28$ of such beams in the whole circumference, which will bear collectively $200000 \times 6 \cdot 28 = 1256000$ lbs., distributed over an area of 1018 square inches, or $1256000 \div 1018 = 1233$ lbs. per square inch bursting pressure. With 10 for the Factor of safety, we obtain $1233 \div 10 = 123$ lbs., or $123 \times 2 \cdot 3 = 283$ feet of water, safe working pressure.

(372.) As an illustration of the effect of placing the ribs injudiciously outside instead of inside, we have in Fig. 86 the same girder as in Fig. 85, but in a reversed position. Then calculating as before, but from the line N. A. we have for the top flange $1^2 \times 9 = 9$; for the vertical web $(5^2 - 1^2) \times 1 = 24$; the sum of the two $= 9 + 24 = 33$, whereas in the other position we had 97. The strengths in the two positions will be in the ratio of those numbers, and without going through the whole calculation again, we obtain for the safe working pressure $283 \times 33 \div 97 = 96$ feet of water, instead of 283 feet as with internal ribs: the ratio being $97 \div 33 = 2 \cdot 94$, or nearly 3 to 1 (343).

(373.) "*Wrought Iron and Steel.*"—With very tough and ductile materials, such as wrought iron, there is great difficulty and uncertainty in determining the ultimate or breaking weight

of a bar loaded transversely. A beam of cast iron or timber breaks more or less suddenly, and the breaking point is thus usually well defined; but it is impossible literally to *break* a beam of really good tough wrought iron, and it is difficult to say with what load the bar breaks down completely.

But if the deflections of a bar of wrought iron or steel under a series of progressively increasing loads be very accurately observed, and the results are plotted in a diagram, as in Figs. 210, 211, it will be found, as with the tensile and compressive strains in the diagram, Fig. 215, that up to a certain point the elasticity is almost perfect, that is to say, the deflections are almost exactly proportionate to the loads, and the permanent set almost inappreciable until that point is reached, when the deflections and sets (757) begin to increase very rapidly, showing that the material is beginning to be crippled or over-strained. It will also be observed that beyond that point the deflections and sets increase rapidly with *time*, even with the same load. Now, the Diagram, Fig. 215, shows that both for the tensile and compressive strains the "limit of Elasticity" is about 12 or 13 tons per square inch, and as, by Table 1, the mean ultimate strength of wrought iron is 25·7 tons, it would appear that the "limit of Elasticity" is *half* the ultimate strength for $25 \cdot 7 \div 2 = 12 \cdot 85$ tons per square inch. Applying that ratio to the Diagram for the transverse strain, Fig. 210, we find that the "limit of Elasticity," or the point beyond which the bar would begin to be crippled, is about 1720 lbs. as indicated by a *, with which Rule (327) gives $(1720 \times 6 \cdot 75) \div (1 \cdot 027^2 \times 5 \cdot 51) = 2000$ lbs. as the value of M_T for the limit of Elasticity: then, admitting that to be *half* the ultimate or breaking-down strain, the value of M_T for the latter $= 4000$ lbs.

(374.) This result is confirmed by the experiments of Mr. E. Clark on three bars of wrought iron, $1\frac{1}{2}$ inch square and 3 feet long, in Table 70. With $M_T = 2000$ lbs., Rule (328) gives $W = 1\frac{1}{2}^3 \times 2000 \div (3 \times 112) = 20 \cdot 1$ cwt. as the "limit of Elasticity," and col. 4 shows that up to that point the deflections are nearly in simple proportion to the weights, as due with perfect elasticity. But as the load is increased, the Ratio of

the deflections rises more and more rapidly, until with 41·9 cwts. it becomes as much as 5·145 inches, or 13·13 times the normal amount, showing that practically the strength of the bar is broken-*down*, although the bar may not be actually *broken*. With $M_T = 4000$ lbs., Rule (328) gives $W = 1\frac{1}{2}^3 \times 4000 \div (3 \times 112) = 40 \cdot 2$ cwts. Breaking-down weight, which agrees with the experimental results in Table 70.

We may therefore admit for plain bars of Wrought-iron $M_T = 4000$ lbs. for the Breaking-down weight; 2000 lbs. for the "Limit of Elasticity," and 1330 lbs. for the safe Working dead load.

TABLE 70.—Of the MEAN DEFLECTION of 3 BARS of WROUGHT IRON, $1\frac{1}{2}$ inch square, 3 feet long between supports, loaded in the centre.

Weight.	Deflection in Inches. By Experiment.	Deflection in Inches. By Rule.	Ratio.	Weight.	Deflection in Inches. By Experiment.	Deflection in Inches. By Rule.	Ratio.	Weight.	Deflection in Inches. By Experiment.	Deflection in Inches. By Rule.	Ratio.
cwts.				cwts.				cwts.			
3·73	·035	·0349	1·00	15·5	·165	·145	1·138	27·1	·575	·253	2·273
5·42	·050	·0507	·99	17·2	·185	·161	1·150	28·7	·820	·268	3·06
7·10	·070	·0664	1·05	18·9	·205	·177	1·158	30·3	1·250	·290	4·31
8·80	·085	·0823	1·03	20·6	·225	·193	1·171	31·9	1·585	·298	5·32
10·48	·110	·0980	1·12	22·3	·250	·209	1·200	33·9	2·110	·310	6·81
12·16	·130	·114	1·14	23·9	·290	·223	1·300	37·9	3·230	·354	9·12
13·85	·145	·130	1·12	25·5	·355	·238	1·492	41·9	5·145	·392	13·13
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)

Calculated Breaking-down weight = 40·2 cwt.: "Limit of Elasticity" = 20·1 cwt.

(375.) When wrought iron is rolled into ⊥, I, and L sections, the properties of the material are somewhat changed: it would appear that the maltreatment experienced by being crushed into these forms damages the fibrous texture, as proved by the fact that it breaks *shorter* or more suddenly than plain rectangular sections. The transverse strength of such iron is also considerably less than that of plain bars, as shown by the experiments in Table 71, which give 3208 lbs. as the mean value of M_T. All these bars were loaded to their ultimate strength; two of

them were literally *broken*, and it will be observed that these gave the maximum and minimum values of M_T, the former being 3720 and the latter 2750 lbs.; the mean of the two = 3235 lbs., or nearly the mean of the six experiments, which was 3208 lbs.

For rolled ⊥ and I bars, plate-iron girders, and tubular beams, we may take the value of $M_T = 3200$ lbs. for the breaking-down weight, 1500 lbs. for the "limit of Elasticity" and Proof Strain, and 1120 lbs. for the safe working dead load.

It is remarkable that taking 3200 lbs. for the Ultimate transverse strength, the Rule (500) or (639) gives $3200 \times 18 \div 2240 = 25 \cdot 7$ tons for the maximum ultimate tensile strain, which happens to be precisely the mean strength of British bar iron by Table 1, &c.

(376.) "*Steel.*"—The transverse strength of steel, like that of wrought iron, may be determined most satisfactorily by a Diagram, as in Fig. 211, which shows that, up to 1450 lbs., the elasticity is almost perfect. Taking that as the "limit of elasticity," we have 5600 lbs. or $2\frac{1}{2}$ tons as the value of M_T, with which Rule (328) becomes $W = 1 \cdot 054^3 \times 5600 \div 4 \cdot 5 = 1457$ lbs., and is indicated by a * on the diagram.

Then, for the value of M_T for the breaking-down load, we have the experiments of Mr. Fairbairn, in Table 107, the mean of which = 6663 lbs., or say 3 tons = 6720 lbs.

We may therefore take, as the value of M_T for Steel bars, 6720 lbs. or 3 tons for the breaking-down weight; 5600 lbs. or $2\frac{1}{2}$ tons for the "limit of elasticity" or "Proof strain;" and say 3360 lbs. or $1\frac{1}{2}$ ton for the safe working dead load.

SPECIAL RULES FOR WROUGHT IRON.

(377.) "*Wrought-iron ⊥ and T Beams.*"—The resistances of wrought iron to tensile and crushing strains, and the corresponding extensions and compressions are nearly equal to one another up to 12 or 13 tons per square inch, as shown by Table 91 and Diagram, Fig. 215. The transverse strength of ⊥ bars might, therefore, be calculated by the ordinary rules in (323) or (510) but for the fact that the great ductility of the metal causes it to have but little stiffness under compressive strains, so that a thin rib becomes undulated or wrinkled with a strain very much less

TABLE 71.—Of the STRENGTH, &c., of

No.	Depth and Width.	Thickness. Top Flange.	Thickness. Vertical Rib.	Length.	Weight in Centre.	Permanent Set, &c.	Calculated Breaking Weight.
				feet.	cwts.		cwts.
1	2	$\frac{1}{4}$	$\frac{3}{8}$	5	$17\frac{1}{2}$	Bent down ..	17·3
2	2	$\frac{5}{16}$	$\frac{5}{16}$	8	9	15 inches Per. Set	8·78
3	$2\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	10	15	$\frac{11}{16}$ Per. Set ..	15·8
4	$2\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{8}$	6	27	Broke	23·2
5	$2\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{8}$	10	14	Failed sideways	13·92
6	3	$\frac{3}{8}$	$\frac{1}{2}$	10	24	Broke	27·9
..	..	..	..	..	..	..	Mean =
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

than the crushing strength of the metal. Thus, while as the Diagram shows, a plain solid bar will bear about 13 tons per square inch; a thin plate may fail by wrinkling with 5 tons or less. Special rules, therefore, become necessary where a thin plate or rib occurs, as in tubular beams of plate-iron (405) and in ⊥ sections, and this arises, not because of the inherent weakness of wrought iron in resisting crushing, but from its tendency to fail by wrinkling.

Tubular beams of plate-iron are frequently made of large dimensions, and are extensively used for the most important structures. The calculation of their strength on exact principles becomes, therefore, highly necessary; the Wrinkling strain, by which that strength is potentially governed, is considered at large (319) in Chapter IX., and the results are applied to such beams in (406), &c. But for ordinary ⊥ and I sections, it will suffice to give Empirical rules by which the strength may be calculated with sufficient precision for practical purposes.

(378.) The best rule we can give is to calculate $D^2 \times B$ from the edge of that part of the section which is subjected to *tension*, or from the bottom in the case of a beam supported at both ends and loaded in the usual way. This, it will be observed, is just the reverse of the mode of calculating Cast-iron beams (341).

Fig. 87 is the section of a beam (No. 5 in Table 71) which,

WROUGHT-IRON T BEAMS, flange uppermost.

Value of M_T.	Calculated Safe Load.	Deflection with Safe Load.		Deflection per Cwt. for Bar 1 Foot long.		No.
		By Experiment.	By Calculation.	By Experiment.	By Calculation.	
lbs.	cwts.					
3237	5·8	·263	·174	·0003776	·000240	1
3278	2·93	·713	·441	·0004752	·000294	2
3052	5·3	·526	·571	·0000933	·000108	3
3720	7·73	·251	·197	·0001506	·000118	4
3215	4·64	·645	·548	·0001390	·000118	5
2750	9·3	·662	·456	·0000712	·000049	6
3208	..	..	..	..	..	..
(9)	(10)	(11)	(12)	(13)	(14)	(1)

with a length of 10 feet between bearings, failed with 14 cwt. in the centre. The flange being uppermost, and under compression, we must calculate from the *bottom* of the section or from the line N. A.; then $D^2 \times B$ becomes for the rib $2\frac{1}{4}^2 \times \frac{3}{8} = 1\cdot9$; and for the top flange $(2\frac{1}{2}^2 - 2\frac{1}{4}^2) \times 2\frac{1}{2} = 2\cdot975$. The sum of the two $= 1\cdot9 + 2\cdot975 = 4\cdot875$, hence with 3200 lbs. for the value of M_T, the rule (324) becomes $4\cdot875 \times 3200 \div 10 =$ 1560 lbs. breaking weight: experiment gave 14 cwt. or 1568 lbs., as in col. 6 of Table 71.

In the reversed position, or flange downwards, as in Fig. 88, we have still to calculate from the bottom of the section or from the line N. A., and for the bottom flange we have $\frac{1}{4}^2 \times 2\frac{1}{2} = 0\cdot156$; for the vertical web $(2\frac{1}{2}^2 - \frac{1}{4}^2) \times \frac{3}{8} = 2\cdot32$. The sum of the two $= 2\cdot476$, and we obtain $2\cdot476 \times 3200 \div 10 = 792$ lbs. breaking weight, or about half the strength in the other position, which we calculated to be 1560 lbs.

(379.) Table 72 gives the safe or working load for Standard sizes of wrought-iron T beams, the application of which is very simple. Thus, say that we require the central safe load for a bar $4 \times 4 \times \frac{3}{8}$ inch thick, with a length of 12 feet; the Table gives 156 cwt. for 1 foot long; hence $156 \div 12 = 13$ cwt. for 12 feet long. The deflection by the rules (697) for this safe load would be $\delta_s = \dfrac{12^2 \times \cdot0235}{4} = \cdot846$ inch: for a load spread

equally all over we should have 26 cwt., and $\cdot 846 \times 10 \div 8 = 1 \cdot 06$ inch deflection, &c.

Again, say that we require a bar to carry 20 cwt. in the centre with a length of $7\frac{1}{2}$ feet:—this is equal to $20 \times 7 \cdot 5 = 150$ cwt. with a length of 1 foot, for which Table 72 gives $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$ inch, or $4 \times 4 \times \frac{3}{8}$ inch. These are the sizes for a dead load; with a moving or rolling load we should require double strength, or say 300 cwt. for 1 foot long, and the bar should then be, say $5 \times 5 \times \frac{1}{2}$ inch.

TABLE 72.—Of the TRANSVERSE STRENGTH of WROUGHT T BEAMS 1 foot long, flange uppermost.

Sizes.	Thickness all over.						
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$
	Safe Load in Centre: in Cwts.						
$1\frac{1}{2} \times 1\frac{1}{2}$	13	16	..	..	..	..	..
2×2	25	30	35	..	..	..	..
$2\frac{1}{2} \times 2\frac{1}{2}$	40	49	57	65	..	..	..
3×3	..	72	85	97	..	..	..
$3\frac{1}{2} \times 3\frac{1}{2}$	..	100	118	135	151	..	..
4×4	..	133	156	178	200	..	..
5×5	..	..	248	285	322	392	..
6×6	..	..	..	418	475	580	677

(380.) The strength of the bar $4 \times 4 \times \frac{3}{8}$ inch in the normal position T is given by Table 72 at 156 cwt.:—in the reversed position ⊥, the bottom flange would give $\frac{3}{8}^2 \times 4 = \cdot 56$; the vertical web $(4^2 - \frac{3}{8}^2) \times \frac{3}{8} = 5 \cdot 94$; the sum of the two $= 6 \cdot 5$, and taking the value of M_T for safe load in cwts. (375) at $9 \cdot 5$, we obtain $6 \cdot 5 \times 9 \cdot 5 = 62$ cwt. breaking weight; hence the ratio of strength in the two positions is $156 \div 62 = 2 \cdot 5$ to $1 \cdot 0$, and this may be taken as a general ratio for all the bars in Table 72. The variations in the position of bar, distribution of load, &c., are rather confusing; the following statement gives

the general Ratios of loads, and actual safe loads of a bar $4 \times 4 \times \frac{3}{8}$, 12 feet long:—

		Ratio of Load.	Load. cwts.
Normal position	T, dead load in centre ..	1·0 ..	13
"	T, distributed dead load ..	2·0 ..	26
"	T, Rolling load	$\frac{1}{2}$ ..	6·5
Reversed position	⊥, dead load in centre ..	$\frac{2}{5}$ ..	5·2
"	⊥, distributed dead load ..	$\frac{4}{5}$ ..	10·4
"	⊥, Rolling load	$\frac{1}{5}$ ..	2·6

(381.) "*Wrought-iron* I *Beams.*"—The strength of rolled beams with double flanges may be calculated on the same principles as those of ⊥ and T section. Figs. 89, 90 are sections of two wrought-iron beams experimented upon by Mr. Fairbairn. Fig. 89 *sunk* with 12,955 lbs. in the centre, the length between bearings being 11 feet. Calculating from the *bottom* as before (378), the reduced value of $D^2 \times B$ becomes for the top flange $(7^2 - 6^2) \times 2\frac{1}{2} = 32 \cdot 5$; for the vertical web $(6^2 - \cdot 38^2) \times \cdot 325 = 11 \cdot 7$; and for the bottom flange, $\cdot 38^2 \times 4 = 0 \cdot 576$. The sum of the whole $= 44 \cdot 776$, which with $M_T = 3200$ lbs. (375) and rule (324) becomes $44 \cdot 776 \times 3200 \div 11 = 13{,}026$ lbs., which is $\frac{1}{2}$ per cent. only in *excess* of 12,955 lbs., the experimental weight.

Fig. 90 *sunk* with 18,962 lbs. in the centre, the length between bearings being 10 feet. Calculating from the bottom as before, the bottom flange gives $\cdot 44^2 \times 4 \cdot 3 = \cdot 83$; for the vertical web $(7^2 - \cdot 44^2) \times \cdot 35 = 17 \cdot 08$; and for the top flange $(8^2 - 7^2) \times 2\frac{3}{4} = 41 \cdot 25$. The sum of the whole $= 59 \cdot 16$, with which the rule (324) becomes $59 \cdot 16 \times 3200 \div 10 = 18931$ lbs., which is only $0 \cdot 17$ per cent. *less* than 18,962 lbs., the experimental weight. See (673) and Table 109 for the Deflections.

(382.) These results show not only the accuracy of the method of calculation, but also the correctness of $M_T = 3200$ lbs. derived from T bars (375) as applied to I sections. Both beams are described as yielding by *lateral* flexure of the top flanges under compression, although their areas were much greater than those of the respective bottom flanges. As shown in (445), with

wrought-iron beams, the flanges subjected to compression should have considerable *width*, to enable them to resist flexure laterally, and this is especially necessary where the length of the beam is great.

(383.) These rolled beams are now made of large sizes, and are deservedly used extensively. In Table 73 are given the sizes of the most useful sections supplied by various makers; col. 14 gives the safe working loads for beams 1 foot long between bearings, from which the load for any length may be easily found. Thus with No. 7, which is $8\frac{3}{4}$ inches deep, the safe dead load, with a length of 12 feet, would be $706 \div 12 = 59$ cwt.; or for a moving load $29\frac{1}{2}$ cwt. (923). Again: say we require a beam to carry a dead load of 20 cwt. with a length of 14 feet, which is equivalent to $20 \times 14 = 280$ cwt. with a length of 1 foot, for which No. 4, $6\frac{1}{2}$ inches deep, 279 cwt. would be used.

The deflections may be found by (674) and by col. 13 in the same Table:—thus with 61 cwt. in the centre, No. 7, 12 feet long would deflect $\cdot000001486 \times 12^3 \times 61 = \cdot1514$ inch. With 20 cwt. in the centre, No. 4, 15 feet long deflects $\cdot000006351 \times 15^3 \times 20 = \cdot34$ inch, &c.

(384.) "*Plate-iron Girders.*"—The investigation of the strength of plate-iron girders may be effected most easily by analysis from elementary principles with the known tensile and crushing strengths of the material, in the manner illustrated for plain rectangular sections in (494). By Table 1, the mean tensile strength of plate-iron is $21 \cdot 6$ tons per square inch; and by (201) the mean crushing strength $= 19$ tons, the ratio being as 8 to 7, which will be the ratios of the areas of the top and bottom flanges also.

(385.) Let Fig. 93 be the section and Fig. 91 the elevation of a girder, say 20 feet or 240 inches long between bearings, loaded in the centre, and 30 inches deep, but the *effective* depth, or that at the centres of gravity of the top and bottom sections may be taken at 29 inches (449). This will evidently be equivalent to a cantilever of half the length or 120 inches, built into a wall at one end and loaded at the other with *half* the central load, as in Fig. 92:—but it is necessary to remember here that in a beam supported at both ends and loaded in the centre, the lower part

TABLE 73.—Of the STRENGTH and STIFFNESS of ROLLED IRON I BEAMS.

No.	Depth.	Top and Bottom Flanges.		Thickness of Vertical Web.	Length.	Load.	Deflection in Inches.				Calculated Safe Load.	Mean Deflection per Cwt. with a Beam 1 Foot long, by Experiment.	Safe Weight in Centre for 1 Foot long.
		Breadth.	Thickness.				By Experiment.	Calculated.					
								Rule (669).	Rule (670).	Rule (673).			
					feet.	cwts.					cwts.		cwts.
1	$4\frac{1}{2}$	$1\frac{7}{8}$	$\frac{3}{8}$	$\frac{5}{16}$	9	16	$\frac{3}{8}$ bare	·2481	·2857	·4228	10·7	·00003688	96
”	”	”	”	”	7	13	$\frac{3}{16}$	·0948	·1092	·1616	13·7	”	”
2	5	$1\frac{3}{4}$	$\frac{5}{16}$	$\frac{1}{4}$	11	17	$\frac{1}{2}$	·3645	·4770	·6032	9·4	·00001965	103
”	”	”	”	”	12	16	$\frac{5}{8}$	·4454	·5828	·7371	8·6	”	”
”	”	”	”	”	13	15	$\frac{3}{4}$	·5309	·6948	·8785	7·9	”	”
”	”	”	”	”	23	8	$1\frac{1}{8}$	1·5680	2·0520	2·5950	4·5	”	”
3	$5\frac{5}{8}$	$1\frac{3}{4}$	$\frac{7}{16}$	$\frac{3}{8}$	$13\frac{1}{2}$	20	$\frac{5}{8}$	·4612	·5258	·7115	13·0	·00001282	175
”	”	”	”	”	14	19	$\frac{11}{16}$	·4887	·5572	·7539	12·5	”	”
”	”	”	”	”	16	17	$\frac{7}{8}$	·6526	·7442	1·0070	10·9	”	”
4	$6\frac{1}{2}$	$2\frac{1}{8}$	$\frac{1}{2}$	$\frac{7}{16}$	10	36	$\frac{1}{4}$	·1921	·2084	·2878	28·0	·000006351	279
”	”	”	”	”	16	40	$1\frac{1}{8}$	·8745	·9484	1·3100	17·4	”	”
”	”	”	”	”	17	21	$\frac{5}{8}$ bare	·5506	·5972	·8246	16·4	”	”
”	”	”	”	”	17	23	$\frac{5}{8}$ full	·6031	·6541	·9031	16·4	”	”
5	$7\frac{1}{4}$	$2\frac{1}{2}$	$\frac{7}{16}$	$\frac{7}{16}$	17	30	$\frac{13}{16}$	·5352	·6183	·8589	20·1	·00000409	341
”	”	”	”	”	19	27	$\frac{5}{8}$ full	·6723	·7767	1·0790	18·0	”	”
”	”	”	”	”	21	24	$\frac{3}{4}$	·8070	·9323	1·2950	16·2	”	”
6	$8\frac{1}{4}$	$2\frac{1}{2}$	$\frac{9}{16}$	$\frac{1}{2}$	17	40	$\frac{9}{16}$	·4422	·5063	·6951	29·5	·00000239	502
”	”	”	”	”	20	35	$\frac{5}{8}$	·6296	·7214	·9903	25·1	”	”
”	”	”	”	”	25	27	$\frac{7}{8}$	·9486	1·0870	1·4920	20·1	”	”
7	$8\frac{3}{4}$	3	$\frac{5}{8}$	$\frac{5}{8}$	22	42	$\frac{5}{8}$	·6250	·7823	1·0650	32·1	·000001486	706
”	”	”	”	”	24	40	$\frac{13}{16}$	·7727	·9673	1·3170	29·4	”	”
”	”	”	”	”	24	50	$1\frac{1}{8}$	·9659	1·2090	1·6460	29·4	”	”
”	”	”	”	”	24	60	$1\frac{3}{16}$	1·1590	1·4510	1·9750	29·4	”	”
”	”	”	”	”	24	70	$1\frac{3}{4}$	1·3520	1·6920	2·3040	29·4	”	”
8	10	$4\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	18	$30\frac{1}{4}$	0·16	·1369	·1643	·2504	53·0	·000000907	953
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)

of the section is subjected to tension and the upper part to compression. In a cantilever loaded in the usual manner this is just reversed:—to prevent confusion, therefore, we have in Fig. 92 taken the weight as acting *upwards* by means of a pulley, thus eliminating that difficulty, and restoring the normal conditions with a beam supported at both ends and loaded in the centre.

The top and bottom flanges in Fig. 93 occupy the position of fulcrum and resistance to one another reciprocally, the strength may therefore be found from *one* of them, say the bottom. We have first to find the reduced or *net* area of the bottom by deducting the metal cut away by the rivets, or by taking the area through the line of rivet-holes:—the rivets being $\frac{5}{8}$ inch diameter, the area of the bottom plate $= (12 - 1\frac{1}{4}) \times \frac{3}{4} = 8 \cdot 06$ square inches. If we admit that the part of the middle web between the two angle-irons compensates for one pair of rivet-holes (which is very nearly the fact), we shall have for the area of the two angle-irons $\left\{3 + 2\frac{1}{2}) - \frac{5}{8}\right\} \times \frac{1}{2} \times 2 = 4 \cdot 875$ square inches. The sum of the two areas $= 8 \cdot 06 + 4 \cdot 875 = 12 \cdot 935$, say 13 square inches.

Then, the tensile strength of plate-iron being $21 \cdot 6$ tons per square inch by Table 1, the resistance of the bottom becomes $13 \times 21 \cdot 6 = 280 \cdot 8$ tons, and the leverages being 29 and 120 inches respectively, as in Fig. 92, we have at the end of the 10-foot cantilever $280 \cdot 8 \times 29 \div 120 = 67 \cdot 86$ tons breaking weight, equivalent to $67 \cdot 86 \times 2 = 135 \cdot 72$ tons in the centre of our beam, 20 feet long.

(386.) It is shown in (464) that the breaking weights of girders of similar or nearly similar sections are in the ratio of the respective areas of their bottom flanges, multiplied by the depth, and divided by the length: hence we have the rules:—

(387.) $W = A \times D \times C \div L.$

(388.) $A = (W \times L) \div (D \times C).$

(389.) $D = (W \times L) \div (A \times C).$

(390.) $C = (W \times L) \div (A \times D).$

In which W = the central load on the beam in lbs., tons, &c., dependent on the terms of C.

A = the area of the bottom plate and angle-irons, in square inches.

D = the depth in inches or feet.

L = the length between supports in the same terms as D.

C = a constant adapted to the particular materials, &c.; for plate-iron Girders = 75 tons.

It will be expedient to take for A, the *gross* area of the bottom (making no deduction for rivet-holes), and for D, the total depth (449). The value of C, as adapted to those conditions, may then be found by rule (390), taking W as found by analysis or as given by direct experiment.

(391.) Thus, the gross area of the bottom plate in Fig. 93, $= 12 \times \frac{3}{4} = 9$, and of the two angle-irons, $(3 + 2\frac{1}{2}) \times \frac{1}{2} \times 2 = 5\cdot5$; the sum being $9 + 5\cdot5 = 14\cdot5$ square inches. Then, the breaking weight W as found in (385) being 135·72 tons, rule (390) gives $C = (135\cdot72 \times 240) \div (14\cdot5 \times 30) = 74\cdot88$, say 75.

The rule (387), namely $W = A \times D \times C \div L$, is the well-known one given by Mr. Fairbairn, the value of C as given by him being 80 for tubular beams, and 75 for ordinary flanged girders, the latter having precisely the value that we found for it by an altogether independent method.

(392.) We must now consider the top flange, in connection with which there are three points requiring attention:—1st, to see that the *area* is sufficient to bear the crushing strain:—2nd, that the *breadth* is sufficient to prevent failure by lateral flexure; and 3rd, that the *thickness* is sufficient to prevent "Wrinkling."

The area which has to sustain the crushing strain is really the *gross* area, for the material lost at the rivet-holes is replaced by the rivets, which being put in and riveted while hot, effectually fill the holes and restore the section to its normal condition so far as compressive strains are concerned:—but of course this does not apply to tensile strains, for which the *net* area at the rivet-holes must be taken.

(393.) The gross area of the top plate $= 12 \times \frac{3}{4} = 9$ square

inches, and of the two angle-irons $(3 + 2\frac{1}{2}) \times \frac{1}{2} \times 2 = 5 \cdot 5$; the total being $9 + 5 \cdot 5 = 14 \cdot 5$ square inches, which having to sustain the same maximum total strain as we found for the bottom (385), namely 280·8 tons, gives $280 \cdot 8 \div 14 \cdot 5 = 19 \cdot 37$ tons per square inch, which is rather in excess of 19 tons (384), but is near enough for the purpose.

(394.) Then, for resistance to lateral flexure. It is shown in (448) that this tendency is a maximum with $\frac{2}{3}$rds of the length of the top plate, the compression strain being then $\frac{1}{3}$rd of the maximum central strain:—thus in our case, this length $= 20 \times \frac{2}{3} = 13 \cdot 3$ feet, and the strain $280 \cdot 8 \times \frac{1}{3} = 93 \cdot 6$ tons. We have, therefore, virtually a pillar $12 \times \frac{3}{4}$ inches forced to bend in the direction of its larger dimensions, with a length of 13·3 feet, and we can calculate the strength by the Rule (234), $F = M_P \times t^{2 \cdot 6} \times b \div L^2$, or $72 \cdot 74 \times 640 \times \cdot 75 \div 176 \cdot 9 = 197 \cdot 3$ tons, or more than double 93·6 tons, the real strain; there is therefore no danger of failure by lateral flexure.

(395.) As to Wrinkling, it is shown in (317) that in a girder, a plate $\frac{3}{4}$ inch thick supported at one side will not fail by wrinkling with less than 19 tons per square inch, unless the plate projects $5\frac{5}{8}$ inches or more beyond the support:—in our case the projection beyond the angle-iron $= 2\frac{7}{8}$ inches only, hence there is no fear of failure by wrinkling.

TABLE 74.—Of the 1·5 POWER of NUMBERS. Wrinkling Strains due to given Ratios of Thickness in Wrought-iron Plates.

No. Thickness.	1½ Power. Strain.	No. Thickness.	1½ Power. Strain.	No. Thickness.	1½ Power. Strain.
1	1·0	11	36·5	21	96
2	2·8	12	41·5	22	103
3	5·2	13	46·8	23	110
4	8·0	14	52·0	24	117
5	11·2	15	58·1	25	125
6	14·7	16	64·0	26	132
7	18·5	17	70·0	27	140
8	22·6	18	76·3	28	148
9	27·0	19	82·8	29	156
10	31·6	20	89·5	30	164

These calculations apply to the centre of the girder, where the Crushing, lateral, and Wrinkling strains are a maximum.

(396.) "*Graduation of Strains.*"—It is shown by (682) and Fig. 155, that in a girder with a central load the Tensile and Compressive strains in the bottom and top flanges respectively, are a maximum at the centre, and diminish in arithmetical ratio towards the end supports, where they become nil. Thus dividing the half-length into say 8 equal parts, and reckoning from one support, we have Tensile and Compressive strains in the ratio:—

0	1	2	3	4	5	6	7	8

and if the top and bottom flanges are parallel or of uniform breadth throughout their length, the thickness to resist the tensile strains would follow the same ratio, and if 1-inch thick is necessary at the centre, we should have a graduated series of thicknesses of:—

0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1-inch.

But to resist the Wrinkling strains, the resistance following the $1\frac{1}{2}$ power of the thickness (315), as given by Table 74, the thickness will follow the $1\frac{1}{2}$ root of the strain as given by Table 75; hence in our case we must have thicknesses in the ratio:—

0	1	1·587	2·08	2·52	2·924	3·302	3·66	4·0

and if in order to resist the maximum Wrinkling strain at the centre, a thickness of 1 inch is required, we should have a graduated series of thicknesses =

0	·25	·393	·52	6·3	·731	·826	·915	1-inch.

(397.) In this way we might find the point between the centre and the end of a girder where any given thickness would be required to avoid wrinkling, supposing that the thickness at the centre is sufficient as calculated by the Rules (307), our present purpose being only to graduate the thicknesses from centre to end, so that the tendency to fail by Wrinkling shall be the same throughout.

Thus for a girder 20 feet long, the half-length = 10 feet;

TABLE 75.—Of the 1·5 ROOT of NUMBERS. Thickness of Wrought-iron Plates to sustain Wrinkling Strain in certain Ratios.

No. Strain.	1½ Root. Thickness.	No. Strain.	1½ Root. Thickness.	No. Strain.	1½ Root. Thickness.
1	1·000	11	4·946	21	7·612
2	1·587	12	5·246	22	7·852
3	2·080	13	5·529	23	8·088
4	2·520	14	5·808	24	8·320
5	2·924	15	6·082	25	8·550
6	3·302	16	6·350	26	8·777
7	3·660	17	6·611	27	8·995
8	4·000	18	6·868	28	9·226
9	4·327	19	7·120	29	9·441
10	4·640	20	7·368	30	9·661

then the thickness in the centre being 1 inch, or eight 8ths, it may be reduced by Table 74 to 7, or $\frac{7}{8}$ inch, at a distance of $10 \times 18·5 \div 22·6 = 8·2$ feet from the end: to 6, or $\frac{3}{4}$ inch, at $10 \times 14·7 \div 22·6 = 6·5$ feet: to 5, or $\frac{5}{8}$ inch, at $10 \times 11·2 \div 22·6 = 5$ feet: to 4, or $\frac{1}{2}$ inch, at $10 \times 8 \div 22·6 = 3·5$ feet: to 3, or $\frac{3}{8}$ inch, at $10 \times 5·2 \div 22·6 = 2·3$ feet: to 2, or $\frac{1}{4}$ inch, at $10 \times 2·8 \div 22·6 = 1·24$ foot: and to 1, or $\frac{1}{8}$ inch, at $10 \times 1 \div 22·6 = ·45$ foot: the lengths and thicknesses for equal *Wrinkling* strains in Fig. 94 have been obtained in this way: Fig. 95 gives the thicknesses for equal *crushing* strains.

We have thus given the theoretical thicknesses for the sake of illustrating principles; but it is not expedient in practice to use very thin plates of wrought iron: where a structure is exposed to the weather more particularly, a considerable extra thickness is necessary to allow for the wear of the elements, irrespective of that required to bear the strain: in most cases plates less than $\frac{1}{4}$ inch thick would not be used in practice.

(398.) The central thickness might be maintained throughout, and the strength duly proportioned to the strain by reducing the *breadth* of the flanges from the centre to the ends; but this would increase the labour and cost of manufacture to an extent that would not be compensated by the saving of material. The question of labour in making, militates very much against *any* method by which the strength may be graduated in proportion

to the strain, and for that reason the top and bottom flanges of small girders are usually made of the same breadth and thickness throughout the length. With large girders, however, the value of the material bears a higher proportion to the labour, and it is commercially expedient to economise by graduating the sizes in proportion to the strains.

(399.) It is shown in (315) that the resistance to Wrinkling *per square inch* is proportional to the square-root of the thickness or $\sqrt{t}$, but the *total* resistance in the ratio $\sqrt{t} \times t$, or $t^{1 \cdot 5}$. Therefore the thickness necessary to bear a given wrinkling strain will be proportional to the $1\frac{1}{2}$ root of the strain: Tables 74 and 75 have been thus calculated.

(400.) "*Vertical Web.*"—The continuous plate forming the web of a girder is in effect an infinite number of diagonals similar to those of a lattice girder, but at infinitely varied angles, and the strains may be the most easily investigated by that analogy.

The first thing to be noted is that, whatever the amount of the strain, it is equal throughout the length of the beam, when that beam is loaded in the centre only; the thickness of the web will therefore be the same throughout for that particular case, see Figs. 136, 155. For this reason the strain on the diagonals is small compared with that on the centre of the top and bottom flanges, which suffer an *accumulated* strain from leverage, see Fig. 155.

It should also be noted that the web alone acts *directly* in sustaining the vertical load, because whatever the strains on the top and bottom flanges may be, they act *horizontally* only, and must act indirectly and through the medium of the vertical web in sustaining the vertical load.

(401.) Taking a cantilever, Fig. 96, and allowing the safe working strain to be 5 tons per square inch, the bar A carrying 10 tons must be $10 \div 5 = 2$ square inches area; the diagonal B carrying a tensile strain of 14 tons must be $14 \div 5 = 2 \cdot 8$ square inches area, and if say $\frac{1}{2}$ inch thick, then $2 \cdot 8 \div \frac{1}{2} = 5 \cdot 6$ inches wide measured at right angles to its own axis, or in the direction C, E. Now, if we would convert the lattice bracing into a continuous web, this may be done by increasing the

width of all the diagonals exposed to tension F, G, H, B, *until they meet*, B for instance being increased to a, b, and H, to c, d, &c., &c. The cross-sectional area being as before 2·8 square inches, and the width a, b or c, d 17 inches, the thickness comes out $2·8 \div 17 = ·165$ inch only, which is very light: in practice it would perhaps be made $\frac{1}{4}$ inch. In some large flanged girders made by Mr. Fairbairn for a Railway bridge, in which the length was 57 feet and depth $4\frac{1}{2}$ feet, top flange $18 \times \frac{7}{8}$, &c., the vertical web was only $\frac{3}{8}$ inch thick in the centre, and $\frac{7}{16}$ inch at the ends (442), but it was stiffened by cover-plates at all the vertical joints, which were 3 feet apart: see also (927) and Fig. 133.

(402.) But we have also to consider the compressive strains in the web of a plate girder, for it will be observed in the lattice girders, that while a set of diagonals is subjected to tensile strains, another and similar set bear crushing strains and require to be of ⊥ section (439) to resist flexure. It might appear at first sight that a similar form would be necessary in the web of a plate-iron girder, which as a very thin plate seems wholly unfit to bear a heavy crushing strain. But if, as in Figs. 141, 155, the diagonals under compression were connected by a rivet with those under tensile strain by which they are crossed, the effect would be to reduce the length of the pillar to half, and as by (147) the strength of a pillar is inversely proportioned to the square of the length, the strength would in that case be quadrupled. Now in a web, we have an infinite number of imaginary diagonals under compression, which are crossed by another similar series under tensile strain, and the two sets being in effect interlaced and interlocked with one another, the length of those under compression is reduced indefinitely, so that flexure or wrinkling is impossible even with a very thin plate. We have therefore to consider the crushing strength of the material only, and as the tensile and crushing strengths of wrought iron are practically equal to one another, it follows that the area which we found necessary to bear the former (401) will suffice for the latter also.

(403.) "*Shearing Strain at Ends of Girders.*"—It is absolutely essential that plate-iron girders with a vertical web of the usual

light description, should be well strengthened at the ends where they rest on the supports, to prevent the web doubling up under the *vertical* strain that it has to bear, which in most cases, even when the load is a central one, amounts to half that load plus half the weight of the girder itself. With a rolling load it may be a great deal more than that; in extreme cases it may amount to the whole load plus half the weight of the girder. Obviously the thin plate of the web, say $\frac{1}{8}$ or even $\frac{3}{8}$, inch thick, Fig. 133, could not bear such a strain as that, without the assistance of vertical, angle, or ⊥ irons; usually the latter would be used, and as they are strained as pillars, their sizes may be determined by the rules in (534), or by Table 82.

(404.) Table 76 gives the strength and stiffness for certain standard sizes of plate-iron girders whose proportions are given by Figs. 101 to 107. Thus, the girder Fig. 103, 2 feet deep, and say 30 feet long, will break by col. 5 with $1665 \div 30 = 55 \cdot 5$ tons in the centre:—the safe dead load may be taken at $55 \cdot 5 \div 3 = 18 \cdot 5$ tons, with which the deflection by col. 6 and (674) would be $18 \cdot 5 \times \cdot 0000007606 \times 27000 = \cdot 38$ inch. With a load equally distributed all over the length, we should have had $18 \cdot 5 \times 2 = 37$ tons, with which the central deflection would have been by col. 7 in the Table, $37 \times \cdot 0000004754 \times 27000 = \cdot 475$ inch, &c.

TABLE 76.—Of the STRENGTH and STIFFNESS of WROUGHT PLATE-IRON GIRDERS, 1 foot long.

Figs.	Area of Plates and Angle-irons.		Reduced Effective Depth.	Breaking Weight in Centre.	Deflection per Ton.	
	Top.	Bottom.			Weight in Centre.	Weight all over.
			inches.	tons.		
101	24·0	20·5	34	4612	·0000001885	·0000001178
102	18·5	15·5	28½	2906	·0000003514	·0000002195
103	13·0	11·1	23	1665	·0000007606	·0000004754
104	11·2	9·5	17	1068	·000001621	·000001013
105	8·5	7·5	14	703	·000003090	·000001931
106	7·1	6·2	11½	465	·000005513	·000003446
107	5·3	4·6	8¼	259	·00001355	·000008470
(1)	(2)	(3)	(4)	(5)	(6)	(7)

(405.) "*Tubular Beams of Plate-iron.*"—The strength of a tubular beam, like that of every other structure, is limited by the strength of the weakest part, this is usually the top plate, which is due to the tendency of thin plates of wrought iron to fail by Wrinkling with a strain much less than the crushing or tensile strength of the material. This, however, is not universally the case, but depends on the relative dimensions of the plates subjected to tension and compression respectively. For example, with No. 1 in Table 77, the top and bottom plates have equal areas, they have also the same total load to carry, occupying as they do the position of fulcrum and resistance to one another reciprocally. But while the bottom will bear 21·6 tons per square inch, namely, the mean tensile strength of *plate* iron by Table 1, the top will fail by wrinkling with 18·38 tons per square inch, as shown by col. (11): the top being the weakest will therefore govern the strength of the beam. The thickness of the bottom plate might in this case be reduced to $\frac{3}{4} \times 18 \cdot 38 \div 21 \cdot 6 = \cdot 6382$, or about $\frac{5}{8}$ inch, without at all affecting the strength of the beam:—both plates would then be strained in proportion to their strength, and they should fail simultaneously. In this case, in order to obtain equality of strength, the areas would be in the ratio of 6 to 5.

(406.) The rule for calculating the strength of tubular plate-iron beams must therefore include the *Wrinkling* Strain as a fundamental datum; the ordinary rules in which that is neglected will not give correct results. Thus, taking as an example No. 1 in Table 77, whose section is given by Fig. 97, taking the value of M_T from col. 6 of Table 66 at 1·786 ton, Rule (330) gives

$$W = \frac{(35 \cdot 75^3 \times 24) - (34 \cdot 25^3 \times 22 \cdot 5)}{35 \cdot 75} \times 1 \cdot 786 \div 45 = 214$$

tons; but by col. 8, the experimental breaking weight = 118 tons; hence $214 \div 118 = 1 \cdot 81$, or an error of + 81 per cent.

(407.) A still more striking illustration would be given by the very thin tube No. 12 in Table 77, with which Rule (330) gives

$$W = \frac{(24^3 \times 15) - (23 \cdot 752^3 \times 14 \cdot 752)}{24} \times 1 \cdot 786 \div 30$$

= 24 tons; but by col. 8, the experimental breaking weight

TABLE 77.—Of the STRENGTH of TUBULAR BEAMS, of THIN WROUGHT-IRON PLATES, Loaded in the Centre.

No.	Length between Bearings.	External Depth.	External Breadth.	Thickness of Plate.			Experimental Breaking Weight, plus Half Weight of Tube.	Calculated Breaking Weight.		Calculating Wrinkling Strain. Tons per Square Inch.	Value of f. Tons per Square Inch.
				Top.	Bottom.	Sides.		Tons.	Error per Cent.		
	feet.	inches.	inches.								
1	45	35·75	24	·75	·75	·75	118·0	122·5	+ 3·8	18·38	17·75
2	45	36	24	·75	·75	·375	105·0	104·2	− 0·7	18·38	18·52
3	45	36	24	·5625	·397	·214	67·0	57·67	− 13·9	15·92	18·48
4	45	36	24	"	"	"	63·8	57·67	− 9·6	15·92	17·60
5	30	24	15·5	·525	·525	·525	58·66	58·48	− 0·4	19·14	19·20
6	30	24	16	·5	·5	·5	55·25	54·91	− 0·6	18·39	18·50
7	30	24	16·5	·75	·5	·25	54·82	58·20	+ 6·2	22·17	17·29
8	"	"	"	"	"	"	51·38	58·20	+ 13·3	22·17	16·20
9	30	23·75	15·75	·437	·272	·272	33·64	33·74	+ 0·3	17·32	17·26
10	30	24	16	·372	·244	·125	26·60	24·23	− 8·9	15·86	18·23
11	30	23·75	15·5	·272	·272	·272	23·33	22·29	− 4·4	13·78	14 42
12	30	24	15	·124	·124	·124	5·78	7·04	+ 21·8	9·46	7·74
13	7½	6	3·9	·1325	·1325	·1325	4·47	3·697	− 17·3	19·17	23·13
14	7½	5 8	3·8	·065	·065	·065	1·417	1·596	+ 12·6	13·60	15·32
15	3¾	3·0	1·95	·061	·061	·061	1·10	·8241	− 24·9	18·40	24·56
16	3¾	3·0	1·9	·03	·03	·03	0·30	·2933	− 2·3	13·07	13·38
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)

$= 5\cdot78$ tons only; hence the calculated strength is $24 \div 5\cdot78 = 4\cdot16$ times the experimental!

These illustrations show that the ordinary rules which may be correct for very thick plates which fail by *crushing* are not correct for thin plates which fail by *Wrinkling*.

(408.) The first step as a basis for correct calculation of the strength of Tubular Beams with thin plates is to find the Wrinkling strain by the Rule (308), which for beams is $W_W = \sqrt{t_W \div b_W} \times 104$, or in our case $\sqrt{\frac{3}{4} \div 24} \times 104$, or $\sqrt{\cdot03125} \times 104$, or $\cdot17677 \times 104 = 18\cdot38$ tons per square inch, col. 11. Having thus found the wrinkling strain, and supposing it not to exceed the *crushing* strength, or 19 tons per square inch (201), we may take it as equivalent to f in rule (514), and may then calculate the strength of the beam by that rule, which in our case becomes

$$W = \frac{18\cdot38 \times 2 \times \left\{35\cdot75^3 \times 24) - (34\cdot25^3 \times 22\cdot5\right\}}{3 \times 540 \times 35\cdot75}$$

$= 122\cdot5$ tons, as in col. 9 of Table 77: experiment gave 118 tons as in col. 8; hence $122\cdot5 \div 118 = 1\cdot038$, showing an error of $+ 3\cdot8$ per cent. only, as in col. 10.

To vary the illustration we may take No. 11 in the same Table, whose section is given by Fig. 98. For the wrinkling strain, the rule (308) becomes $\sqrt{\cdot272 \div 15\cdot5} \times 104$, or $\sqrt{\cdot01755} \times 104$, or $\cdot1325 \times 104 = 13\cdot78$ tons per square inch. Taking this as the value of f, the rule (514) becomes

$$W = \frac{13\cdot78 \times 2 \times \left\{23\cdot75^3 \times 15\cdot5) - (23\cdot206^3 \times 14\cdot956\right\}}{3 \times 360 \times 23\cdot75}$$

$= 22\cdot29$ tons: experiment gave $23\cdot33$ tons, col. 8; hence $22\cdot29 \div 23\cdot33 = \cdot956$, giving $1\cdot0 - \cdot956 = \cdot044$, or an error of $- 4\cdot4$ per cent., as in col. 10.

(409.) In some cases the wrinkling strain comes out in excess of the absolute crushing strength of the material, or 19 tons per square inch (201), and of course it will not be realised, the metal failing by crushing; in that case we must take $f = 19$ tons, whatever the wrinkling strain may be. An illustration of this is given by Nos. 7 and 8, in Table 77, whose section is shown by Fig. 99:—the rule (308) gives $W_W = \sqrt{\cdot75 \div 16\cdot5}$

$\times$ 104 = 22·17 tons per square inch, as in col. (11), which being in excess of 19 tons must be rejected; and the value of f being taken as 19 tons, the rule (514) becomes

$$W = \frac{19 \times 2 \times \left\{24^3 \times 16 \cdot 5) - (22 \cdot 75^3 \times 16\right\}}{3 \times 360 \times 24} = 58 \cdot 2 \text{ tons}:$$

experiment gave 54·82 tons; hence 58·2 ÷ 54·82 = 1·062, or an error of + 6·2 per cent., col. 14. Obviously, if we had erroneously taken 22·17 tons, the error would have been greater, the breaking weight coming out 58·2 × 22·17 ÷ 19 = 67·91 tons, giving 67·91 ÷ 54·82 = 1·24, or an error of + 24 per cent.

(410.) The cols. 9, 10, and 11 of Table 77 have been calculated throughout in this manner:—the general result shown by col. 10 is that the sum of all the + errors is 58, and of all the − errors is 83, giving on 16 experiments an average of (83 − 58) ÷ 16 = 1·56, or − 1·56 per cent. (959). The greatest + error = 21·8, and the greatest − error = 24·9 per cent., thus showing nearly an equal range; and it should be observed that this range of error may possibly be due to the natural variation in the strength of the material:—thus, taking the simple tensile strength of plate-iron, where, of course, the case is not complicated by possible errors in the rules, &c., Table 149 shows a variation of + 29 and − 33 per cent. respectively. The range in the dimensions of the beams is worthy of note; the extreme sizes, or those of Nos. 2 and 16, are shown by Fig. 100, and it should be observed that the errors of these two sizes differ but little from one another, being − 0·7 per cent. in the largest (No. 2), and − 2·3 per cent. in the smallest (No. 16).

Extreme cases are crucial tests of the accuracy of any rules, and, as we have seen, the rules we have given bear that test satisfactorily. Other illustrations of the different methods of calculating tubular beams are given in (320), (412), (414).

(411.) It is shown in (405) that there should be equality in the resistances to tension by the bottom, and compression by the top of a tubular beam, and failing that, the weaker of the two will govern the strength of the beam. With most of the beams in Table 77 the *top is the* weaker, but in Nos. 7, 8, 9, and 10

the bottom is the weaker, and *should* govern the strength, but does not seem to do so. For example, with No. 9, the wrinkling strain by col. 11 is 17·32 tons per square inch, hence 17·32 × ·437 × 15·75 = 119·2 tons, the resistance to compression: but for the bottom we have 21·6 × ·272 × 15·75 = 92·54 tons only as the resistance to tension. In col. 9 the strength, as calculated from the resistance of the top plate, came out 33·74 tons, the error being only + 0·3 per cent. Now, if the strength were dominated by the bottom, as it should be by theory, we obtain 33·74 × 92·54 ÷ 119·2 = 26·19 tons only, an error of − 22·15 per cent.

(412.) We should obtain a result but little more satisfactory by analysis, as in (320), (414), &c. Let Fig. 108 be a section of the same beam reduced to the case of a cantilever of half the length fixed at one end and loaded at the other, as in Fig. 92; the Neutral axis will not now be in the centre, but must occupy such a position as to give equality to the resistances of tension and compression. We must assume a position for it by judgment, say as in the figure: for the top plate we found in (411) 119·2 tons, which with leverages of 11·182 and 180 inches gives at W, 119·2 × 11·182 ÷ 180 = 7·406 tons. Then, for the sides above the line N. A., the maximum strain at the top being 17·32 tons per square inch, that at *o* becomes 17·32 × 5·482 ÷ 11·4 = 8·329 tons. The area is 10·963 × ·544 = 5·964 square inches; hence 5·964 × 8·329 × 5·482 × $\frac{4}{3}$ ÷ 180 = 2·016 tons at W, which added to the resistance of the top plate = 7·406 + 2·016 = 9·422 tons at W due to compression alone.

Similarly, for the part of the section subjected to Tension, we have for the bottom plate 15·75 × ·272 × 21·6 × 12·214 ÷ 180 = 6·277 tons at W:—the strain at *p* is 21·6 × 6·039 ÷ 12·35 = 10·56 tons per square inch, which at W becomes 10·56 × ·272 × 2 × 6·039 × 1·333 ÷ 180 = 3·106 tons, giving for the total resistance to tension 6·277 + 3·106 = 9·380 tons at W, which is rather less than 9·422 tons due to compression, but is sufficiently near equality for our purpose. The sum of Tension and Compression is thus 9·380 + 9·422 = 18·802 tons at W, and is equivalent to 18·802 × 2 = 37·6 tons in the centre of the beam 30 feet long: experiment gave 33·64 tons, hence the

error is $37 \cdot 6 \div 33 \cdot 64 = 1 \cdot 117$, or $+ 11 \cdot 7$ per cent. By rule (514) the error was $+ 0 \cdot 3$ per cent. only, as in col. 10.

It is remarkable that the rule (514) by which col. 9 of Table 77 is calculated, which is based on the value of f, or the wrinkling strain, and is therefore not strictly applicable to beams which fail by tension, gives, notwithstanding, more correct results than any other rule:—the mean error of Nos. 7, 8, 9, and 10 being $+ 2 \cdot 7$ per cent. only.

(413.) "*Lateral Strength of Beams.*"—In designing large Tubular beams for bridges, &c., it is necessary to consider and provide for the *lateral* strain due to the wind impinging on the side. The proportions of beams are usually fixed principally with a view to sustain a vertical load, and as a result they are frequently very weak in resisting horizontal pressure. With a view to obtain experimental information on this matter, Mr. Hodgkinson took the beam, No. 10 in Table 77, which in the ordinary vertical position broke with a load of $26 \cdot 6$ tons, and laid it on its side, as in Fig. 109, when it failed with $14 \cdot 3$ tons. In this abnormal position the whole of the conditions are so greatly changed, that the strength cannot be calculated correctly by the usual rule (408) for Tubular beams; thus, in our case, the wrinkling strain of the thin top plate by the rule in (308) becomes $\sqrt{\cdot 125 \div 24} \times 104 = 7 \cdot 505$ tons per square inch, and the rule (514) then gives

$$W = \frac{7 \cdot 505 \times 2 \times \left\{16^3 \times 24) - (15 \cdot 75^3 \times 23 \cdot 384\right\}}{3 \times 360 \times 16}$$

$= 6 \cdot 028$ tons only, whereas experiment gave $14 \cdot 3$ tons. But this rule supposes that the beam is of the ordinary form and proportions, and that the strength is governed by the resistance to wrinkling or by f, but obviously the proportions might be such that the wrinkling strain would have little effect on the strength: for example, in Fig. 110 the great strength of the side plates a, b render them independent of the weak top plate c, and the rule which takes *that* plate as the exponent of the strength of the whole beam must necessarily fail to give correct results.

(414.) The best method of calculating a beam of such an abnormal form is by analysis, as illustrated in (320) and (412). We shall assume that the *top* plate fails by wrinkling with

7·5 tons per square inch, as calculated in (413), and the bottom plate with 21·6 tons, namely, the mean tensile strength of *plate* iron as given by Table 1. Then, for the sides, we shall take the *maximum* tensile strain at T and compressive strain at C = 21·6 tons per square inch also: the neutral axis N. A. must then be placed in such a position as to give equality to the tensile and compressive forces above and below that line respectively, and this position must be fixed tentatively by judgment. Reducing the case for the purposes of calculation to the equivalent one of a cantilever of half the length of the beam fixed at one end and loaded at the other, as in Fig. 92, we obtain the dimensions given by Fig. 109. For the top plate we obtain $24 \times \cdot 125 \times 7 \cdot 5 = 22 \cdot 5$ tons, which with the leverages of 9·438 and 180 inches respectively, gives $22 \cdot 5 \times 9 \cdot 438 \div 180 = 1 \cdot 18$ ton at W. Then for that part of the side plates above N. A., the strain of 21·6 tons at C becomes $21 \cdot 6 \times 4 \cdot 688 \div 9 \cdot 5 = 10 \cdot 66$ tons per square inch at *o*, and their area being $9 \cdot 375 \times \cdot 616 = 5 \cdot 775$ square inches, with leverages of 4·688 and 180 inches gives at W, $5 \cdot 775 \times \frac{4}{3} \times 4 \cdot 688 \div 180 = 2 \cdot 137$ tons for the sides (495), making with that due to the top $1 \cdot 18 + 2 \cdot 137 = 3 \cdot 317$ tons from compression alone.

Similarly, for the bottom plate, we obtain $24 \times \cdot 125 \times 21 \cdot 6 = 6 \cdot 48$ tons, which at W becomes $6 \cdot 48 \times 6 \cdot 438 \div 180 = 2 \cdot 318$ tons. The strain at *p* becomes $21 \cdot 6 \times 3 \cdot 188 \div 6 \cdot 5 = 10 \cdot 59$ tons per square inch. Then $6 \cdot 375 \times \cdot 616 \times \frac{4}{3} \times 10 \cdot 59 \div 180 = \cdot 982$ ton at W due to the sides, making a total of $2 \cdot 318 + \cdot 982 = 3 \cdot 3$ tons due to Tension, or nearly the same as 3·317 tons due to compression. The sum of the two $= 3 \cdot 3 + 3 \cdot 317 = 6 \cdot 617$ tons at the end of the cantilever 15 feet long, equivalent to $6 \cdot 617 \times 2 = 13 \cdot 234$ tons in the centre of the beam 30 feet long, as in our case: experiment gave 14·3 tons, hence $13 \cdot 234 \div 14 \cdot 3 = \cdot 9255$, showing an error of $1 \cdot 0 - \cdot 9255 = \cdot 0745$, or $- 7 \cdot 45$ per cent. Considering the extremely abnormal form of the beam in this position, the error is perhaps not greater than might be expected.

(415.) It will be observed that we have taken the full tensile strength of the iron, allowing nothing for rivet-holes, &c., on the supposition that there is no joint at or near the centre of

the beam, where the strain is a maximum. With small and moderate-sized beams this condition is easily obtained, and with large structures we can secure practically the same condition by the adoption of chain-riveting (36).

FORM FOR EQUAL STRENGTH THROUGHOUT.

(416.) We have so far assumed that the beams were parallel, or of the same cross-section from end to end, and the load in the centre. In that case, the transverse strain is a maximum at the centre, and is progressively reduced towards the two supports, where it becomes nothing; to obtain throughout the length an equality between the strain and the strength it would be necessary to reduce the section of the beam toward each prop in proportion to the strain at each point, which can be effected by regulating the depths alone, while the breadths remain constant; or, on the other hand, by graduating the breadths, while the depths remain constant; or by a combination of the two methods.

Let Fig. 111 be a beam 16 feet long, with a central weight of 10, producing of course a strain of 5 on each prop: this is obviously equivalent to a cantilever, Fig. 112, of half the length built into a wall at one end, and loaded with a weight of 5 at the other end. The ratio of the strains at each point along the beam is evidently proportional to the leverage with which the load of 5 acts, or to the distances 1, 2, 3 . . 8, &c., as in Fig. 112, and as by (324) the strength is proportional to D^2 when the breadth is constant, it follows that D must be proportional to the square-roots of the respective strains, or $\sqrt{1} = 1$; $\sqrt{2} = 1 \cdot 41$; $\sqrt{3} = 1 \cdot 73$, &c., as in the figure, these being of course proportional, and not real dimensions. We have thus obtained the depths in Fig. 112, which again give us the depths in Fig. 111.

(417.) If we would obtain a uniform depth throughout, as in Fig. 113, which again is equivalent to the cantilever, Fig. 114, the breadths being simply proportional to the strains, the latter being 1, 2, 3, 4, &c., the former will be 1, 2, 3, 4, &c., also, and we thus obtain Fig. 115, giving a uniform taper from the wall to the end, and from this we obtain Fig. 116.

To apply this to practice:—Let Fig. 117 be a beam of Beech, 14 inches deep, 10 inches wide, and 12 feet long, which by rule (324) with $M_T = 585$ by Table 66 breaks with $14^2 \times 10 \times 558 \div 12 = 91140$ lbs. in the centre. The square of the depth at each point being proportional to the distance of that point from the nearest prop as shown by (416), and that at 6 feet being 14 inches, we have $14^2 \div 6 = 3 \cdot 27$, a constant, which multiplied by the distance of each point, will give the square of the depth at that point, thus:—

At A	we get	$32 \cdot 7 \times 6$	or	$\sqrt{196 \cdot 0} = 14 \cdot 00$	inches.
B	„	$32 \cdot 7 \times 5$	„	$\sqrt{163 \cdot 0} = 12 \cdot 76$	„
C	„	$32 \cdot 7 \times 4$	„	$\sqrt{131 \cdot 0} = 11 \cdot 44$	„
D	„	$32 \cdot 7 \times 3$	„	$\sqrt{98 \cdot 9} = 9 \cdot 9$	„
E	„	$32 \cdot 7 \times 2$	„	$\sqrt{65 \cdot 4} = 8 \cdot 1$	„
F	„	$32 \cdot 7 \times 1$	„	$\sqrt{32 \cdot 7} = 5 \cdot 7$	„
G	„	$32 \cdot 7 \times 0$	„	$\sqrt{0 \cdot 0} = 0 \cdot 0$	„

We thus obtain the depths in Fig. 117, the breadth being 10 inches throughout. If, on the other hand, we maintain the depth at 14 inches throughout, the breadth would taper off uniformly from 10 inches at the centre to nothing at the props, as in Fig. 116.

(418.) It may seem anomalous that the size at the end should be *nothing*, whereas a strain of 45,570 lbs., or half the central weight, has to be borne by it; but we have been considering the *transverse* strain only, which is really *nil* at G; the strain of 45,570 lbs. is a *shearing* or cross-strain, and must be provided for, but is a matter quite foreign to the proper subject of this chapter (123); see (403).

(419.) "*Load out of Centre.*"—Having thus found the forms of beams with a single central load, we may proceed to consider 1st, the effect of a single load out of the centre; and 2nd, of two or more loads variously distributed. In order to give precision to the investigation we will take a case, say that of a beam of Beech 12 inches square and 16 feet long; taking the value of M_T from col. 6 of Table 66 at $\cdot 25$ ton, the rule (324) gives for a central load $12^2 \times 12 \times \cdot 25 \div 16 = 27$ tons breaking weight, as in Fig. 118.

The central breaking weight being thus found, we require 1st, the extent to which that same weight would strain the parallel beam if placed at a given point out of the centre, and 2nd, the breaking weight at any other point in the same parallel beam.

(420.) A weight placed anywhere on a beam divides the whole length into two imaginary lengths—equal if in the centre, unequal if elsewhere—and when the weight is constant the strain produced by it at each point is proportional to the *product* of those two imaginary lengths, and will be found to be a maximum when the load is in the centre. Thus in Fig. 118, with 27 tons in the centre the beam is divided into two equal 8-foot lengths, the product of which is $8 \times 8 = 64$; now if that load of 27 tons is removed to 4 feet from one prop, therefore 12 feet from the other, as in Fig. 119, the product becomes $4 \times 12 = 48$, and the beam is strained at A to $\frac{48}{64}$ths only of the breaking weight at that point. Hence the breaking weight there would be $27 \times 64 \div 48 = 36$ tons. Calculating in this way we have obtained the ratios in col. 4 of Table 104 and in curve C of Fig. 213, which give the equivalent load out of the centre corresponding to a central load of $1 \cdot 000$.

(421.) For general purposes perhaps a simpler course is to find a reduced or imaginary *length* to be used in the ordinary rules in (323), &c., instead of the actual length as for central loads. For this purpose :—divide the product of the two lengths into which the load divides the beam by $\frac{1}{4}$th the actual length, and the quotient is the reduced length to be used in calculation. Thus in Fig. 119 we have $4 \times 12 \div (16 \div 4) = 12$ feet :—then with this reduced length, the rule $W = d^2 \times b \times M_T \div L$ becomes in our case $12^2 \times 12 \times \cdot 25 \div 12 = 36$ tons, as before. The same result would have been found by using the ratios 64 to 48 as in (420); for $16 \times 48 \div 64 = 12$ feet, the reduced length, &c.

(422.) The contour of the beam in elevation may now be found as in (417), the breadth being 12 inches throughout, by making the square of the depth proportional to the distance from the props, but so that the depth at A shall be 12 inches. Thus from A to E we have four divisions :—then $12^2 \div 4 = 36$

is a constant which multiplied by the distance of each point E, D, C, B, A from A in Fig. 120 will give the square of the depth at that point.

Thus at E	we have	36×4	or	$144 \sqrt{} = 12$	inches.
D	„	36×3	„	$108 \sqrt{} = 10 \cdot 4$	„
C	„	36×2	„	$72 \sqrt{} = 8 \cdot 5$	„
B	„	36×1	„	$36 \sqrt{} = 6 \cdot 0$	„
A	„	36×0	„	$0 \sqrt{} = 0 \cdot 0$	„

Similarly from E to Q we have 12 divisions, hence $12^2 \div 12 = 12$ is a constant which multiplied by the distances of the points F, G, &c., &c., from Q will give the square of the depth at each point from E to Q.

Thus at E	we have	12×12	or	$144 \sqrt{} = 12$	inches.
F	„	12×11	„	$132 \sqrt{} = 11 \cdot 5$	„
G	„	12×10	„	$120 \sqrt{} = 11 \cdot 0$	„
H	„	12×9	„	$108 \sqrt{} = 10 \cdot 4$	„
I	„	12×8	„	$96 \sqrt{} = 9 \cdot 8$	„
J	„	12×7	„	$84 \sqrt{} = 9 \cdot 2$	„
K	„	12×6	„	$72 \sqrt{} = 8 \cdot 5$	„
L	„	12×5	„	$60 \sqrt{} = 7 \cdot 7$	„
M	„	12×4	„	$48 \sqrt{} = 6 \cdot 9$	„
N	„	12×3	„	$36 \sqrt{} = 6 \cdot 0$	„
O	„	12×2	„	$24 \sqrt{} = 4 \cdot 9$	„
P	„	12×1	„	$12 \sqrt{} = 3 \cdot 5$	„
Q	„	12×0	„	$0 \sqrt{} = 0 \cdot 0$	„

We have thus found the depth at every foot in the length of the beam; for ordinary purposes a smaller number of points would have sufficed, at least between E and Q, but we have a special purpose in view presently for which we require the depth at numerous points.

(423.) "*Effect of Two or more Loads.*"—We may now proceed to consider the effect on the form of a beam, of two or more loads variously distributed. We will take the case of Fig. 122, where we have two weights each of 36 tons, both being 4 feet from one prop and 12 feet from the other, and we will take it first as composed of two similar beams, as in Figs. 120, 121.

Obviously Fig. 121 is the counterpart of Fig. 120, and if we imagine those two beams placed side by side, as in Fig. 122, we should have a compound beam that would fulfil the given conditions. Now in the beam Fig. 122, the depth squared multiplied by the breadth, must at each and every point be equal to the *sum* of the depths squared multiplied by the breadths of the corresponding points in the two beams Fig. 122, which are given by Figs. 120, 121. The breadths of all the beams we are considering being alike, or 12 inches, we may eliminate b and shall deal only with d^2. Then, the square of the depth at b in Fig. 122 must be equal to the sum of the squares of the depths at B in Fig. 120 and at P in Fig. 121, or $6^2 + 3\cdot5^2 = 48\cdot25$, and $\sqrt{48\cdot25} = 6\cdot95$ inches, the depth at b in Fig. 122.

(424.) Calculating in this way we obtain the depths at each foot of length as follows:—

			Inches.		
$(B^2 + P^2)\surd$	or	$(6\cdot0^2 + 3\cdot5^2)\surd =$	$6\cdot95$	at b	in Fig. 122.
$(C^2 + O^2)\surd$	,,	$(8\cdot5^2 + 4\cdot9^2)\surd =$	$9\cdot81$	c	,,
$(D^2 + N^2)\surd$	,,	$(10\cdot4^2 + 6\cdot0^2)\surd =$	$12\cdot00$	d	,,
$(E^2 + M^2)\surd$	,,	$(12\cdot0^2 + 6\cdot9^2)\surd =$	$13\cdot85$	e	,,
$(F^2 + L^2)\surd$	,,	$(11\cdot5^2 + 7\cdot7^2)\surd =$	$13\cdot85$	f	,,
$(G^2 + K^2)\surd$	,,	$(11\cdot0^2 + 8\cdot5^2)\surd =$	$13\cdot85$	g	,,
$(H^2 + J^2)\surd$	,,	$(10\cdot4^2 + 9\cdot2^2)\surd =$	$13\cdot85$	h	,,
$(I^2 + I^2)\surd$	,,	$(9\cdot8^2 + 9\cdot8^2)\surd =$	$13\cdot85$	i	,,
$(J^2 + H^2)\surd$	,,	$(9\cdot2^2 + 10\cdot4^2)\surd =$	$13\cdot85$	j	,,
$(K^2 + G^2)\surd$	,,	$(8\cdot5^2 + 11\cdot0^2)\surd =$	$13\cdot85$	k	,,
$(L^2 + F^2)\surd$	,,	$(7\cdot7^2 + 11\cdot5^2)\surd =$	$13\cdot85$	l	,,
$(M^2 + E^2)\surd$	,,	$(6\cdot9^2 + 12\cdot0^2)\surd =$	$13\cdot85$	m	,,
$(N^2 + D^2)\surd$	,,	$(6\cdot0^2 + 10\cdot4^2)\surd =$	$12\cdot00$	n	,,
$(O^2 + C^2)\surd$	,,	$(4\cdot9^2 + 8\cdot5^2)\surd =$	$9\cdot81$	o	,,
$(P^2 + B^2)\surd$	,,	$(3\cdot5^2 + 6\cdot0^2)\surd =$	$6\cdot95$	p	,,
$(Q^2 + A^2)\surd$	,,	$(0\cdot0^2 + 0\cdot0^2)\surd =$	$0\cdot00$	q	,,

It will be observed that between the points e and m the beam is a parallel one, and it is curious to see how this is brought about by the fact that the sum of the squares of the depths at the corresponding points in the two parabolas in Figs. 120, 121 is constant between those points.

(425.) The analytical method of investigation and calculation adopted in the above may be laborious, but it has two great advantages: 1st, it covers and includes all possible cases, of any number of weights, and any method or order of distribution, for obviously it is only necessary to calculate the form and dimensions of beam for each weight independently, and then combine them in the manner we have illustrated; 2nd, this method has the great advantage of allowing every step in the process of calculation to be *seen* by the operator, and the rationale to be understood.

We have applied the method of finding the theoretical forms to *timber* beams for convenience of illustration: for that material the parallel form would almost always be adopted in practice from motives of economy of labour, &c.; the saving of material which would accrue from the adoption of the theoretical form would not compensate for the labour and trouble required to obtain it. With cast- and wrought-iron beams the case is different.

(426.) "*Equally Distributed Load.*"—When a load is spread equally all over the length of a beam, it may be considered as divided into any given number of equal weights equidistant from each other, and then the effect of each of these imaginary weights may be separately calculated. Let Fig. 125 be a beam 16 feet long supported at each end, with its load divided into 16 parts:—this is equivalent to a cantilever, Fig. 126, of half the length with its load divided into 8 parts, and we require 1st the depth at the points *a*, *b*, *c*, &c., the breadth being constant, and 2nd we require to find the breadth at those points when the depth is constant. For this purpose we must suppose each weight to rest on the beam at two points, each giving half the strain due to the whole weight. Thus the weight W being $1 \cdot 0$ will give a pressure of $\frac{1}{2}$ at *a* and $\frac{1}{2}$ at *b*; then the weight *w* being also $1 \cdot 0$ will give $\frac{1}{2}$ at *b*, and $\frac{1}{2}$ at *c*, &c.; the combined effect of W, and *w* at *b* is therefore $\frac{1}{2} + \frac{1}{2} = 1 \cdot 0$.

(427.) The transverse strain at every point may now be calculated; thus the strain at *b* is that due to the weight of $\frac{1}{2}$ acting with a leverage of $1 \cdot 0$, or $\frac{1}{2} \times 1 = 0 \cdot 5$, which is the *ratio* of the breadth at that point when the depth is constant;

but if the breadth is constant, we have $\sqrt{0\cdot5} = \cdot707$, which is the *ratio* of the depth at that point. Again at c we have a weight of $\frac{1}{2}$ with leverage of 2, plus the weight B or $1\cdot0$ with a leverage of $1\cdot0$ or $(\frac{1}{2} \times 2) + (1 \times 1) = 2$, which is the ratio of the breadth at that point when the depth is constant; but if the breadth is constant, then $\sqrt{2} = 1\cdot414$ is the ratio of the depth at c: again at d, we have $(\frac{1}{2} \times 3) + (1 \times 2) + (1 \times 1) = 4\cdot5$, the ratio of breadth when the depth is constant, and $\sqrt{4\cdot5} = 2\cdot12$, the ratio of the depth when the breadth is constant, &c. Calculating in this way we find that at

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*k*

where the depth is constant, the breadths should be in the ratios Fig. 129, or

0·0	0·5	2·0	4·5	8·0	12·5	18·0	24·5	32

but when the breadth of the beam is constant throughout its length, then the depths should be in the ratios Fig. 127, or

0·0	0·707	1·414	2·12	2·83	3·53	4·24	4·95	5·65

It will be observed that when the depth of the beam is constant, the breadths are in the ratio 1, 4, 9, 16, &c., or as the squares of the distances from the *end* of the beam, as in Fig. 128. But when the breadth is constant, the depths follow the simple arithmetical ratio 1, 2, 3, 4, &c., and the profile of the beam is then a triangle, as in Fig. 127. Applying this to a beam supported at each end and with the load equally distributed, then, when the depth is constant, the breadths are as given by Fig. 129, but when the breadth is constant, the profile is that of two triangles united at the base, as at x x in Fig. 127. The proportions of depths to breadths may be varied to any extent so long as $d^2 \times b$ follow the ratio of the middle line above, or $0\cdot5$, $2\cdot0$, $4\cdot5$, &c.

(428.) "*Form, as governed by Taste, &c.*"—The forms of beams which we have thus obtained, although theoretically correct for the transverse strains, are not such as to satisfy the requirements of taste, moreover, they do not provide for the shearing strain at the ends (403) nor for a fair area of bearings at the supports so as to spread over a large surface the insistent

weight which otherwise would crush the material,—stone,—brick, &c., on which the beam rests. To meet these requirements, the theoretical forms may be modified at pleasure, care being taken, however, that the sizes demanded by theory are not curtailed by the lines required by taste, convenience, or other considerations. Thus Fig. 117 might be modified to Fig. 130, in which the length is increased by the supplementary pieces *m*, *n*, the amount of which must be fixed by judgment so as to give a good bearing. The two semi-parabolas *o*, *p*, &c., are the same as in Fig. 117, and the curve *r*, *s*, *t* is an ellipsis which is perhaps the most beautiful of simple curves, and may be easily described by taking a piece of paper with a perfectly straight edge P, making the distance *a*, *b*, equal to S, V, and *b*, *c*, equal to U, V; then passing it over the latter so that *b* and *c* are always in contact with the major and minor axes of the ellipse respectively, and making dots with a pencil or needle point at *b*, *b*, &c., a sufficient number of guide-dots is obtained, through which the perfect ellipse may be drawn by a French curve, &c.

(429.) The same principles may be applied to find the section at different points in the length of I girders, whose profile has been determined by taste or convenience. Let Fig. 131 be a girder 16 feet between bearings, resting 18 inches on the wall at each end, 30 inches deep in the centre, whose section there is given at A, and the load being a central one, or by the Pillar E. The strength at the centre (350), or the reduced value of $d^2 \times b$, is for the top flange $2^2 \times 8 = 32$; for the vertical web $(28^2 - 2^2) \times 1\frac{1}{2} = 1170$: and for the bottom flange $(30^2 - 28^2) \times 18 = 2088$:—altogether $32 + 1170 + 2088 = 3290$. Dividing the *extreme* half-length of the girder into four parts, we can now determine the strength, or $d^2 \times b$ at each point B, C, D, that at A being 3290 on the principles explained in (423) and by Fig. 122. Thus, at D it will be $3290 \div 4 = 822$; at C, $3290 \div 2 = 1645$; and at B, $3290 \times 3 \div 4 = 2467$. The depth of the girder at those points having been predetermined by Fig. 131, we have now to find the breadths of the bottom flange necessary to give the required strength, that of the top flange being maintained at 8 inches throughout. Thus, at B, the depth being 25 inches, we have:

top flange, $2^2 \times 8 = 32$; vertical web $(23^2 - 2^2) \times 1\frac{1}{2} = 787$; or $32 + 787 = 819$ together; and as we require 2467 at that point, the bottom flange must yield $2467 - 819 = 1648$, and as the depth there squared is $25^2 - 23^2 = 96$, the breadth of the bottom flange must be $1648 \div 96 = 17 \cdot 1$ inches. Similarly, at C we have: top flange, $2^2 \times 8 = 32$; vertical web $(19^2 - 2^2) \times 1\frac{1}{2} = 535$; or $32 + 535 = 567$ together; hence the bottom flange must yield $1645 - 567 = 1078$, and the depth squared being $21^2 - 19^2 = 80$, the width must be $1078 \div 80 = 13\frac{1}{2}$ inches. Finally, at D we have for the top flange and vertical web $(2^2 \times 8) + (16^2 - 2^2] \times 1\frac{1}{2}) = 410$, and as we require 822, the bottom flange must yield $822 - 410 = 412$, and the depth squared being $18^2 - 16^2 = 68$, the width must be $412 \div 68 = 6 \cdot 06$ inches, &c.

In many cases the form of the bottom flange as thus found would need modification to meet the requirements of taste, care of course being taken that the calculated sizes are not curtailed (428).

(430.) "*Effect of Modes of Fixing and Loading.*"—There are three principal methods of fixing beams:—1st, when supported at the two ends; 2nd, when *fixed*, or built into walls at the two ends; and, 3rd, when fixed or built into a wall at one end only, the other end being free, and the beam then becomes a cantilever.

With each of these modes of fixing beams there are two principal methods of arranging the load, namely, 1st, a single weight in the centre of beams that are fixed or supported at the two ends; and, 2nd, when the load is distributed equally all over the length. Similarly, with a cantilever, the load may be, 1st, a single one at the remote end, and, 2nd, it may be equally distributed all over the length.

(431.) The ratios of the loads in these various cases are as follow:—

	Ratio of Loads.
Supported at two ends, and loaded in the centre ..	1·0
Supported at two ends, load spread equally all over	2·0
Fixed at two ends, and loaded in the centre	1·5
Fixed at two ends, load spread equally all over ..	3·0
Fixed at one end, and loaded at the other	0·25
Fixed at one end, load spread equally all over ..	0·50

Of course the distribution of the load may be varied endlessly; in (419), &c., the whole matter is fully investigated, and the effect on the sizes and forms of beams is considered in detail on a principle that admits of universal application.

The ratios given above are easily applied in practice:—Say that we require the depth of a cantilever of Riga Fir, projecting 5 feet from the wall, to carry safely a load of 1900 lbs. distributed all over, the thickness being 3 inches. We find first from the ratio 0·50 given above that 1900 lbs. equally distributed over a cantilever is equivalent to $1900 \div \cdot 5 = 3800$ lbs. in the centre of a similar beam of the same length supported at the two ends as in the rules in (323), and the value of M_T for safe load being for Riga Fir, 78 lbs. by col. 3 of Table 67, the rule $d = \sqrt{\{W \times L) \div (M_T \times b\}}$ becomes in our case $\{3800 \times 5) \div (78 \times 3\}\sqrt{} = 9$ inches, the depth required, &c.

LATTICE GIRDERS.

(432.) The investigation of the strains on the several parts of lattice girders is an interesting study on its own account, and is also instructive as illustrating the internal strains in girders of other kinds, where the phenomena are often very obscure. In lattice girders the tensile and compressive strains are confined to certain definite lines formed by the different members of the structure, and this fact enables us to estimate the force, direction, and resultants of those strains with a facility and precision not attainable with girders of other kinds.

The forms of lattice girders are so very variable, that in most cases the strains on the various parts must be found by analysis and reasoning rather than by set rules; but for the main question of the Load which can be borne, the Rules for Plate-iron Girders in (386), &c., will equally apply to Lattice Girders.

(433.) Let A, B, C, Fig. 134, be a triangular frame loaded with a weight W of 1 ton, the angle of the strut B being 45°. The weight W may be resolved into two forces on A and B respectively by the well-known parallelogram of forces; making the diagonal *a* equal to the weight W by a scale of equal parts, the strain on B will be equal to the length *d* or *e*, and will be

in our case 1·414, or say 1·4 ton by the same scale. The strain on A will be equal to the length *f* or *g*, namely = 1·0. Then the strain on B, or 1·4 ton, may be resolved by another parallelogram into the two strains on C and *h*, by making the diagonal R = 1·4, when the length of *m* or *n* will give the strain on C = 1·0, and *o* or *p* the strain on *h* = 1·0 also. The strain on B is a compressive one, and those on A and C are tensile ones: the former are represented by full lines, the latter by dotted lines.

Let Fig. 135 be a system of framed rods, in which A, B, C are obviously under precisely the same conditions as in Fig. 134, and bear the same strains. We may now find the other strains by reasoning, thus:—the strain on A or 1·0 must evidently be transmitted to D, and be borne by that member of the system; but D has also to bear the extra strain from the thrust of E, and as the strain of 1·4 on B caused a strain of 1·0 on A, so the strain of 1·4 on E will cause a strain of 1·0 on D in addition to that transmitted from A; hence the total strain on D = 2·0. The strain on E is known to be 1·4, because, as in Fig. 134, the weight W = 1·0 produced a strain of 1·4 on B, and 1·0 on C, so in Fig. 135 will the strain of 1·0 on C produce a strain of 1·4 on E and 1·0 on *r*. Similarly, as B gave a strain of 1·0 on C, so will E cause a strain of 1·0 on F, while G simply taking the place of *h* in Fig. 134 bears the same strain, namely 1·0, but the point *r* having received 1·0 from E, has a total strain on it = 2·0, namely, 1·0 from G and 1·0 from E.

(434.) Fig. 136 is a long girder or cantilever, composed of a series of frames, as in Fig. 135, and the strains throughout may be found by pursuing the same reasoning:—thus we have seen that every diagonal, B, E, &c., causes an extra strain of 1·0 on those members of the top and bottom which receive it, the total strains on the top become 1, 2, 3, 4, 5, and 6; and those on the bottom 1, 2, 3, 4, and 5. After H has passed the foot of the last diagonal K and received its thrust, the strain at J becomes 6 also. The strain at the wall, or at J and L, might be found direct from the leverage with which the weight W acts, for as the length L, M is six times the depth J, L, the

weight of $1 \cdot 0$ at M will evidently give a strain of 6 at J and L, as found by the preceding analysis. It should also be observed that, whatever the length of the girder may be, the strain on the verticals and diagonals is constant from end to end. In all the figures solid lines represent compressive strains, and dotted lines tensile ones.

In Fig. 137 we have a similar girder, but in a reversed position, the principal effect being that the strains on the diagonals which were compressive in Fig. 136 become tensile in Fig. 137, while the tensile strains on the verticals become compressive ones.

(435.) If we now combine Fig. 136 with Fig. 137 by superimposing one upon the other we obtain Fig. 138; the strains on the top and bottom members will evidently be the *sum* of those in Figs. 136 and 137; those on the diagonals will remain unchanged in character and amount, but those on the verticals, being tensile in one case and compressive in the other, will neutralise each other, showing that there will be no strain upon them. They may therefore be omitted altogether as useless, as is done in Fig. 138.

The strain on top and bottom members is of course a maximum at the wall, and is equal to 12 tons, as shown by Fig. 138; we may check this result in another way. Thus, the depth of the girder being taken as $= 1 \cdot 0$, the length in our case $= 6 \cdot 0$, therefore the load of 2 tons becomes a strain of $2 \times 6 = 12$ tons at the wall, as found before.

It will be observed that when, as in Figs. 136, 137, there is only one set of diagonals, the strains on the top and bottom members increase towards the wall in the simple arithmetical ratio 1, 2, 3, &c.; but where there are two sets at similar angles, as in Fig. 138, those strains increase in the order of the odd numbers 1, 3, 5, 7, &c. Another important fact is, that the strain on the diagonals is constant throughout, whatever may be the length of the girder. But these statements are true only where the strain is taken as a single load at the end, the weight of the girder itself being neglected. The effect of the latter, being equivalent to an equally distributed load, is considered in (444).

(436.) The strain on the diagonals will vary with their angle; an equilateral triangle, as in Fig. 139, is commonly used, and the strains on A, B, &c., may be found by the parallelogram of forces as before. We have to determine first, the ratio between a, b, c, &c.: let M in Fig. 140 be an equilateral triangle whose sides are all $= 1\cdot 0$; we want to know the height of the vertical line f. Dividing the triangle into two equal parts, we obtain the right-angled triangle N, having its two sides $= 1\cdot 0$ and $\frac{1}{2}$ respectively; then by the well-known rule for right-angled triangles, namely, that the square of the hypothenuse is equal to the sum of the squares of the other two sides, we have $f, = \sqrt{} (a^2 - c^2)$, which in our case becomes $(1^2 - \frac{1}{2}^2)\sqrt{} = \cdot 866$; therefore when, as with O, $f = 1\cdot 0$ ton, c will be $1 \div \cdot 866 = 1\cdot 155$ ton, and as e is half of c, we have $e = 1\cdot 155 \div 2 = \cdot 5775$ ton. Transferring these numbers to Fig. 139, we obtain the strains on A and B respectively.

The strains on the diagonals will be constant, as we found in (435) and Fig. 136, &c., and will be $1\cdot 155$ ton throughout. The strain on G in Fig. 139 will be the resultant of the thrust of B and the pull of C, and may be found by the parallelogram P in Fig. 140, where the compressive strain of B is converted into an equivalent tensile one B′ at the same angle. Making B′ and C each $= 1\cdot 155$, G is in our case $1\cdot 155$ also, from which we find that each *pair* of diagonals adds $1\cdot 155$ ton to the top and bottom members, which receive their combined strains; hence H, Fig. 139, becomes $1\cdot 155 + 1\cdot 155 = 2\cdot 31$ tons. After F has been passed, and the thrust of *one* more diagonal has been received, the strain becomes $2\cdot 31 + \cdot 5775 = 2\cdot 8875$ tons. Again; in the top member, the strain on A $= \cdot 5775$ ton, as found in (435); on J it becomes $\cdot 5775 + 1\cdot 155 = 1\cdot 7325$ ton; and on K, having received the thrust of F, and the pull of E, it becomes $1\cdot 7325 + 1\cdot 155 = 2\cdot 8875$ tons, or the same as we found for the maximum strain on the bottom members.

(437.) "*Beams with Load in Centre, &c.*"—The case of a cantilever, as in Fig. 139, may be easily converted into that of a girder of double length, supported at the ends and loaded in the centre with 2 tons. Obviously, in order to obtain a

strain of 1 ton at each end, the central load must be 2 tons, and the cantilever must be inverted. See Figs. 91, 92.

(438.) "*Rolling Load.*"—The effect of change of position in the load, such as occurs where it rolls slowly without shock from end to end of a girder, may be illustrated by Figs. 144 to 148 inclusive, the strains throughout being calculated as in the preceding examples, starting in each case with the load on each prop as a datum. From these figures we may obtain some useful general facts.

(439.) First,—it will be observed that the strains on the diagonals change with the varying position of the load, not only in amount, but also in character, or from tensile to compressive, and *vice-versâ*. Taking G for example, the strain changes from a tensile one of 5·77 tons in Fig. 146, to a compressive one of 3·46 tons in Fig. 147. A strain thus alternating or acting in opposite directions, is known (915) to be very trying to any material which in fact suffers from *fatigue*; the effect is equivalent to the *sum* of the two strains, or in our case to $5 \cdot 77 + 3 \cdot 46 = 9 \cdot 23$ tons acting in one direction only, but being alternately laid on and relieved; this, again, is more trying than a steady and constant load. These facts should be remembered in fixing the "Factor of safety" (880) for the particular case; it will also be seen that in a girder for carrying a rolling load, all the diagonals must be adapted to sustain both tensile and compressive strains, and in many cases should be of ⊥ iron, or some such form of section, rather than simple thin bars.

(440.) Secondly,—the figures show that while the strains on the top and bottom members never change their *character* with a rolling load, the amount of the strain varies considerably; for instance, from 5·2 tons tensile strain on M in Fig. 144, to ·5755 ton, also tensile, in Fig. 148; and again, from 8·08 tons on N in Fig. 145, to 1·155 ton in Fig. 148, both compressive strains. This partial relief of the strain every time the load passes is much more trying to the material than even the maximum load would be if it were a dead and constant strain (912), and the "Factor of safety" should be higher than for an equal statical load.

It should be observed that the strains on the diagonals being *alternate*, or in both directions, while those on the top and bottom are simply *intermittent*, should in strictness lead to the use of a higher Factor of Safety for the former than for the latter, in the ratio of 2 to 1 (915).

(441.) "*Effect of Distribution of Load.*"—The effect of two or more equal loads may be found by adding together the respective strains in two or more of the figures; thus, in Fig. 149 we have the strains with 10 tons on the apex of every triangle, which were obtained by adding together all the corresponding strains in the figures from 144 to 148 inclusive. Thus, on A A, &c., they are all compressive strains, and their sum is 28·86 tons; but on D, two are compressive, with a sum of $1{\cdot}155 + 3{\cdot}46 = 4{\cdot}615$ tons, and three are tensile, with a sum of $5{\cdot}77 + 3{\cdot}46 + 1{\cdot}155 = 10{\cdot}385$ tons, so that the tensile, being $10{\cdot}385 - 4{\cdot}615 = 5{\cdot}77$ tons in excess of the compressive, is the final strain on D, as in Fig. 149.

(442.) It is remarkable that with an equally distributed load, as in Fig. 149, while the strain on the top and bottom members is a maximum at the centre, where it is 37·48 tons, and is reduced progressively toward the ends, where it becomes 14·42 tons; it is just the reverse with the diagonals, being a minimum, or 5·77 tons, in the centre, increasing progressively to a maximum of 28·86 tons at the ends, &c.

(443.) In Fig. 150 we have the strains with two weights each of 10 tons, placed at equal distances on each side of the centre, which were obtained by combining those in Figs. 145, 147. It will be thus found that the strains on the diagonals between the two weights neutralise one another, being tensile in one case and compressive in the other; they are therefore useless, and are omitted altogether in Fig. 150.

The same principles will apply to any other equal or unequal loads on the several points by simple proportion. Thus, taking for illustration the diagonal D in all the figures, say we have 5 at Z in Fig. 149, then Fig. 144 gives $1{\cdot}155 \times {\cdot}5 = 0{\cdot}578$ at D; say 2 at Y gives by Fig. 145, $3{\cdot}46 \times {\cdot}2 = 0{\cdot}692$ at D, both being *crushing* strains, and their sum ${\cdot}578 + {\cdot}692 = 1{\cdot}27$. Then all the rest are *tensile* strains on

D; say 8 at X by Fig. 146 becomes $5 \cdot 77 \times \cdot 8 = 4 \cdot 616$ at D; say 12 at V becomes by Fig. 147, $3 \cdot 46 \times 1 \cdot 2 = 4 \cdot 152$ at D; and say 6 at U gives by Fig. 148, $1 \cdot 155 \times \cdot 6 = 0 \cdot 693$ at D. The sum of the three tensile strains $= 4 \cdot 616 + 4 \cdot 152 + 0 \cdot 693 = 9 \cdot 461$, and the sum of the two crushing strains being $1 \cdot 27$, we have as a final result $9 \cdot 461 - 1 \cdot 270 = 8 \cdot 191$ tensile strain on D. Applying the same process to the other parts of the girder, we may obtain the strains throughout for loads of 5, 2, 8, 12, and 6, at Z, Y, X, V, and U respectively.

(444.) "*Effect of the Weight of the Girder itself.*"—We have so far considered only those strains which a given load would produce in the different parts of a lattice girder, irrespective of those due to the weight of the girder itself. In short girders the latter is usually very small in proportion to the load, and may in such cases be neglected; but in long girders it becomes important, as shown in (488), &c., and must not be overlooked.

The weight of the girder itself is in effect equivalent to a load equally distributed all over the length; it will not, however, suffice to take it as borne on the top only, as in Fig. 149, but rather by the top and bottom equally, as in Fig. 151. Calculating, as before, successively for the loads at R, S, T, Q, we obtain another series of strains which, added to those given by Figs. 144 to 148, give the strains in Fig. 151, which may be taken as representing the ratios of the strains on a girder arising from its own weight. It should be observed that we have not only 10 tons at the apex of every triangle, but also 5 tons at each end; but as these latter impinge direct on the bearings, they cause no strain on the members of the girder. The total weight on the girder thus becomes 100, or 50 on each bearing, but the *straining* weight is 45 tons only.

We have here assumed that the weight of the girder per foot run is equal from end to end, which it usually is in ordinary cases; where, however, the strength and corresponding weight of the several parts are graduated from the centre to the ends, as would or should be the case in large and important structures, the method of calculating must be modified accordingly.

The general effect of the weight of a beam on the load which

it will bear is shown most clearly and fully in the case of "Similar" beams (488).

(445.) "*Top Flange.—Lateral Stiffness.*"—The *form* of section at the top of a lattice girder, or rather at that part subjected to compression, is not arbitrary; there must not only be area sufficient to bear the crushing strain, but also a flange of considerable breadth, so as to give lateral stiffness sufficient to prevent flexure sideways. The top flange of a girder is virtually a pillar, and while yielding by flexure vertically is prevented by the diagonals, there is nothing to resist flexure sideways except the resistance of the top flange. With long girders particularly, considerable breadth is necessary to prevent the top yielding by flexure with a strain much less than that necessary to crush the material.

(446.) For the more perfect investigation of the matter let us take the case of the long girder in (681) and Fig. 155: the total length being 32 feet with 16 bays, each bay is 2 feet. Now, obviously, the part of the top flange between the points *a*, *a*, is a pillar 4 feet long compressed with 15 tons; between *b*, *b*, a pillar 8 feet long loaded with 13 tons, &c., &c.; hence we have a series of pillars with lengths varying by increments of 4 feet, namely:—

32	28	24	20	16	12	8	4

feet long, the corresponding compressive strains being

1	3	5	7	9	11	13	15

tons, and the question is, with which of those lengths and corresponding strains the lateral strength required to resist flexure at the centre becomes a maximum.

By (147) it is shown that the tendency of a pillar of wrought iron to break by flexure is proportional to $W \times L^2$, which in our case becomes:—

1024	2352	2880	2800	2304	1584	832	240

which being a maximum with 5 tons and a length of 24 feet, or *c*, *c*, in Fig. 155, shows that with a flange of the same size throughout, this is the weakest pillar of the set, and that the

breadth required for that length and weight being greater than any other, governs the case (448).

These rectangular pillars are under peculiar conditions, being forced to yield by flexure in the direction of their larger dimension; of course an ordinary rectangular pillar of unequal dimensions, left to itself, would fail by bending in the direction of its least dimension: in a lattice girder this is prevented by the diagonals. By col. 4 of Table 34, M_P = 162900 lbs., which with Factor of Safety = 3 becomes $162900 \div (112 \times 3) =$ 485 cwt.: then the rule (235) may be modified into:—

(447.) $$B^{2 \cdot 6} = (W \times L^2) \div (480 \times t).$$

In which W = the safe working dead load in cwts.; L = the length of the pillar in feet; B = the breadth or greatest dimension of a rectangular pillar, and t = the thickness or least dimension of the same, both in inches. For 24 feet and 5 tons, or 100 cwt., as in our case, and assuming $t = \frac{1}{2}$ inch, we obtain $B^{2 \cdot 6} = (100 \times 576) \div (480 \times \frac{1}{2}) = 240$, which is the $2 \cdot 6$ power of B. By logarithms log. $240 = 2 \cdot 38 \div 2 \cdot 6 = \cdot 915$, which is the log. of $8 \cdot 22$ inches, the breadth B, required. To show that the breadth is a maximum with the length we have taken, we may calculate it for the next greater and next less lengths: thus, with 20 feet, or d, d, in Fig. 155, the strain by (446) is 7 tons, or 140 cwt., and we have $\left\{140 \times 400) \div (480 \times \frac{1}{2}\right\}^{\frac{1}{2 \cdot 6}} = 8 \cdot 142$ inches. For 28 feet, or e, e, in Fig. 155, and 3 tons, or 60 cwt., we obtain $\left\{60 \times 784) \div (480 \times \frac{1}{2}\right\}^{\frac{1}{2 \cdot 6}} = 7 \cdot 615$ inches; both being less than $8 \cdot 22$ inches as found for 24 feet. The actual breadth of the top flange in Fig. 141 given by two angle-irons and the $\frac{1}{2}$-inch diagonals was $8\frac{1}{2}$ inches.

(448.) In an ordinary plate-iron girder with a continuous vertical web the strains would be found to increase, not by steps of 2 tons, as in Fig. 155, but in regular arithmetical progression from the ends where it is nothing to the centre where it attains a maximum, and it will be found that the tendency to yield by flexure at the centre is a maximum with *one-third* of the maximum or central strain, which by the conditions of the

case is exerted on an acting length of *two-thirds* of the extreme length of the girder. Thus, let Fig. 142 be a girder 9 feet long and 1 foot deep loaded with 4 tons in the centre, giving 2 tons on each prop, which is equivalent to a cantilever, Fig. 143, $4\frac{1}{2}$ feet long, fixed in a wall at one end, and loaded with 2 tons at the other. The strain on the top and bottom members at the wall is therefore $2 \times 4\frac{1}{2} \div 1 = 9$ tons, equivalent to 9 tons at the centre of the girder, Fig. 142. Then we have the series of longitudinal strains given in the figure, increasing in arithmetical progression, and we thus obtain a series of imaginary pillars with lengths of:—

0 1 2 3 4 5 6 7 8 9

feet, the corresponding longitudinal compressive strains being

9 8 7 6 5 4 3 2 1 0

tons. Then the tendency to break by flexure at the centre being in the ratio $W \times L^2$, becomes

0 8 28 54 80 100 108 98 64 0

which is a maximum with 6 feet, or two-thirds of the extreme length, and the corresponding strain of 3 tons, or one-third of the maximum or central strain. The practical application of these laws to plate-iron girders is illustrated by (394).

(449.) "*Effective, and Extreme Depth.*"—It should be observed that in the diagrams, Figs. 136–151, the strains are taken as acting on mathematical lines, or centres of strain, and that in dealing with practical cases, the depth taken as a basis for calculation should not be the extreme depth of the girder, but rather the distance between centres of the resisting forces of cohesion and crushing, which may be taken as coincident with the centres of gravity of the sections of the top and bottom members of the girder (385), (684). The difference between the effective depth thus measured and the extreme depth is sometimes considerable, but in practical rules it is allowed for in fixing the value of the constant (390).

CHAPTER XI.

ON "SIMILAR" BEAMS.

(450.) Beams are termed "Mathematically Similar" when *all* the dimensions of one bear a given proportion to *all* the corresponding dimensions of the others. Thus, in Fig 152, A, B, C are three similar tubular beams of plate-iron, or other material, the length, depth, breadth, and thickness of B, being double the corresponding dimensions of A, and in C, triple those of A. Again, in Fig. 153, D, E, F are three similar cast-iron girders, all the corresponding dimensions being in the ratio 1, 2, 3, as before.

We may now complete the illustrations by calculating the breaking weight of these beams. For the wrought-iron tubular beams A, B, C, we may take the value of M_T at 3200 lbs. (375), then rule (330) becomes with—

$$A = \frac{(2^3 \times 1) - (1{\cdot}8^3 \times {\cdot}8)}{2} \times 3200 \div 4 = 1333{\cdot}76 \text{ lbs.,}$$

$$B = \frac{(4^3 \times 2) - (3{\cdot}6^3 \times 1{\cdot}6)}{4} \times 3200 \div 8 = 5335{\cdot}04 \text{ lbs.,}$$

$$C = \frac{(6^3 \times 3) - (5{\cdot}4^3 \times 2{\cdot}4)}{6} \times 3200 \div 12 = 12003{\cdot}84 \text{ lbs.}$$

Now, it will be observed that the linear dimensions being in the ratio 1, 2, 3, the loads are in the ratio of the squares of those numbers, or 1, 4, 9. We should obtain the same *Ratios* by the special rules (377), or even by the incorrect old rule (337), although the calculated loads would differ in amount.

(451.) For the cast-iron girders D, E, F, we may take the value of M_T at ·92 ton (335); then calculating by the special rule in (378), we obtain:—

$$D = \left\{\tfrac{1}{2}^2 \times 2) + (3^2 - \tfrac{1}{2}^2] \times \tfrac{1}{2}) + (4^2 - 3^2] \times 4\right\} \times {\cdot}92 \div 5 = \overset{\text{Tons.}}{6{\cdot}049}$$

$$E = \left\{1^2 \times 4) + (6^2 - 1^2] \times 1) + (8^2 - 6^2] \times 8\right\} \times {\cdot}92 \div 10 = 24{\cdot}196$$

$$F = \left\{1\tfrac{1}{2}^2 \times 6) + (9^2 - 1\tfrac{1}{2}^2] \times 1\tfrac{1}{2}) + (12^2 - 9^2] \times 12\right\} \times {\cdot}92 \div 15 = 54{\cdot}441$$

Here again, the linear dimensions being in the ratio 1, 2, 3, the loads are in the ratio of the squares of those numbers, or 1, 4, 9, &c.

(452.) "*General Laws for Similar Beams.*"—Mr. Tate and others have shown that there are some general laws governing the relations of similar beams, which are very useful, by enabling us to reason from one whose strength is known by experiment, to another of very different sizes, but similar proportions, whose strength we desire to know. Thus, as an extreme case, it is shown in (475) that the strength and sizes of the great tubular bridge at Conway, 400 feet span, and weighing 1080 tons, might be calculated with approximate accuracy from the experimental strength of a little model tube $3\frac{3}{4}$ feet span, weighing only $4\frac{1}{3}$ lbs. Figs. 152, 153, will enable us to illustrate the principal laws for similar beams.

(453.) 1st. The breaking loads of similar beams are to each other as their sectional areas:—thus, the areas of A, B, C are 0·56, 2·24, and 5·04, and of D, E, F, 6·25, 25·0, and 56·25 square inches respectively, which in both cases are in the ratios of the breaking weights 1, 4, 9.

(454.) 2nd. In similar beams, the *cubes* of the breaking loads are to each other as the *squares* of the weights of the beams between supports. Thus, the breaking loads in our figures being in the ratio 1, 4, 9, we have 1^3, 4^3, 9^3, or 1·0, 64, and 729, which are the ratios of the squares of the weights of the beams, for 1^2, 8^2, and 27^2 are = 1, 64, and 729.

(455.) 3rd. The breaking weights of beams of similar sections, but of varying lengths, are equal to the continued product of the whole cross-sectional area, the depth, and a constant derived from experiment for the particular form of beam, material, &c., divided by the length, or distance between supports, hence we have the rules:—

(456.) $$\mathrm{W} = a \times d \times \mathsf{M} \div l.$$

(457.) $$a = (\mathrm{W} \times l) \div (d \times \mathsf{M}).$$

(458.) $$\mathsf{M} = (\mathrm{W} \times l) \div (a \times d).$$

In which W = the weight or load on the beam in lbs., tons, &c., dependent on the terms of M; a = the whole cross-sectional

area in square inches; d = the depth in inches; l = the length or distance between supports in inches; and M = a constant from experiment, &c. Thus taking the girder E, Fig. 153, we may find the value of M: here W = 24·196 tons, as found in (451); a = 25 square inches; d = 8 inches, and l = 120 inches. Then M = (24·196 × 120) ÷ (25 × 8) = 14·5176. Now applying this, for example, to the girder F, where a = 56·25, d = 12, and l = 180, we obtain W = 56·25 × 12 × 14·5176 ÷ 180 = 54·441 tons, as before, &c. (451).

(459.) 4th. The breaking weights of beams of similar *sections*, but all of the *same length*, are as the cubes of the linear ratios of the sections. Thus, if the beams A, B, C had all been say 4 feet long, or the length of A, the breaking weights would evidently have been in the ratio 1, 8, 27, instead of 1, 4, 9:—then the linear ratio of the sections being 1, 2, 3, we have 1^3, 2^3, 3^3, or 1, 8, 27, which is also the ratio of the breaking weights.

(460.) 5th. In beams having similar sections but different lengths, L and l, corresponding to breaking weights W and w, the relations are expressed by the equation $w = R^3 \times W \times L \div l$, in which R = the linear ratio of the two sections. Thus, with the cast-iron girder D in Fig. 153, we have W = 1, and L = 5; then comparing with the girder F, we have R = 3, from which we obtain the relative breaking weight of F = $3^3 \times 1 \times 5 \div 15 = 9$; that is to say, if the breaking weight of D = 1, that of F will be 9. Similarly, reasoning from D to E, we have R = 2, and w comes out $2^2 \times 1 \times 5 \div 10 = 4$, the relative breaking weight of E, &c.

(461.) 6th. The breaking weights of similar beams are to each other as the squares of their linear dimensions. Thus, with Figs. 152, 153, in A, B, C, or in D, E, F, the ratio of the linear dimensions is 1, 2, 3, and 1^2, 2^2, 3^2, or 1, 4, 9, is the ratio of the breaking weights, as given by the figures and shown by (450), (451).

(462.) 7th. In beams of similar sections the cube-root of the breaking weights multiplied by the respective lengths, is in the simple ratio of the linear dimensions, or $\sqrt[3]{W \times L} = R$. Thus, in the tubular beams A, B, C, we have breaking weights in the ratio 1, 4, 9, corresponding to lengths in the ratio 4, 8,

12; then we have $\sqrt[3]{1 \times 4} = 1 \cdot 587$; $\sqrt[3]{4 \times 8} = 3 \cdot 174$; and $\sqrt[3]{9 \times 12} = 4 \cdot 762$, which are in the linear ratio of the sections, or 1, 2, 3. Similarly with the cast-iron girders D, E, F, in Fig. 153, we have $\sqrt[3]{1 \times 5} = 1 \cdot 71$; $\sqrt[3]{4 \times 10} = 3 \cdot 42$; and $\sqrt[3]{9 \times 15} = 5 \cdot 13$; which are still in the linear ratio of the sections, or 1, 2, 3. This rule will be found useful in giving the sizes for "Unit" girders, or girders for a load $= 1 \cdot 0$ with a length $= 1 \cdot 0$, from which the sizes necessary for any load and length are easily obtained from direct experiment on beams of any size or form (485).

(463.) 8th. The breaking weights of similar cast-iron girders are to each other as the areas of their bottom flanges. Thus, in the girders D, E, F, Fig. 153, the areas of the bottom flanges are 4, 16, and 36 square inches respectively, or in the ratio 1, 4, 9, which is also the ratio of the breaking weights.

(464.) 9th. The breaking weights of girders of similar sections are to each other as the areas of their bottom flanges, multiplied by the respective depths, and by a constant adapted to the particular form of section, divided by the length between supports, or—

(465.) $$W = a \times d \times \mathsf{M} \div l.$$

(466.) $$a = (W \times l) \div (d \times \mathsf{M}).$$

(467.) $$\mathsf{M} = (W \times l) \div (a \times d).$$

In which $a =$ the area of the bottom flange in square inches, and the rest as in (458). The value of M will vary with the form of section; for the section given by Fig. 153 it is $22 \cdot 68$ for tons:—thus with F we have $(54 \cdot 441 \times 180) \div (36 \times 12) = 22 \cdot 68 = \mathsf{M}$.

For the form of section recommended by Mr. Hodgkinson (351) and Fig. 79, with top and bottom flanges having areas in the ratio 1 to 6, he gives the value of M at 26, for finding the breaking weight in tons. With that value for M, we obtain for the girder D, $4 \times 4 \times 26 \div 60 = 6 \cdot 933$ tons; for E, $16 \times 8 \times 26 \div 120 = 27 \cdot 73$ tons; and for F, $36 \times 12 \times 26 \div 180 = 62 \cdot 4$ tons, and those breaking weights are in the ratio 1, 4, 9, as in Fig. 153.

TABLE 78.—Of the RELATIVE STRENGTH of "MATHEMATICALLY SIMILAR" TUBULAR BEAMS.

No.	Length.	Experimental Breaking Weight, plus ½ Weight of Beam.	Depth.	Breadth.	Thickness all over.	Ratio of the Dimensions.					Power of R.	Strength of large Beam, calculated from its small fellow.			
						Lengths.	Depths.	Breadths.	Thicknesses.	Mean of the whole, R.		By Experimental Ratio, $R^{1\cdot9}$.		By Theoretical Ratio, R^2.	
												Tons.	Error per Cent.	Tons.	Error per Cent.
	feet.	tons.	inches.	inches.	inches.										
1	30	58·605	24·0	15·5	·525	4	4·0	3·975	3·958	3·983	1·862	61·76	+ 5·4	70·91	+21·0
2	7½	4·47	6·0	3·9	·1325										
3	30	23·417	23·75	15·5	·272	4	4·095	4·08	4·195	4·090	1·991	20·59	−12·07	23·70	+ 1·20
4	7½	1·417	5·8	3·8	·065										
5	30	58·605	24·0	15·5	·525	8	8·0	7·949	8·607	8·139	1·884	60·54	+ 3·3	74·66	+27·4
6	3¾	1·127	3·0	1·95	·061										
7	30	23·417	24·0	16·5	·272	8	8·0	8·864	9·07	8·439	2·029	17·79	−24·0	22·02	− 5·97
8	3¾	0·3009	3·0	1·9	·030										
9	45	118·02	35·75	24·0	·75	12	11·92	12·31	12·29	12·13	1·864	129·2	+ 9·5	165·8	+40·5
10	3¾	1·127	3·0	1·95	·061										
11	45	118·02	35·75	24·0	·75	6	5·96	6·16	5·656	5·944	1·837	132·1	+12·0	157·9	+33·8
12	7½	4·47	6·0	3·9	·1325										
13	45	118·02	35·75	24·0	·75	1·5	1·49	1·50	1·50	1·498	1·877	119·1	+ 1·0	124·0	+ 5·1
14	30	55·275	24·0	16·0	·50										
	..	..	..	..	..	..	..	..	..	Means =	1·9062	..	− 0·7	..	+17·58
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)

(468.) 10th. In similar beams the weights of the beams between supports are proportional to the cubes of the linear ratio of the dimensions. Thus, in Figs. 152, 153, the linear ratio being 1, 2, 3, we have 1^3, 2^3, 3^3, or 1, 8, 27, which is also the ratio of the weights of the beams.

We may now give some illustrations of the application of these laws to cases in practice.

(469.) The 6th law in (461) states that the breaking weights of similar beams are to each other as the squares of their linear dimensions. Mr. Hodgkinson made some experiments with the special object of testing the accuracy of this law as applied to tubular beams. These experiments had particular reference to the great Britannia and Conway bridges, in which tubular beams were used for the first time on a large scale. Little or nothing was at that time known of the strength of beams of the tubular form, and it was highly necessary that the theoretical laws should be checked by experiment. In Table 78 the beams are grouped together *in pairs*, selected so as to be "similar" in all respects as nearly as possible, but also very different in dimensions, the special object being to show how far the strength of a large Tubular beam could be calculated with accuracy from that of its small fellow. Of course it was impossible in practice to preserve precisely any given ratio between *all* the dimensions; for instance, in Nos. 9 and 10 the ratio intended was 1 to 12, as in col. 6, but the actual ratios of the depths, breadths, and thicknesses of plate were 11·92, 12·31, and 12·29 respectively, as shown by cols. 7, 8, and 9. The best that can be done under these circumstances is to take the *mean* ratio of the four dimensions as the general ratio or value of R, and this is given by col. 10 = 12·13.

(470.) Now, the experimental breaking weight of the small beam of this pair, as given by col. 2, is 1·127 ton, and $12 \cdot 13^2$ being 147·1, we obtain $1 \cdot 127 \times 147 \cdot 1 = 165 \cdot 8$ tons as the strength of the large beam, col. 14. But experiment gave 118·02 tons only, hence we have $165 \cdot 8 \div 118 \cdot 02 = 1 \cdot 405$, or an error of $+ 40 \cdot 5$ per cent., as in col. 15. Calculating in this way throughout, the mean error of the whole series of experiments is $+ 17 \cdot 58$ per cent., col. 15, showing that the theoretical

ratio R^2 is too high, and we have to find the power of R which will agree with the experiments. Thus, with Nos. 9 and 10, the logarithm of $118\cdot02 = 2\cdot071889$; of $1\cdot127 = 0\cdot051924$; and of $12\cdot13 = 1\cdot083861$. Using these numbers logarithmically we obtain $(2\cdot071889 - 0\cdot051924) \div 1\cdot083861 = 1\cdot864$, as in col. 11, which is the power of the ratio of the linear dimensions, or R, agreeing with the experimental strengths of this pair of beams, so that instead of R^2 as due by theory, we have $R^{1\cdot864}$ due to experiment, as given in col. 11, which gives throughout the power of R for each pair of beams, the mean of the whole being $1\cdot9062$.

(471.) We may therefore admit that the mean power of the ratio governing the strength of "similar" tubular beams of wrought plate-iron is $R^{1\cdot9}$ instead of R^2 due by theory: the effect of this apparently small difference is in extreme cases very great, as shown in (482).

Col. 12 has been calculated by that ratio throughout; thus, in Nos. 9, 10, the mean ratio $R = 12\cdot13$, the logarithm of which, or $1\cdot083861 \times 1\cdot9 = 2\cdot059336$, the natural number due to which is $114\cdot65$:—then the breaking weight of the small tube being $1\cdot127$ ton, we obtain $1\cdot127 \times 114\cdot65 = 129\cdot2$ tons, the breaking weight of the large tube, as in col. 12. Experiment gave $118\cdot02$ tons, hence we have $129\cdot2 \div 118\cdot02 = 1\cdot095$, or an error of $+ 9\cdot5$ per cent., col. 13: the mean error of the whole series as thus calculated is $- 0\cdot7$ per cent. only.

(472.) With the modification of $R^{1\cdot9}$ instead of R^2, the correctness of the theoretical law is fairly borne out by the experiments. The rule was thus tested because there was some doubt whether it would hold in the case of tubular beams made with thin plates of wrought iron, which have a tendency to fail by wrinkling with a strain much lower than is necessary to crush the material. But it is shown in (316) that when the thickness and breadth of the plate subjected to compression are both increased in the same ratio, which of course is the case with "similar" beams, the Wrinkling strain per square inch remains constant, the rule (308) being $W_W = \sqrt{t_W \div b_W} \times 104$. Thus, with three plates of the respective thicknesses $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$ inch, and breadths 10, 20, 30 inches, we have $\sqrt{\cdot1 \div 10} \times 104$

$= 10\cdot4$; $\sqrt{\cdot2 \div 20} \times 104 = 10\cdot4$; and $\sqrt{\cdot3 \div 30} \times 104 = 10\cdot4$ tons per square inch, W_w being constant. Table 78 shows that this law is correct as applied to tubular beams, even with such a ratio in the sizes as 12 to 1, and as illustrated by Fig. 100.

(473.) "*Conway Tube.*"—We may now apply this law to the calculation of the strength of the great tube of the Conway Bridge. A large model tube was made, 75 feet long between bearings, and when the best proportions of the areas at the top and bottom had been decided by many experiments, the tube was finally broken with $86\cdot25$ tons in the centre; but the weight of the tube itself was $5\cdot8$ tons, which, being a distributed load, is equivalent to $5\cdot8 \div 2 = 2\cdot9$ tons in the centre; the *total* breaking load was therefore $86\cdot25 + 2\cdot9 = 89\cdot15$ tons.

The breaking weight of similar beams being by the modified rule (471) proportional to $R^{1\cdot9}$, the length of the model tube 75 feet, and of the Conway tube 400 feet, we get $89\cdot15 \times 400^{1\cdot9} \div 75^{1\cdot9} = 2145$ tons, the details of the calculation being as follows. The logarithm of 400, or $2\cdot602 \times 1\cdot9 = 4\cdot9438$, the natural number due to which, or 87,870, is the $1\cdot9$ power of 400: then the log. of 75, or $1\cdot875 \times 1\cdot9 = 3\cdot5625$, the natural number due to which, or 3652, is the $1\cdot9$ power of 75. The breaking weight of the model tube, or $89\cdot15 \times 87870 \div 3652 = 2145$ tons in the centre, the total gross breaking weight of the Conway tube.

(474.) By the 10th law (468) the weights of similar beams are proportional to the cubes of the linear dimensions:—in our case we have $5\cdot8 \times 400^3 \div 75^3 = 880$ tons: the actual weight of the Conway tube, 424 feet long, was 1146 tons, but between bearings 400 feet apart, it would be about $1146 \times 400 \div 424 = 1080$ tons, which, being a distributed load, is equivalent to 540 tons in the centre, hence the *useful* load would be $2145 - 540 = 1605$ tons in the centre. It was found necessary, in order to resist the shearing strain (403) at the bearings, to introduce about 300 tons of cast-iron frames, but as these were principally at the ends they would not sensibly increase the load upon the beam.

(475.) The model tube was in this case made of the largest

practicable dimensions, the ratio to the large tube being as 1·0 to 5·34, and this was doubtless the most prudent course to adopt. But as a matter of fact, and a wonderful illustration of the correctness of the law $R^{1 \cdot 9}$, we may show that the strength of the great tube at Conway, 400 feet span and weighing 1080 tons, may be calculated with approximate accuracy from the experimental strength of a little model $3\frac{3}{4}$ feet span, weighing only $4\frac{1}{3}$ lbs. In this case the ratio of the dimensions is $400 \div 3 \cdot 75 = 106 \cdot 7$ to $1 \cdot 0$: the log. of 106·7 or $2 \cdot 028164 \times 1 \cdot 9 = 3 \cdot 8535$, the natural number due to which, or 7137 ! ! is the ratio of the breaking weights. By col. 2 in Table 78 the breaking load of the little model No. 8 was 0·3009 ton, hence the strength of the great tube would be $0 \cdot 3009 \times 7137 = 2147$ tons, differing very little from 2145 tons as found in (473) from the strength of the 75-foot model. Then following out the ratio we may obtain the general dimensions of the great tube from those of the little one; thus, the depth of the little tube being 3 inches or ·25 feet, that of the great one comes out $\cdot 25 \times 106 \cdot 7 = 26 \cdot 7$ feet: the actual depth was 25·5 feet. The breadth would be $106 \cdot 7 \times 1 \cdot 9 \div 12 = 16 \cdot 8$ feet, the actual breadth = 14·7 feet. The thickness of plate comes out $\cdot 03 \times 106 \cdot 7 = 3 \cdot 2$ inches, and the width of the top plate being 16·8 feet or 202 inches, the area at the top would be $202 \times 3 \cdot 2 = 646$ square inches: the actual area was 565 square inches, &c., which for practical reasons was arranged as cells of thin plate-iron $\frac{3}{4}$ inch thick.

If, in calculating the strength of the Conway tube from the 75-foot model, we had taken the theoretical ratio R^2, we should obtain $89 \cdot 15 \times 400^2 \div 75^2 = 2536$ tons in the centre breaking weight, or $2536 - 540 = 1996$ tons net, exclusive of the weight of the tube itself.

(476.) It is an inconvenience of this 6th law (461) that *all* the dimensions should be in an exact given ratio, a condition that is sometimes difficult to fulfil in practice. The third law (455) is more elastic, for though the value of M is adapted strictly only to beams that are mathematically "similar," still small departures from that primary condition are unimportant, and variations in the dimensions are met by the rule.

Applying this 3rd law to the 75-foot model tube and the Conway Bridge as before, we have the areas of the whole cross-section 55·47, and 1530 square inches respectively. Then, the model tube being 4·5 feet deep, we get the value of M by the rule $M = (W \times L) \div (a \times D)$, which, taking L and D both in feet, becomes in our case $(89 \cdot 15 \times 75) \div (55 \cdot 47 \times 4 \cdot 5) = 26 \cdot 8$ the value of M. In the Conway tube $L = 400$, $D = 25 \cdot 5$, and $a = 1530$ square inches; hence the rule $W = a \times D \times M \div L$, becomes $1530 \times 25 \cdot 5 \times 26 \cdot 8 \div 400 = 2614$ tons gross, or $2614 - 540 = 2074$ tons net central breaking weight. This differs but little from 1996 tons as found by the 6th law in (475), where the theoretical ratio R^2 is taken as a basis, but is much in excess of 1605 tons as found in (473) with the corrected experimental ratio $R^{1 \cdot 9}$, which is unquestionably the most correct.

(477.) The 1st law in (453), that the breaking weights of similar beams are simply proportional to their respective cross-sectional areas, may be illustrated from the same examples. Thus, the areas being 55·47, and 1530 square inches, and the gross breaking weight of the small tube being 89·15 tons, that of the large one will be $89 \cdot 15 \times 1530 \div 55 \cdot 47 = 2459$ tons gross, or $2459 - 540 = 1919$ tons net.

(478.) The 9th law (464) gives us valuable practical rules for cast-iron girders more particularly:—thus for the proportions recommended by Mr. Hodgkinson, where the flanges have areas in the ratio of 6 to 1, he gives, from his own experiments, the value of M at 26 for the breaking weight in tons. Fig. 79 gives the section of large girders experimented upon by Mr. Owen (see Table 68): the mean breaking weight by thirteen experiments was 38·3 tons with a length of 16 feet between bearings. The area of the flanges was 6 to 1, and that of the bottom one $1 \cdot 75 \times 12 = 21$ square inches; then the rule $W = a \times d \times M \div l$, becomes $21 \times 14 \times 26 \div 192 = 39 \cdot 81$ tons. The multiplier 26 is strictly applicable only to girders with flanges in the proportions of 6 to 1. (See 351.)

(479.) This law may also be applied to plate-iron tubular beams; but some caution is necessary here, because the rule, taking the bottom flange alone as the index of the strength, pre-

supposes that the other parts of the beam, such as the top plate, &c., are not weaker than the bottom flange or plate, &c. In cast-iron girders this condition is generally realised, because the tensile strength of cast iron is only $\frac{1}{6}$th of its compressive strength, and for that reason the bottom flange is usually the weak part. But with wrought iron, although the tensile and crushing strains are nearly equal (377), the tendency of thin plates to wrinkle or corrugate with less than the crushing strain, causes the top flange or plate to be frequently the weak member, and therefore to govern the case.

(480.) Taking again the 75-foot model tube: after the top plate had been duly strengthened until the beam failed by the bottom plate giving way under the tensile strain, we had an area of 22·45 square inches at the bottom, with a depth of 4·5 feet, and a length of 75 feet. In the Conway tube the bottom area was 500 square inches, the depth 25·5 feet, and the length 400 feet: hence the rule $\mathsf{M} = (W \times L) \div (a \times D)$, taking L and D in feet, becomes $(89 \cdot 15 \times 75) \div (22 \cdot 45 \times 4 \cdot 5) = 66$. Then for the Conway tube, the rule $W = a \times D \times \mathsf{M} \div L$, becomes $500 \times 25 \cdot 5 \times 66 \div 400 = 2104$ tons gross, or $2104 - 540 = 1564$ tons net.

(481.) We have thus obtained by different rules various values for the strength of the Conway tube; the variations are not very great, and any of the results are sufficiently correct for the requirements of practice, a small error being in any case amply covered by the Factor of Safety, which for a Railway Bridge would not be less than 6. Collecting these results:—

By Rule	6, with	$R^{1 \cdot 9}$	we found in	(473) 2145	tons, gross.
„	6, „	R^2	„	(475) 2536	„ „
„	3,	..	„	(476) 2614	„ „
„	1,	..	„	(477) 2459	„ „
„	9,	..	„	(480) 2104	„ „

(482.) The difference between R^2 and $R^{1 \cdot 9}$ in the 6th law is of course most considerable when R, or the difference in the sizes of the model and the beam, is very great, and in such cases

more particularly the 6th Rule (461) will no doubt with $R^{1\cdot 9}$ give more correct results than any of the others. For instance, if we calculate the strength of the Conway tube from that of the little model as in (475) by the 6th law as before, but with R^2 instead of $R^{1\cdot 9}$, we should have a difference or error of 59 per cent. The ratio R is in this case 106·7 to 1, hence we obtain $0\cdot3092 \times 106\cdot7^2 = 3520$ tons gross, instead of 2207 tons as in (475); a difference of $3520 \div 2207 = 1\cdot59$, or + 59 per cent.; this being due to the difference between $106\cdot7^2 = 11385$, and $106\cdot7^{1\cdot 9} = 7137$.

(483.) The 7th law in (462) is a very useful one, enabling us to reason direct from a girder of any form whose strength is known by experiment, to another similar girder for any span and load whose sizes we require. For instance, Fig. 156 is the section of a cast-iron girder 4½ feet between bearings, whose breaking weight was found by Mr. Hodgkinson to be 6·456 tons in the centre. Now, say that we require to find from this the sizes for a girder of similar section, to bear safely 20 tons in the centre, with a length of 25 feet. With Factor 3, we have $20 \times 3 = 60$ tons breaking weight:—then by Rule 7, putting w and l for the breaking weight and length of the experimental girder, and W and L for the breaking weight and length of the girder required, we have:—

(484.) $$M_G = \sqrt[3]{W \times L} \div \sqrt[3]{w \times l},$$

which in our case becomes $\sqrt[3]{60 \times 25} \div \sqrt[3]{6\cdot456 \times 4\cdot5} = 3\cdot71$, the value of M_G, a multiplier, which, applied to all the cross-sectional dimensions of Fig. 156, will give the corresponding dimensions of Fig. 157. Thus, for the depth we have $5\cdot125 \times 3\cdot71 = 19$ inches; for the breadth of the bottom flange $4\cdot16 \times 3\cdot71 = 15\cdot43$, or say 15½ inches, &c., &c., and we thus obtain all the sizes in Fig. 157 *from direct experiment* on a Similar girder.

The most useful application of this method is in cases where for particular reasons the section required is of unusual form, such as to be not easily calculated by the ordinary rules. In such a case the most satisfactory course is to make a model girder, as large as conveniently possible, which, being experi-

mentally broken, will supply data for calculating the sizes of the girder required (901).

(485.) "*Unit Girders.*"—The 7th law in (462) will also enable us to find, from direct experiment or otherwise, the sizes of "Unit" girders 1 foot long with a breaking weight of 1 ton, from which we may easily obtain the sizes necessary for *similar* girders with other loads and lengths by the Rule:—

(486.) $$M_U = \sqrt[3]{W \times L}.$$

In which W = the load on the centre of the girder in lbs., tons, &c., dependent on M_U; L = the length in feet, and M_U = a constant. Thus, taking again the girder, Fig. 156, the rule in our case becomes $M_U = \sqrt[3]{6 \cdot 456 \times 4 \cdot 5} = 3 \cdot 074$:—now dividing all the cross-sectional dimensions of Fig. 156 by 3·074, we obtain the corresponding sizes of the "UNIT" girder, Fig. 160: thus for the depth we obtain $5 \cdot 125 \div 3 \cdot 074 = 1 \cdot 667$: for the bottom flange $4 \cdot 16 \div 3 \cdot 074 = 1 \cdot 353$ inch, &c., &c.

(487.) Then, to apply this to other cases, say for a girder as in (483), 60 tons breaking weight and 25 feet long, the rule $\sqrt[3]{W \times L} = M_U$ becomes $\sqrt[3]{60 \times 25} = 11 \cdot 4$, and by multiplying all the cross-sectional sizes of the "Unit" girder, Fig. 160, by 11·4 we obtain the corresponding sizes for the girder required. Thus, the depth will be $1 \cdot 667 \times 11 \cdot 4 = 19$ inches; the width of bottom flange, $1 \cdot 353 \times 11 \cdot 4 = 15 \cdot 4$, say $15\frac{1}{2}$ inches, &c., &c., as in Fig. 157.

We have thus obtained the series of "Unit" girders, Figs. 158 to 163, any one of which may be used in the way we have illustrated: all of these Unit sections have been calculated from girders experimented upon by Mr. Hodgkinson. In making a selection we should be guided by the special requirements of practice:—for instance, where great stiffness is required, say for carrying a water-tank, where considerable deflection would strain and endanger the joints, we should select a deep one, such as Fig. 159; in other cases great depth might be inadmissible, and say Fig. 162 would be selected; in other cases we may require a wide top flange for a Bressummer to carry a wall, and Fig. 158 would be the most suitable, &c. See (618) and Fig. 208.

(488.) "*Effect of the Weight of the Beam itself.*"—The laws for "Similar Beams" will enable us to show most clearly the effect of the weight of the beam itself on the total load which it can bear. It is important to remember that the breaking weight of a beam must always be composed of two different loads, one being the weight of the beam itself, and the other the extra load laid upon it; moreover, that the proportion between these two is very variable (490). In short beams, the weight bears a very small proportion to the load, and in most ordinary cases may be safely neglected, but with long beams, the proportion rises with the length, until the whole strength of the beam is required to carry its own weight, and it can bear no extra load whatever (492). In the large tube of the Conway Bridge the weight was 1080 tons (474), which is equivalent to $1080 \div 2 = 540$ tons in the centre, and this is about $\frac{1}{4}$th of the total strength of the beam, or 2145 tons, as calculated in (473).

(489.) If we take a series of "Similar" beams, with lengths and all other dimensions in the ratio 1, 2, 3 . . . . 10, the 6th law in (461) shows that the breaking weights will be proportional to the *squares* of those numbers, and by the 10th law in (468) the weights of the beams will be as the *cubes*. Thus, with beams having all their dimensions, lengths, depths, breadths, and thicknesses in the ratio:—

1	2	3	4	5	6	7	8	9	10

the *total* breaking weights taken as a distributed load, including the strain from the weight of the beam itself, would be in the *ratio* of the squares of those numbers (461), say

10	40	90	160	250	360	490	640	810	1000

Now, say that the weights of the beams between supports, being by (468) in the ratio of the *cubes* of the dimensions, are:—

1	8	27	64	125	216	343	572	729	1000

Then, the useful load borne, over and above the weights of the beams themselves, would obviously be found by deducting the

weight of the beam from the total breaking load, and become:—

9	32	63	96	125	144	147	128	81	0

Hence, with the smallest beam of the series, the weight of the beam itself is $\frac{1}{10}$th, and the useful load $\frac{9}{10}$ths of the total strength:—with the length, &c. = 5, the weight of the beam is $\frac{1}{2}$, and the useful load $\frac{1}{2}$ the total strength. But with the largest beam of the series, the weight of the beam itself is equal to the total strength, and it can bear no useful load whatever.

(490.) We have seen (488) that with the large Conway tube the weight of the beam was $\frac{1}{4}$th of the total strength. With the 75-foot model tube (473) it was $2 \cdot 9 \div 89 \cdot 15 = \cdot 0325$, or $\frac{1}{30}$th of the total strength. With the little model tube No. 8 in Table 78, the weight between supports was $4 \cdot 34$ lbs., equivalent to $4 \cdot 34 \div 2 = 2 \cdot 17$ lbs. in the centre, and the total breaking weight being $672 + 2 = 674$ lbs., the weight of the beam is $2 \cdot 17 \div 674 = \cdot 00322$, or $\frac{1}{310}$th of the total strength.

(491.) "*Effect with Large Beams.*"—With very large beams the effect of the strain due to the weight of the tube is to complicate considerably the application of the "Factor of Safety." The majority of our leading Engineers admit 6 as the value of that Factor for Railway Bridges, or that the beam may be strained to $\frac{1}{6}$th of its breaking weight only, for a moving load such as a train. But we have seen that in the case of the Conway tube the weight of the tube alone strains the structure far beyond that limit, in fact to $\frac{1}{4}$th of the breaking weight: hence the highest *possible* Factor = 4. Thus the calculated breaking weight of that tube by the most correct method of calculation (473) being 2145 tons, the safe load with Factor 6 is $2145 \div 6 = 358$ tons, but as we have seen (474) the strain due to the weight of the tube itself is 540 tons.

It is shown, however, in (839) that in most cases the strain produced by a rapid train is very little greater than that due to a dead load, for which, as shown by (885), (886), the Factor for cast or wrought iron may be taken at 3. Then, with the Conway tube we obtain $2145 \div 3 = 715$ tons total safe load,

and as the strain due to the weight of the tube is 540 tons, we have 715 − 540 = 175 tons for the weight of the train.

(492.) It may easily be shown that if the length of the Conway tube were doubled, the span becoming 800 feet, and all the cross-sectional dimensions were retained, the beam would break with its own weight; for obviously the breaking load would be reduced to half or to 2145 ÷ 2 = 1073 tons, while the weight of the beam being doubled becomes 540 × 2 = 1080 tons.

If, on the other hand, *all* the dimensions were increased in the same ratio, the length with which the beam would break with its own weight would be 1600 feet, or 4 times its present span. In that case, the breaking weight increasing as the square of the dimensions becomes $2145 \times 4^2 = 34320$ tons, and the weight of the tube itself increasing as the cube of the dimensions becomes $540 \times 4^3 = 34560$ tons, &c. We have here, however, taken the theoretical ratio R^2 instead of the more correct experimental ratio $R^{1 \cdot 9}$ (471).

CHAPTER XII.

THE CONNECTION OF THE TENSILE AND CRUSHING STRAINS WITH THE TRANSVERSE STRAIN.

(493.) When a beam is fixed at one end and loaded at the other, the transverse strain is resolved into a tensile one at the upper part of the section, and a crushing one at the lower part. There is therefore an intimate connection between the transverse strength of a material, and its strength in resisting tensile and crushing strains, and our present object is to investigate the nature of that connection, and to obtain rules that will enable us to calculate any one of those three strains, when the other two are known by experiment.

(494.) Let Fig. 165 be a beam or cantilever built into a wall and unloaded, the depth being 20 inches, the breadth 1 inch, and the length 100 inches. Fig. 166 is the same beam loaded

at the end with a weight W, the tendency of which is to cause the beam to rotate round the "neutral axis" N. A., the fibres above that line being stretched in a ratio increasing in simple arithmetical progression from the neutral axis, where it is nothing, to the upper edge of the section, where it is a maximum, and if we admit that the strains are simply proportional to the extensions (which is not strictly true (604)), those strains will obviously increase in arithmetical progression also, and it will follow that if the beam is loaded up to the point of rupture, it will give way at first by the fracture of the upper fibres, the rest following in succession. Obviously, the fibres below the neutral axis are compressed and subjected to a crushing strain increasing from the neutral axis to the lower edge of the section, where it becomes a maximum.

(495.) If the tensile and crushing strengths are equal to one another, the neutral axis will be in the centre of the section as in Fig. 166: assuming for illustration that they are thus equal, and that the *maximum* strength is 10 tons per square inch, we have in Fig. 164 a section of the beam to a larger scale, and can calculate the strains on each square inch of that section, also the leverage with which it acts; and its effect in sustaining a load at W, in Fig. 166.

Thus, the square inch B is strained with 10 tons per square inch at its upper edge, and 9 tons at the lower edge; the mean is 9·5 tons, which, acting with a leverage of 9·5 inches (namely, the distance from its centre of gravity to the neutral axis), will exert, in sustaining a load at W, a strain of $9 \cdot 5 \times 9 \cdot 5 \div 100 = \cdot 9025$ ton. Similarly, D gives $8 \cdot 5 \times 8 \cdot 5 \div 100 = \cdot 7225$ ton: following out the calculation in this way throughout, the sum of the whole series of strains for that half of the section subjected to tension will come out 3·325 tons: if the series had been infinite the sum of the resistances would have been $3\frac{1}{3}$ tons.

We might have obtained the same result more easily by multiplying the whole area 10, by the mean strain 5, and by the mean leverage 5, and taking $\frac{4}{3}$ of that product; thus $10 \times 5 \times 5 \times \frac{4}{3} \div 100 = 3\frac{1}{3}$ tons, as before.

This strain is due to tension alone, and that due to com-

pression being the same we obtain $6\frac{2}{3}$ at W. If we would convert this case of a cantilever loaded at one end, Fig. 166, into an equivalent beam supported at each end and loaded in the centre, Fig. 167, we have evidently a length of 200 inches, a strain of $6\frac{2}{3}$ tons on each prop, and a central load of $13\frac{1}{3}$ tons.

This mode of calculation may be expressed by general Rules which become:—

(496.) $$W = \left(\frac{\sqrt{C}}{\sqrt{C}+\sqrt{T}} \times d\right)^2 \times b \times T \div (L \times 4{\cdot}5).$$

(497.) $$W = \left(\frac{\sqrt{T}}{\sqrt{T}+\sqrt{C}} \times d\right)^2 \times b \times C \div (L \times 4{\cdot}5).$$

(498.) $$C = \frac{W \times L \times 4{\cdot}5}{\left\{d - \sqrt{W \times L \times 4{\cdot}5 \div (T \times b)}\right\}^2 \times b}.$$

(499.) $$T = \frac{W \times L \times 4{\cdot}5}{\left\{d - \sqrt{W \times L \times 4{\cdot}5 \div (C \times b)}\right\}^2 \times b}.$$

In which T = the maximum tensile strain, or that at the extreme edge of the section, per square inch.

C = the maximum crushing strain, or that at the extreme edge of the section, per square inch.

d = the depth of rectangular beam, in inches.

b = the breadth ,, ,, ,,

L = length of beam, supported at both ends, in feet.

W = weight in centre of beam.

Of course, T, C, and W must all be taken in the same terms, tons, lbs., &c.

(500.) Applying rule (496) to the case of the beam whose strength was investigated analytically in (494), the length

being 200 inches, or 16·667 feet, T and C each = 10 tons, d = 20 inches, b = 1 inch, we obtain

$$\left(\frac{\sqrt{10}}{\sqrt{10}+\sqrt{10}}\times 20\right)^2 \times 1 \times 10 \div (16\cdot667 \times 4\cdot5) = 13\cdot333 \text{ tons},$$

as before.

With a bar 1 inch square and 1 foot long of the same material, we obtain

$$W = \left(\frac{\sqrt{10}}{\sqrt{10}+\sqrt{10}}\times 1\right)^2 \times 1 \times 10 \div (1 \times 4\cdot5) = \cdot555 \text{ ton}$$

breaking weight in the centre. This it should be observed is $\frac{1}{18}$th of T or C, for ·555 × 18 = 10 tons:—this, however, is true only where the tensile and crushing strengths are equal to one another (638).

(501.) But the Rules in (496), &c., will apply to cases where T and C are unequal, which, as shown by Table 79, is the case with most materials. Thus, with cast iron the mean value of T = 7·142 tons and of C = 43 tons per square inch: then a bar 1 inch square and 1 foot long will break transversely with

$$\left(\frac{\sqrt{43}}{\sqrt{43}+\sqrt{7\cdot142}}\times 1\right)^2 \times 1 \times 7\cdot142 \div (1 \times 4\cdot5) = \cdot8016 \text{ ton},$$

or 1795 lbs. in the centre. The mean transverse strength by experiment is 2063 lbs. (335), hence 2063 ÷ 1795 = 1·15, so that the calculation shows an error of + 15 per cent.

(502.) The equation $\frac{\sqrt{C}}{\sqrt{C}+\sqrt{T}}\times d$ gives the depth of that part of the section subjected to tension; thus with cast iron

$$\frac{\sqrt{43}}{\sqrt{43}+\sqrt{7\cdot142}}\times 1, \text{ or } \frac{6\cdot557}{6\cdot557+2\cdot673}, \text{ or } \frac{6\cdot557}{9\cdot23} = \cdot7104 \text{ inch};$$

hence the depth of the part subjected to compression must be 1·0 − ·7104 = ·2896 inch, and thus we obtain the position of the neutral axis, as in Fig. 168. Similarly, the equation $\frac{\sqrt{T}}{\sqrt{T}+\sqrt{C}}\times d$ gives the depth of that part of the section

subjected to compression, or in our case $\frac{2\cdot673}{2\cdot673 + 6\cdot557}$, or $\frac{2\cdot673}{9\cdot23} = \cdot2896$ inch, as before.'

Again, with Ash, T = 16576 lbs., and C = 9023 lbs. per square inch, and a bar 1 inch square will have as the depth subjected to tension $= \frac{\sqrt{9023}}{\sqrt{9023} + \sqrt{16576}}$, or $\frac{95}{95 + 128\cdot7}$, or $\frac{95}{223\cdot7} = \cdot4247$ inch, hence the depth subjected to compression must be $1\cdot0 - \cdot4247 = \cdot5753$ inch, and we thus obtain the position of the neutral axis, as in Fig. 169.

Of all known materials, glass gives the greatest inequality of T and C, their ratio to one another being 1 to 11·78, as shown by col. 3 of Table 79. With T = 2560 lbs., and C = 30150 lbs. per square inch, the equation $\frac{\sqrt{C}}{\sqrt{C} + \sqrt{T}}$ becomes $\frac{\sqrt{30150}}{\sqrt{30150} + \sqrt{2560}}$, or $\frac{173\cdot6}{173\cdot6 + 50\cdot6}$, or $\frac{173\cdot6}{224\cdot2} = \cdot7734$ inch, which is the depth subjected to tension, hence $1\cdot0 - \cdot7734 = \cdot2266$ inch for the depth subjected to crushing, and we thus obtain the neutral axis, as in Fig. 170.

(503.) The rule (498) for finding C, is of special value as applied to malleable materials such as Wrought Iron, Steel, Gun-metal, and Brass, whose resistance to crushing cannot be determined with precision by experiments conducted in the usual manner, namely, by crushing small specimens by direct pressure, because the ductile and semi-fluid character of such metals enables them to *flow* or expand laterally under pressure to an almost unlimited extent, instead of crushing suddenly into fragments as cast iron and similar materials do (133).

(504.) "*Wrought Iron.*"—In applying the Rules (496), &c., to such materials we obtain the *apparent* rather than the real tensile and crushing strengths; at least this is true with heavy strains approaching the ultimate or breaking loads. It is assumed in the analytical investigation in (494) that the exten-

sions and compressions are proportional to the distance from the neutral axis, and that the tensile and crushing strains are simply proportional to the respective extensions and compressions. This last assumption is practically correct for light strains or even up to half the ultimate weight, with wrought iron and steel, but is very far from the truth with heavier strains as shown by Table 95: plotting col. 3 in a diagram and eliminating the anomalies of experiment by a mean curve we obtain for

14	15	16	17	18	19	20	21	22	23	24

tons per square inch, the approximate extensions in parts of the length, cleared as far as possible from the effect of time (621), are

·002	·0035	·0054	·0076	·0102	·0132	·0164	·0198	·024	·03	·038

The extensions for lower strains are given by col. 4 of Table 96.

(505.) Let Fig. 203 be the section of a bar of wrought iron below the neutral axis when strained as a beam up to the point of rupture, the maximum strain or that at B being say 24 tons per square inch. Now, if the strains were simply as the extensions, we should have the series given by col. A in that figure, proceeding with which as in (495) we obtain column X.

Area.		Lever.		Strain.		X.	Area.		Lever.		Strain.		Z.
1	×	1	×	1·5	=	1·5	1	×	1	×	14·25	=	14·25
1	×	3	×	4·5	=	13·5	1	×	3	×	16·70	=	50·10
1	×	5	×	7·5	=	37·5	1	×	5	×	18·53	=	92·65
1	×	7	×	10·5	=	73·5	1	×	7	×	19·55	=	136·85
1	×	9	×	13·5	=	121·5	1	×	9	×	21·28	=	191·52
1	×	11	×	16·5	=	181·5	1	×	11	×	22·25	=	244·75
1	×	13	×	19·5	=	253·5	1	×	13	×	23·00	=	299·00
1	×	15	×	22·5	=	337·5	1	×	15	×	23·70	=	355·50
						1020·0							1384·62

Thus taking B C and assuming the area = 1·0, as we require proportional numbers only, the mean strain at D becomes 22·5, and the leverage = 15, hence we obtain 1 × 15 × 22·5 = 337·5, &c., &c., as in column X.

But by (504) the extension at B due to 24 tons $= \cdot 038$, therefore at E, or half the distance from the neutral axis, we obtain $\cdot 038 \div 2 = \cdot 019$, which, by the series of strains and corresponding extensions in (504), is due to a strain somewhere between 20 and 21 tons: by interpolation we find the exact strain to be $20 \cdot 76$ tons, as in the figure. Similarly at F, which is $\frac{1}{4}$th of the distance of B from the neutral axis, we obtain $\cdot 038 \div 4 = \cdot 0095$, which is between 17 and 18 tons: by interpolation we obtain $17 \cdot 73$ tons, as in the figure. Calculating in this way, we have obtained the column N: then taking B, C as before, we have $1 \times 15 \times 23 \cdot 7 = 355 \cdot 5$, &c., &c., as in col. Z.

(506.) The result is, that although the maximum strain at B is the same in both cases, namely 24 tons, the mean resistance by col. Z is greater than in col. X in the ratio 1 to $1384 \cdot 62 \div 1020 = 1 \cdot 36$ nearly, or 36 per cent. Therefore, if we would calculate wrought-iron bars by the rules in (496), &c., we must take a *fictitious* value for the maximum strain at B, which would become in effect $24 \times 1 \cdot 36 = 32 \cdot 64$ tons per square inch. Thus, while the real maximum tensile and crushing strain $= 24$, the apparent strain by the ordinary rules is $= 32 \cdot 64$ tons per square inch: see (133).

The near approach to equality in the strains throughout the section, as shown by col. N in Fig. 203, is remarkable: thus at G, $\frac{1}{16}$th of the distance of B from the neutral axis, the strain instead of being $24 \div 16 = 1 \cdot 5$, is $14 \cdot 25$ tons per square inch.

By (374) it is shown that a bar of wrought iron 1 inch square and 1 foot long between bearings, and loaded as a beam, will break down (so far as that point can be definitely fixed) with 4000 lbs. or $1 \cdot 786$ ton in the centre. By the rule in (638) this is equivalent to $1 \cdot 786 \times 18 = 32 \cdot 15$ tons per square inch maximum strain at the top and bottom edges of the section, which far surpasses $25 \cdot 7$ tons, the mean tensile strength of British bar-iron, as determined by direct experiment and given by Table 1. But $32 \cdot 15$ is the *apparent*, not the real, strength of the iron, which by the ratio in (506) is reduced to $32 \cdot 15 \div 1 \cdot 36 = 24$ tons, differing $24 \div 25 \cdot 7 = \cdot 934$ or $1 \cdot 0 - \cdot 934 = \cdot 066$, namely $6 \cdot 6$ per cent. only from the tensile strength by direct experiment (520).

With lower strains, say half the ultimate strength or $24 \div 2 = 12$ tons per square inch, the elasticity of wrought iron is practically perfect, that is to say, the strains are simply proportional to the extensions or to the distances from the neutral axis N. A. We should then have the *ratios* given by col. X in (505), which gives 1020 with the full strain, therefore $1020 \div 2 = 510$ with half the ultimate strength, and by col. Z $1384 \cdot 62$ with the full ultimate strain: hence the ratio becomes $1384 \cdot 62 \times 510 = 2 \cdot 72$ to $1 \cdot 0$: while, therefore, we double the maximum strain (namely 12 to 24 tons), we increase the mean apparent strain not to double only or to 2, but to $2 \cdot 72$ or to $2 \cdot 72 \times 12 = 32 \cdot 64$ tons, being the same as in (506).

(507.) "*Steel.*"—Applying the rules in (496), &c., to steel, we obtain results analogous to those we have found for wrought iron: for example, a bar 1 inch square and 1 foot long breaks down with 6720 lbs., or 3 tons in the centre, as shown in (376). By Table 1, the mean tensile strength of a steel bar $= 47 \cdot 8$ tons per square inch: then the Rule (498) becomes

$$C = \frac{3 \times 1 \times 4 \cdot 5}{\left\{1 - \sqrt{3 \times 1 \times 4 \cdot 5 \div (47 \cdot 8 \times 1)}\right\}^2 \times 1} = 61 \cdot 48 \text{ tons per}$$

square inch *apparent* crushing strength. What the real crushing strength may be we have no means of determining exactly, because the compression of steel by that strain is not accurately known; but by the experiments on steel pillars, the mean resistance to crushing seemed to be 52 tons per square inch (268). If we admit this to be the *real* strain, and $61 \cdot 48$ tons the *apparent* strength under transverse strains, we have the ratio $61 \cdot 48 \div 52 = 1 \cdot 18$, or 18 per cent. difference, being half of that obtained for wrought iron under similar strains.

(508.) Table 79 gives the results of the rules in (496), &c., as applied to many different kinds of materials, compared with the transverse strength, &c., as found by direct experiment. The values of T, C, and W are taken from Tables 1, 31, 32, 66, &c.;—the ratios of T to C, as given by col. 3, vary from 1 to $\cdot 34$ with Willow, to 1 to $11 \cdot 78$ with Glass, and although there are in col. 6 departures from uniformity, they are not greater than might be expected under such extremely variable condi-

TABLE 79.—Of the CONNECTION between the ultimate TRANSVERSE STRENGTH of MATERIALS, and the TENSILE, and CRUSHING STRENGTHS.

Material.	Longitudinal Strength in Lbs. per Square Inch.			Transverse Strength of a Bar 1 Inch Square, 1 Foot Long.		
	Tensile, T.	Crushing, C.	Ratio, C ÷ T.	By Experiment, E.	By Rule, R.	Error, per cent.
Cast Iron	16,000	96,320	6·02	2063	1795	−15·0
" Stirling's No. 2	25,764	119,457	4·64	2835	2666	− 6·0
" " No. 3	23,461	129,876	5·54	2272	2568	+13·0
Wrought Iron: working load	23,940	23,940	1·00	1330	1330	0·0
Steel: working load ..	60,480	60,480	1·00	3360	3360	0·0
Gun-metal	31,360	34,652*	1·105	1830	1830	0·0
Brass	17,970	24,000*	1·335	1150	1150	0·0
Glass	2,560	30,150	11·78	262	340	+30·0
Slate	2,666*	12,062	4·52	421	421	0·0
Alder	14,186	6,896	·486	530	545	+ 2·85
Ash	16,576	9,023	·544	681	673	− 1·17
Beech	14,822	8,548	·577	558	613	+10·00
Deal	17,850	6,602	·370	615	567	− 7·8
Larch	9,560	4,385	·459	380	346	− 9·0
Mahogany	10,818	8,198	·758	589	514	−12·73
Oak, English	12,332	8,271	·670	509	556	+ 9·2
Pine, yellow	13,300	5,410	·407	428	448	+ 4·7
" red	13,300	6,457	·486	491	498	+ 1·4
" pitch	13,300	6,790	·510	577	514	−10·92
Sycamore	13,000	8,144	·626	535	566	+ 5·8
Teak	15,090	10,706	·709	724	700	− 3·32
Willow	13,250	4,513	·340	365	347	− 4·93
	(1)	(2)	(3)	(4)	(5)	(6)

NOTE.—The values marked * have been *calculated* from the transverse strengths in col. 4, &c., &c.

tions. The sum of all the + errors in col. 6 is 76·95, and of the − errors is 70·87, giving an average of (76·95 − 70·87) ÷ 22 = + 0·276 per cent. on the 22 experiments: the greatest + error was + 30 per cent. with Glass, and the greatest − error was − 15 per cent. with Cast Iron.

(509.) It will be interesting and instructive to observe the effect on the transverse strength, of variations in the Tensile and Crushing strengths:—for instance, if by mixture of metals

or otherwise we could double the value of *both* strains, no doubt the transverse strength would be doubled also; but the question is, what would be the effect of a given alteration in one of those strains only?

Table 80 has been calculated by the rules in (496), &c., and shows the effect of changing the value of T and C from 7 and 42 tons per square inch respectively (which are nearly the mean strengths of ordinary cast iron) to 14 and 7 tons. Thus if T could be doubled or increased to 14 tons, while C remained at its normal value of 42 tons, the increase in the Transverse strength, as shown by col. 5, would be 63·3 per cent. If, on the other hand, with T at its normal value of 7 tons per square inch, C is reduced to 7 tons also, the transverse strength would be reduced to ·4962, or about half its normal value; when, as in (500) and (638), the transverse strength is $\frac{1}{18}$th of T or C; for by col. 4, $\cdot 389 \times 18 = 7$ tons, &c. A practical example of this is given by Stirling's iron in (939), where it is shown that the effect of Stirling's process is to increase the tensile strength 74 per cent., and the crushing strength 30 per cent., the result being an increase of 60 per cent. in the transverse strength by experiment, and 59 per cent. by calculation with the rules in (496), &c.

TABLE 80.—Of the TRANSVERSE STRENGTH of CAST IRON, &c., as affected by varying Tensile and Crushing Strength.

Tensile, Tons.	Crushing, Tons.	Transverse Strength.		Ratios.	Ratios.
		Lbs.	Tons.		
7	7	871	·3890	·4962	1·000
7	14	1195	·5344	·6816	1·372
7	21	1400	·6251	·7973	1·674
7	28	1548	·6909	·8812	1·777
7	35	1664	·7427	·9473	1·911
7	42	1756	·7840	1·0000	2·016
8	42	1926	·8600	1·097	2·211
10	42	2258	1·008	1·286	2·592
12	42	2538	1·133	1·445	2·914
14	42	2867	1·280	1·633	3·291
(1)	(2)	(3)	(4)	(5)	(6)

Theoretical Rules.

(510.) Theoretical writers have given rules connecting the transverse strain on a beam with the tensile and crushing strength of the material, based on the assumption that the two latter are equal to one another, and that the extensions and compressions under those strains are also equal. This, however, is not true of any material (616) when the strains are very heavy or approach the breaking weight; but with the *working* loads commonly adopted in practice, say ⅕th to ⅓rd of the breaking weight, those rules are nearly correct, and become of considerable value. With very heavy strains other rules become necessary, and are given in (323).

(511.) For solid square sections of beams we have the rules:—

$$W = \frac{f \times 2 \times D^3}{3 \times l}.$$

$$f = \frac{3 \times W \times l}{2 \times D^3}.$$

(512.) For hollow square sections:—

$$W = \frac{f \times 2 \times (D^4 - d^4)}{3 \times l \times D}.$$

$$f = \frac{3 \times W \times l \times D}{2 \times (D^4 - d^4)}.$$

(513.) For solid Rectangular sections:—

$$W = \frac{f \times 2 \times D^2 \times B}{3 \times l}.$$

$$f = \frac{3 \times W \times l}{2 \times D^2 \times B}.$$

(514.) For hollow Rectangular sections:—

$$W = \frac{f \times 2 \times \{D^3 \times B) - (d^3 \times b\}}{3 \times l \times D}.$$

$$f = \frac{3 \times W \times l \times D}{2 \times \left\{ D^3 \times B) - (d^3 \times b \right\}}.$$

(515.) For solid circular sections:—

$$W = \frac{3 \cdot 1416 \times f \times R^3}{l}.$$

$$f = \frac{W \times l}{3 \cdot 1416 \times R^3}.$$

(516.) For hollow circular sections:—

$$W = \frac{3 \cdot 1416 \times f \times (R^4 - r^4)}{R \times l}.$$

$$f = \frac{W \times l \times R}{3 \cdot 1416 \times (R^4 - r^4)}.$$

(517.) For solid elliptical sections:—

$W = 3 \cdot 1416 \times f \times R_D^2 \times R_B$.

[redacted]

W = the transverse load in the centre of a beam supported at both ends in lbs., tons, &c., including the weight of the beam itself reduced to an equivalent central load (785).

D = the external and d = the internal depth in inches.

B = „ b = „ breadth „

R = „ r = „ radius „

R_D = „ r_D = „ radius or vertical semi-diameter.

R_B = „ r_B = „ radius or horizontal semi-diameter.

l = the length or distance between supports, in inches.

(519.) We may now give some illustrations of the application of these rules:—Say we have a bar of wrought iron 1 inch square and 1 foot long, and assuming that the maximum strain f shall not exceed 12 tons per square inch, which, as shown by the Diagram 215, is about the limit of perfect elasticity for both the tensile and compressive strains, we may find the equivalent transverse strain W by rule (511), which becomes $\frac{12 \times 2 = 1^2}{3 \times 12}$ or $\frac{24}{36}$ = ·6667 ton, or 1500 lbs. in the centre.

If the bar had been a round one, then R = 0·5, and $\cdot 5^3$ being ·125, Rule (515) becomes $\frac{3 \cdot 1416 \times 12 \times \cdot 125}{12} = \cdot 3927$ ton in the centre. Hence the ratio of the strengths of square to round bars is ·6667 ÷ ·3927 = 1·7 to 1·0. This is probably the correct ratio for light strains, and nearly so for all strains with materials whose elasticity is nearly perfect, such as steel and wrought iron, but with cast iron and timber, as shown in (361), (362), the ratio with the breaking weights is more nearly 1·5 to 1·0.

(520.) By col. 6 of Table 66, the working load for a plain bar of wrought iron 1 inch square and 1 foot long = ·594 ton in

the centre; by Rule (511) f becomes in that case $\frac{3 \times \cdot 594 \times 12}{2 \times 1^3}$ or $\frac{\cdot 594 \times 36}{2}$, or $\cdot 594 \times 18 = 10 \cdot 7$ tons per square inch.

But if we apply the same rule to extreme strains in wrought or cast iron bars, we obtain a fictitious value for f, for reasons given in (504), &c.; for example, Table 66 gives 4000 lbs. or $1 \cdot 786$ ton for the value of M_T for the breaking-down load; hence $1 \cdot 786 \times 18 = 32 \cdot 15$ tons per square inch, the *apparent* value of f, which being 36 per cent. in excess of the *real* value, as shown in (506), the latter becomes $32 \cdot 15 \div 1 \cdot 36 = 24$ tons per square inch.

It will be observed that in these cases $f = W \times 18$ simply (639), but that rule will apply to those cases only where the tensile and compressive strains are equal to one another for bars 1 inch square and 1 foot long, &c., but will not be correct for cast iron where those strains are very unequal. For example, a bar of cast iron 1 inch square and 1 foot long breaks with $\cdot 92$ ton in the centre (335); hence $f = \cdot 92 \times 18 = 16 \cdot 56$ tons per square inch. But this is neither the true tensile nor crushing strength of cast iron, which, as found by direct experiment, is $7 \cdot 142$ and 43 tons respectively. In applying these rules to cast iron, f must be taken at the *apparent* value of $16 \cdot 56$ tons per square inch in calculating the breaking loads (504).

(521.) The meaning of this *fictitious* value of f is, that if the transverse strength $= \cdot 92$ ton, and the tensile and crushing strengths are equal to one another, then the value of both would be $16 \cdot 56$ tons per square inch; by Rule (496) we then obtain

$$\left(\frac{\sqrt{16 \cdot 56}}{\sqrt{16 \cdot 56} + \sqrt{16 \cdot 56}} \times 1\right)^2 \times 1 \times 16 \cdot 56 \div 4 \cdot 5 = \cdot 92 \text{ ton.}$$

By taking $16 \cdot 56$ tons for the value of f, the rules in (510) coincide in their results with those in (323), with $\cdot 92$ for the value of M_T in rectangular sections, and $\cdot 92 \div 1 \cdot 7 = \cdot 5412$ for circular and elliptical sections. Thus, for the hollow rectangular beam, Fig. 75, we found in (347) and by rule (330) the breaking weight $= 3 \cdot 558$ tons: by Rule (514) we have

$$W = \frac{16 \cdot 56 \times 2 \times \left\{4 \cdot 04^3 \times 2 \cdot 21) - (3 \cdot 29^3 \times 1 \cdot 46\right\}}{3 \times 72 \times 4 \cdot 04} = 3 \cdot 558$$

tons also.

(522.) Fig. 201 gives the section of six elliptical beams experimented upon by Mr. E. Clark; the length was 6 feet, the maximum breaking weight = 3·595 tons; the minimum = 2·918 tons, and the mean of the whole = 3·207 tons. By rule (518) we obtain

$$W = \frac{3 \cdot 1416 \times 16 \cdot 56 \times \left\{2 \cdot 33^3 \times 1 \cdot 165) - (1 \cdot 955^3 \times \cdot 79\right\}}{72 \times 2 \cdot 33} =$$

2·738 tons. By Rule (334) we have

$$W = \frac{(4 \cdot 66^3 \times 2 \cdot 33) - (3 \cdot 91^3 \times 1 \cdot 58)}{4 \cdot 66} \times \cdot 5412 \div 6 = 2 \cdot 736$$

tons, or practically the same as the other; if we take ·92 ÷ 1·5 = ·6133 for the value of M_T, the ratio 1·5 to 1·0 being, as we have shown (361), more correct than 1·7 to 1·0 for the *breaking weight*, W comes out 3·099 tons; experiment gave 3·207 tons, hence 3·099 ÷ 3·207 = ·9664, giving an error of 1·0 − ·9664 = ·0336 or − 3·36 per cent.

For some reason the analytical method followed in (348) does not give correct results in this case; the area of the section = 3·676 square inches, the maximum tensile strain at A = 7·14 tons per square inch, and the mean at B = 3·57 tons; hence (3·76 × 3·57 × $\frac{4}{3}$ × 2·33 ÷ 36) × 2 = 2·265 tons, showing an error of − 30 per cent.

(523.) If we take for cast iron the working or safe tensile strain at $\frac{1}{3}$rd of the breaking weight, f becomes 7·142 ÷ 3 = 2·381 tons per square inch. Table 92 shows that with 2·355 tons the extension and compression are precisely equal to one another. In that case the neutral axis of a rectangular section will be in the centre and the strain at the top and bottom, or f will also be equal, namely, 2·355 tons per square inch. Under these conditions the rules in (510) are quite correct, as we have shown (617), and by Rule (511) we obtain $W = \frac{2 \cdot 355 \times 2 \times 1^3}{3 \times 12}$ or 2·355 ÷ 18 = ·1314 ton in the

centre only. The *breaking* load by experiment = ·92 ton, or ·92 ÷ ·1314 = 7 times the working load as thus found, so that to secure the equality of strains on which the rules in (510) are based, the transverse load must not exceed $\frac{1}{7}$th of the breaking weight, the tensile strain being then $\frac{1}{3}$rd and the compressive strain $\frac{1}{18}$th of their respective breaking weights.

(524.) As the transverse load is increased, the neutral axis moves towards the edge under compression until the breaking weight is reached, when it becomes as in Fig. 168. When the bar is loaded to $\frac{1}{3}$rd of the breaking weight or ·3067 ton, it is not correct to assume that the tensile and crushing strengths are also strained to $\frac{1}{3}$rd of their respective ultimate resistances, but that, on the contrary, the tensile resistance is much more, and the crushing much less than $\frac{1}{3}$rd of their ultimate values (356), (617).

(525.) In calculating the breaking weights on beams by the rules in (510) it is therefore necessary to take a *fictitious* value for f as we have done for cast iron (520); thus for glass, col. 4 of Table 79 gives W = 262 lbs., therefore f must be taken at 262 × 18 = 4716 lbs., whereas, by cols. 1 and 2, the real values of T and C = 2560 and 30,150 lbs. respectively. Again, for Ash by the same Table, W = 681 lbs. by col. 4, therefore f must be taken at 681 × 18 = 12260 lbs., the real values of T and C being 16,576 and 9023 lbs. respectively.

CHAPTER XIII.

ON ROOFS.

(526.) "*Load on Roofs.*"—The load on roofs is, 1st, the weight of the roof itself, and 2nd, the *vertical* pressure due to the Wind. The weight of the materials of the roof is not simply proportional to the span or area, but increases more rapidly than either; it is composed 1st, of the weight of the principals or trusses; 2nd, of the purlins, &c.; and 3rd, of the slates or other covering. It is shown in (490) that the ratio of

the weight of a beam or truss to its safe load rises with the dimensions, and that to such an extent that there is with every beam a length with which it would break with its own weight and could carry no extra load whatever. Then the ratio of the weight of the purlins, &c., to the area rises with the span also, because it is found expedient to increase the distance between the principals for large roofs; for instance, with roofs, say, 60 feet span, the pitch of the principals would be from 7 to 10 feet, but with very large roofs it might be 30 feet; with such a distance the strength of the cross-beams, purlins, &c., must be very great as compared with those for small roofs. The weight of the slates, &c., would be constant for all spans, and may be taken at 10 lbs. per square foot.

"*Wind.*"—The *horizontal* force of the wind in the greatest hurricane in this country is 80 lbs. per square foot, but the *vertical* pressure, with which we have to deal, is a very uncertain one: it has usually been taken at 40 lbs. per square foot.

(527.) Say we take a roof 60 feet span with trusses 8 feet apart, each truss therefore carries $60 \times 8 = 480$ square feet of roof; now the weight of a truss for such a case would be about 22 cwt. or 2464 lbs., and is equivalent to a pressure of $2464 \div 480 = 5$ lbs. per square foot; the weight of the purlins may be taken at 5 lbs. and of the slates = 10 lbs. per square foot; then allowing 40 lbs. for wind, we obtain a total pressure of $5 + 5 + 10 + 40 = 60$ lbs. per square foot.

(528.) For very large roofs we may take as an example the St. Pancras Station roof, where the straining weight of the truss, 240 feet span, = 35 tons or 78,400 lbs., the distance between the trusses was 30 feet, the area of roof borne by each truss, *measured on plan*, becomes $240 \times 30 = 7200$ square feet, equivalent to $78{,}400 \div 7200 = 9 \cdot 2$, or say 10 lbs. per square foot.

The weight of the covering of slates, purlins, &c., with large roofs is very great, for reasons already given; thus at

New Street, Birmingham,	Charing Cross,	Cannon Street,	Lime Street, Liverpool,	St. Pancras,

the spans being:

211	166	190	212	240 feet

respectively, the estimated weight of the coverings, as given by Mr. W. H. Barlow, was:—

20	37	37	38	36 lbs.

per square foot. The weight of the principals being:—

25	27	37	44	54 tons.

(529.) In the St. Pancras roof the estimated total pressure, exclusive of the strain due to the weight of the truss itself, was taken as 70 lbs. per square foot; the truss, as we have seen, = 10 lbs., hence the total = 80 lbs. per square foot, leaving 34 lbs. for the *vertical* pressure of the wind. We thus have, truss = 10, covering = 36, wind = 34, and the total = 80 lbs. per square foot.

For ordinary roofs, say 60 to 70 feet span, we may, for convenience of calculation, take the total pressure at 56 lbs. or $\frac{1}{2}$ cwt., or $\frac{1}{40}$th ton per square foot, being 5 lbs. for weight of truss, 5 lbs. for purlins, &c., 10 lbs. for slates, and 36 lbs. for wind. Large roofs, however, should in all cases be subjected to special calculation.

(530.) "*Strains on Roofs.*"—Let Fig. 179 be an outline of a truss of the form commonly adopted for wooden roofs of small span, in which, for the purposes of calculation, we have taken the strain as concentrated at certain points: for example, if the weight of that part of the rafter, purlins, slates, &c., between n and p = 10, then half of that weight or 5 is discharged at n, and 5 at p. Similarly the part between m, n gives 5 at m and 5 at p, &c., &c.: we thus obtain 10 at each point, except at the ends where we have 5 only, which last, being discharged *direct* on the supports, will have no effect in straining the truss.

In order to analyse the combined effect of these weights, we may take them separately. In Fig. 176, the weight A = 10, evidently gives 5 at B, and 5 at C: then by the parallelogram of forces, making a, b = 5 by a scale of equal parts, that force is resolved into two forces a, d = 11·18, and a, c = 10, hence the strain on the rafter E = 11·18, and that on the tie-rod D = 10 by the same scale.

In Fig. 177, a weight of 10 at G gives 7·5 at H, and 2·5 at

J, being $1\frac{1}{2}$ and $\frac{1}{2}$ respectively of C and B, we therefore obtain $11 \cdot 18 \times 1\frac{1}{2} = 16 \cdot 77$ at K; $10 \times 1\frac{1}{2} = 15$ at L; $11 \cdot 18 \div 2 = 5 \cdot 59$ at M: and $10 \div 2 = 5$ at P. As N has to bear the thrust of M, and at the same angle, the strain on it will be 5·59 also. Of the strain on K, 5·59 is evidently due to N, leaving $16 \cdot 77 - 5 \cdot 59 = 11 \cdot 18$ due to O, which, being at the same angle as K, will bear 11·18 also. To find the strain on Q we make $e, f = 11 \cdot 18$, and drawing h, f horizontal, $h, e = 5$, which is the strain on Q.

Fig. 178 is a counterpart of Fig. 177, except that the load being now at R, the several strains are as before, but in reversed order.

Now, combining the corresponding strains in Figs. 176, 177, 178, we obtain the combined effect of the whole in Fig. 179: thus, at S we have $11 \cdot 18 + 5 \cdot 59 + 5 \cdot 59 = 22 \cdot 36$: at T, $11 \cdot 18 + 16 \cdot 77 + 5 \cdot 59 = 33 \cdot 54$, &c., &c.

(531.) To facilitate the application of these strains to practice, we may easily find what they would be for a total weight = 100, including the weight of the truss and of the other parts of the roof plus the vertical pressure of the wind. Thus, in Fig. 179, the total load = 40, then by proportion, for a total load of 100, we should have, as in Fig. 180, $33 \cdot 54 \times 100 \div 40 = 84$ at T: $22 \cdot 36 \times 100 \div 40 = 56$ at S, &c., &c.

Calculating in this way, we have obtained the comparative strains in all the Figures, 180 to 184 inclusive. The method of determining the strains we have followed and illustrated is very laborious: Mr. Timmins has published a useful series of designs for Iron Roofs, the speciality of which is that the strains are found by Diagrams of an ingenious but rather complex kind: however, they give with much less labour the same strains as those we have found by analysis.

(532.) We may now apply Figs. 180 to 184 to practice: say we take a roof 30 feet span and 10 feet between principals. The area *on plan* $= 30 \times 10 = 300$ square feet, and with 56 lbs. or $\frac{1}{40}$th ton per square foot, the actual load $= 300 \div 40 = 7 \cdot 5$ tons: then the strain on $T = 84 \times 7 \cdot 5 \div 100 = 6 \cdot 3$ tons: at $S = 56 \times 7 \cdot 5 \div 100 = 4 \cdot 2$ tons: at $O = 28 \times 7 \cdot 5 \div 100 = 2 \cdot 1$ tons, &c., &c.

Having thus found the strains on the several members of the truss, we have now to determine the proper sizes for the Rafters, Struts, and Tie-rods.

"*Rafters.*"—There are two kinds of strain on the principal rafter, namely, the compressive strain as given by the figures, and the transverse strain due to the direct dead weight of the purlins, slates, &c. The latter is comparatively small, and the ordinary position of the ⊤-iron with flange uppermost being that in which a wrought-iron beam of that section is the strongest, as shown in (378), we may safely neglect it in most cases, and have then only to consider the compressive strain.

The rafter is virtually a pillar, or rather *a series* of pillars end-to-end with a varying series of strains, as, for example, in Fig. 184, where the strains are in the ratios 116, 99, 83, and 66: but this pillar is otherwise under peculiar conditions, being supported by the purlins at frequent intervals, and failure by horizontal flexure being thus prevented. Then it is not likely to fail by flexure upwards, because flexure in that direction is resisted by the transverse load. Lastly, it is not likely to fail by flexure downward in a vertical direction, for, as shown in (243), the strength as a pillar is no less than three times that in the other direction.

We may therefore admit that the Rafter is not likely to fail as a pillar in any direction, and we have simply to consider it as subjected to a crushing strain, and duly to proportion the area thereto. The resistance of wrought iron to crushing is very difficult to determine, as shown in (133), but for pillars we found it to be about 19 tons per square inch, ultimate strain (210). Taking 3 for the "Factor of Safety" (886), we should have 6·3 tons safe working load, but in order to avoid the remotest probability of failure by flexure, especially with large roofs where the length is necessarily great, it will be expedient to take for roofs a higher Factor of Safety, say 4, and we then obtain $19 \div 4 = 4\frac{3}{4}$, or say 5 tons per square inch safe strain. Table 81 has been calculated on that basis, and its application to practice is shown in (537).

(533.) "*Struts.*"—The struts are simple pillars unsupported

TABLE 81.—FOR ROOFS: SAFE COMPRESSIVE STRAIN ON WROUGHT T IRON RAFTERS: 5 Tons per Square Inch.

Depth and Width.	Thickness all over in Inches.								
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$
	Safe Load, in Tons.								
$1\frac{1}{2}$	3·44	4·3	..	..	..	..	..	..	..
$1\frac{3}{4}$	4·06	5·1	..	..	..	..	..	..	..
2	4·70	5·6	6·7	..	..	..	..	..	..
$2\frac{1}{4}$	..	6·5	7·7	9·0	..	..	..	..	..
$2\frac{1}{2}$	..	7·2	8·6	10·1	..	..	..	..	..
$2\frac{3}{4}$	..	..	9·6	11·1	12·5	..	..	..	..
3	..	..	10·5	12·1	13·7	..	..	..	..
$3\frac{1}{4}$	..	..	11·5	13·2	15·0	17·0	..	..	..
$3\frac{1}{2}$	..	..	..	14·2	16·2	18·1	..	..	..
$3\frac{3}{4}$	..	..	..	15·5	17·5	19·6	..	..	..
4	..	..	..	16·5	18·7	21·1	23·5	..	..
$4\frac{1}{2}$	..	..	..	18·6	21·2	24·0	26·9	..	..
5	..	..	..	..	23·7	26·5	29·2	..	..
6	..	..	..	..	28·7	32·1	35·5	39·0	42·1
7	..	..	..	..	..	37·7	41·7	45·7	49·6

throughout the length, differing essentially from the Rafters in that respect. Their strength must therefore be calculated by the rules for ordinary ⊥ pillars (243). Being riveted at the ends, they might be regarded as pillars flat at both ends, but that condition supposes that the pillar is pressed between two flat and parallel planes (149), which is more favourable to strength than the actual conditions of an ordinary strut. It will therefore be safer to regard it as a pillar flat at one end, and rounded at the other, for which the value of M_r for rectangular wrought-iron pillars is 150 by Table 34. It is shown in (243) that the strength of a long T pillar is practically equal to that of the top flange alone, forced to fail by flexure in the direction of its largest dimension, flexure in the other being prevented by the rib. Adopting 3 for the "Factor of Safety," $M_p = 50$, then the Rule (234) becomes

(534.) $$W = 50 \times t^{2 \cdot 6} \times b \div L^2.$$

In which t = the largest and b = the smallest dimension of the

top flange of the T iron: L = the length of the strut in feet: and W = the safe load in Tons.

Thus for a wrought T iron strut say $3 \times 3 \times \frac{3}{8}$ we have to calculate simply for a rectangular pillar in which $t = 3$, and $b = \frac{3}{8}$ inch. By col. 3 of Table 35, $3^{2 \cdot 6} = 17 \cdot 4$, and we obtain for a length L = 6 feet, $W = 50 \times 17 \cdot 4 \times \frac{3}{8} \div 36 = 9 \cdot 06$ tons = the safe resistance to flexure. This result requires correction for incipient crushing by the rules in (163): admitting that the *ultimate* resistance of wrought-iron pillars to crushing = 19 tons per square inch (201), and with "Factor" 3, we obtain $19 \div 3 = 6 \cdot 3$ tons per square inch: the area of the *whole* cross-section in our case = 2·11 square inches: hence $C_P = 2 \cdot 11 \times 6 \cdot 3 = 13 \cdot 3$, and $\frac{3}{4}\, C_P = 9 \cdot 97$ tons. Then Rule (164) gives $9 \cdot 06 \times 13 \cdot 3 \div (9 \cdot 06 + 9 \cdot 97) = 6 \cdot 33$ tons, the reduced and correct safe load on the strut. Table 82 has been calculated in this way throughout, and the application to practice is illustrated in (537). The necessity of the correction for incipient crushing is clearly shown by the Table, for instance, the T iron we have just considered, with a length of

TABLE 82.—FOR ROOFS: SAFE COMPRESSIVE

Sizes.	Resistance to Crushing, in Tons.		LENGTH OF STRUT					
			2		3		4	
			SAFE WORKING					
	C_P.	$\frac{3}{4}\, C_P$.	By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.
$1\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{4}$	4·33	3·25	9·0	3·18	4·0	2·38	2·24	1·77
$2 \times 2 \times \frac{5}{16}$	7·26	5·45	23·7	5·93	10·5	4·78	5·92	3·78
$2\frac{1}{2} \times 2\frac{1}{2} \times \frac{5}{16}$	9·26	6·94	42·2	7·95	18·7	6·75	10·5	5·57
$3 \times 3 \times \frac{3}{8}$	13·30	9·97	..	..	36·2	10·43	20·4	8·93
$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$	15·65	11·74	..	..	54·1	12·37	30·5	11·3
$4 \times 4 \times \frac{7}{16}$	20·85	15·6	..	..	..	..	50·2	15·9
$4\frac{1}{2} \times 4\frac{1}{2} \times \frac{1}{2}$	26·8	20·1	..	..	..	..	77·8	21·3
$5 \times 5 \times \frac{1}{2}$	30·0	22·5	..	..	..	..	..	..
$6 \times 6 \times \frac{5}{8}$	44·8	33·6	..	..	..	..	..	..

3 feet, gives 36·2 tons, the resistance to flexure, but when reduced for incipient crushing we have 10·43 tons only. On the other hand, very long pillars of the same section require no such correction: thus, with lengths of 10 and 12 feet, the resistance is that due to flexure simply, as shown by the Table.

(535.) "*Tie-rods.*"—The mean Tensile strength of Wrought iron = 25·7 tons per square inch, by Table 1; but for *welded* bars = 21 tons only; taking the latter and "Factor" 3, we obtain 21 ÷ 3 = 7 tons per square inch safe load, and Table 83 gives the strength of round bars on that basis. The areas are given also, so as to enable rectangular bars to be substituted for round ones if preferred: it should be observed that when tie-rods are screwed at the ends, or where they are punched for keys, they should be bulked up to compensate for the area lost by the key-way, &c.

(536.) "*Strength of Keys, Rivets, &c.*"—The keys and rivets by which the junctions are effected are subjected to a shearing strain (123), namely, single-shear when supported on one side

LOAD on WROUGHT T IRON "STRUTS."

UNSUPPORTED, IN FEET.									
5		6		8		10		12	
LOAD, IN TONS.									
By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.	By Flexure.	Reduced.
1·43	1·32	1·00	1·00	0·56	0·56	0·36	0·36	0·25	0·25
3·79	2·98	2·63	2·36	1·48	1·48	0·95	0·95	0·66	0·66
6·75	4·57	4·69	3·73	2·64	2·55	1·69	1·69	1·17	1·17
13·0	7·53	9·06	6·33	5·10	4·50	3·26	3·26	2·26	2·26
19·5	9·77	13·9	8·48	7·62	6·16	4·88	4·60	3·38	3·38
32·1	14·0	22·3	12·3	12·5	9·27	8·03	7·08	5·58	5·50
49·8	19·1	34·6	17·0	19·5	13·2	12·5	10·3	8·65	8·06
65·6	22·3	45·6	20·1	25·6	16·0	16·4	12·6	11·4	10·1
131·2	35·7	91·2	32·7	51·2	27·0	32·8	22·1	22·8	18·1

TABLE 83.—FOR ROOFS: SAFE TENSILE STRAIN on ROUND BARS: 7 Tons per Square Inch.

Diam.	Area.	Strain.	Diam.	Area.	Strain.	Diam.	Area.	Strain.
		tons.			tons.			tons.
$\frac{1}{2}$	·20	1·37	$1\frac{3}{8}$	1·48	10·4	$2\frac{3}{4}$	5·94	41·5
$\frac{9}{16}$	·25	1·74	$1\frac{1}{2}$	1·77	12·4	$2\frac{7}{8}$	6·49	45·4
$\frac{5}{8}$	·31	2·15	$1\frac{5}{8}$	2·07	14·5	3	7·07	49·4
$\frac{11}{16}$	·37	2·60	$1\frac{3}{4}$	2·40	16·8	$3\frac{1}{8}$	7·67	53·7
$\frac{3}{4}$	·44	3·09	$1\frac{7}{8}$	2·76	19·3	$3\frac{1}{4}$	8·30	58·1
$\frac{13}{16}$	·52	3·63	2	3·14	22·0	$3\frac{3}{8}$	8·95	62·6
$\frac{7}{8}$	·60	4·21	$2\frac{1}{8}$	3·55	24·8	$3\frac{1}{2}$	9·62	67·3
$\frac{15}{16}$	·69	4·83	$2\frac{1}{4}$	3·98	27·8	$3\frac{5}{8}$	10·32	72·2
1	·78	5·50	$2\frac{3}{8}$	4·43	31·0	$3\frac{3}{4}$	11·04	77·3
$1\frac{1}{8}$	·99	6·96	$2\frac{1}{2}$	4·91	34·3	$3\frac{7}{8}$	11·79	82·5
$1\frac{1}{4}$	1·23	8·60	$2\frac{5}{8}$	5·41	37·8	4	12·56	87·9

only; and double-shear when supported on both sides. It is shown in (123) that the tensilè and shearing strains are equal to one another; therefore Table 83 may be used for rivets and keys, as well as for tie-rods; for instance, a $\frac{5}{8}$-inch rivet gives 2·15 tons for single, and 4·3 tons for double-shear, &c.

(537.) "*Practical Application.*"—We may now illustrate the application of the Figures and Tables to practice: say we have a truss like Fig. 184, 60 feet. span, with 12 feet between each truss; then the area of roof *on plan* = 60 × 12 = 720 square feet and the total load being 56 lbs., or $\frac{1}{40}$ ton per square foot, as in (529), we have 720 ÷ 40 = 18 tons on the truss. Then on Rafter D we have 116 × 18 ÷ 100 = 20·5 tons, requiring by Table 81 a T iron $4\frac{1}{2} \times 4\frac{1}{2} \times \frac{1}{2}$: for C, 99 × 18 ÷ 100 = 18·8 tons, or $4 \times 4 \times \frac{1}{2}$: for B, 83 × 18 ÷ 100 = 15 tons, or $3\frac{1}{4} \times 3\frac{1}{4} \times \frac{1}{2}$: and for A, 66 × 18 ÷ 100 = 11·9 tons, or say $3 \times 3 \times \frac{7}{16}$. In practice, however, the rafter would usually be in one piece from end to end, and in that case the maximum sizes, or those for D, would be used throughout.

Then, for the struts; the strain on G = 23 × 18 ÷ 100 = 4·14 tons, and the length being in our case about 12 feet, we require by Table 82 a T iron say $3\frac{3}{4} \times 3\frac{3}{4} \times \frac{3}{8}$. To show how serious an error would ensue if we had disregarded the *length*,

and provided for the crushing strain simply, Table 81 would have given for 4·14 tons a T iron $1\frac{1}{2} \times 1\frac{1}{2} \times \frac{5}{16}$. But Table 82 shows that the safe strain on a bar nearly of those sizes, and 12 feet long, would be 0·25 ton only, or less than $\frac{1}{16}$th of 4·14 tons, the actual strain.

(538.) It has been sometimes proposed to substitute for T iron struts two flat bars of equal area, but in the case of long struts this is a most unsafe proceeding: for example, by exact calculation the angle-iron we have proposed for G, namely $3\frac{3}{4} \times 3\frac{3}{4} \times \frac{3}{8}$ gives by Rule (534), $W = 50 \times 3\frac{3}{4}^{2\cdot 6} \times \frac{3}{8} \div 144 = 4\cdot 05$ tons, which is near enough to 4·14 tons, the actual strain, for our purpose. But a flat bar left to itself would of course fail by bending in the direction of its *least* dimension (177): hence $t = \frac{3}{8}$, and $b = 3\frac{3}{4}$, and the 2·6 power of $\frac{3}{8}$, being ·0781 by Table 35, the same rule gives $W = 50 \times \cdot 0781 \times 3\frac{3}{4} \div 144 = 0\cdot 1017$ ton for one bar, or 0·2034 ton for a pair having the same area as the T iron, but $\frac{1}{20}$th! only of the strength as a pillar.

To resume: for the strut F we obtain $19 \times 18 \div 100 = 3\cdot 42$ tons, and the length being about 10 feet, we require a T iron say $3 \times 3 \times \frac{3}{8}$: for E we have $16 \times 18 \div 100 = 2\cdot 88$ tons, and the length being about 8 feet, we require a T iron, say $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{5}{16}$.

Then for the tie-rods: the strain on $L = 105 \times 18 \div 100 = 18\cdot 9$ tons, or $1\frac{7}{8}$ inch diameter by Table 83: for M, $89 \times 18 \div 100 = 16$ tons, or $1\frac{3}{4}$ inch; for N, $74 \times 18 \div 100 = 13\cdot 3$ tons, or $1\frac{5}{8}$ inch: for K, $46 \times 18 \div 100 = 8\cdot 28$ tons, or $1\frac{1}{4}$ inch: for J, $12\cdot 5 \times 18 \div 100 = 2\cdot 25$ tons, or $\frac{11}{16}$ inch: and for H, $6\cdot 5 \times 18 \div 100 = 1\cdot 17$ ton, or $\frac{1}{2}$ inch diameter, &c.

Any of the different forms of truss shown by Figs. 180 to 184 may be used in the manner we have illustrated: it will be found that in order to obtain convenient sections for rafters and struts, Fig. 181 should be restricted to say 30 to 40 feet span: Fig. 182 from 40 to 50 feet: Fig. 183 from 50 to 60 feet: and Fig. 184 from 60 to 70 feet, &c.

(539.) "*Curved Roofs.*"—For large spans, such as Railway Stations, &c., Curved Roofs are now very extensively used: Fig. 185 gives an outline of such a roof with the strains

throughout as due to a total weight = 100, or 50 on each support, in order to assimilate the case to the other examples we have given, and to facilitate calculation of such roofs for various spans.

Say we have a Roof 80 feet span, with 16 feet between the principals: then measured on plan, we have $80 \times 16 = 1280$ square feet on each principal: taking the load at 56 lbs. or $\frac{1}{40}$ ton per square foot as in (529), we have $1280 \div 40 = 32$ tons total load.

The maximum compressive strain on the main rafter or upper ribs A, B, C, will then be $137 \times 32 \div 100 = 43{\cdot}8$ tons, for which Table 81 gives a ⊤ section $7 \times 7 \times \frac{11}{16}$ inches, and in most cases that same section would be used throughout, although the strains on D, E are somewhat less.

				Tons.		In. diam.	
Then for	F,	we have	$32 \times 137 \div 100 =$	$43{\cdot}8$,	requiring	$2\frac{7}{8}$.	Table 83.
„	G,	„	$32 \times 125 \div 100 =$	40	„	$2\frac{3}{4}$.	„
„	H,	„	$32 \times 110 \div 100 =$	35	„	$2\frac{1}{2}$.	„
„	J,	„	$32 \times 78 \div 100 =$	25	„	$2\frac{1}{8}$.	„
„	N,	„	$32 \times 9 \div 100 =$	$2{\cdot}88$	„	$\frac{3}{4}$.	„
„	P,	„	$32 \times 15 \div 100 =$	$4{\cdot}8$	„	$\frac{15}{16}$.	„
„	R,	„	$32 \times 30 \div 100 =$	$9{\cdot}6$	„	$1\frac{3}{8}$.	„

The struts M, O, Q, are pillars of the approximate lengths, in our case, of 8, 8, and 5 feet respectively: then

	Tons.				
$M = 32 \times 25 \div 100 =$	8,	requiring	⊤ iron	$4 \times 4 \times \frac{7}{16}$,	by Table 82.
$O = 32 \times 18 \div 100 =$	$5{\cdot}76$	„	„	$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$	„
$Q = 32 \times 13 \div 100 =$	$4{\cdot}16$	„	„	$2\frac{1}{2} \times 2\frac{1}{2} \times \frac{5}{16}$	„

(540.) "*Wooden Roofs.*"—Iron is now used almost exclusively for large and important Roofs; but for ordinary purposes wood is still extensively employed, and is likely to be so for reasons of convenience and economy. The form of truss commonly used for small spans, say up to 30 feet, is shown by Fig. 179, another convenient form is shown by Fig. 181. The strains may be found by the methods already explained and illustrated with Iron roofs. The proportions of the different parts and the details of construction, are essentially practical questions, which are fully considered in most of the works on Carpentry, such as that of Tredgold and others.

CHAPTER XIV.

ON THE TORSIONAL STRAIN.

(541.) It will be expedient to consider the Torsional strain under two different heads, namely "Torsional Strength," and "Torsional Elasticity." It is the more necessary to follow that course because the laws governing the Strength, differ entirely from those dominating the Stiffness. We shall deal with the Torsional Strength in this chapter and with the Torsional Elasticity in Chapter XVII.

"*Torsional Strength.*"—The fundamental laws for Torsional Strength may be expressed by the Rules:—

For Circular sections,

(542.) $$W \times L = Q \times R^3 \times 3 \cdot 1416 \div 2;$$

(543.) $$\text{or, } W \times L = Q \times R^3 \times 1 \cdot 5708.$$

For Square sections,

(544.) $$W \times L = Q \times S^3 \times \sqrt{2} \div 6;$$

(545.) $$\text{or, } W \times L = Q \times S^3 \times \cdot 2357.$$

For Rectangular sections,

(546.) $$W \times L = Q \times d^2 \times b^2 \div (\sqrt{d^2 \times b^2 \times 3}).$$

In which R = radius of circular sections in inches: S = side of square in inches: d and b, depth and breadth of Rectangular sections in inches: L = the leverage in inches with which the twisting weight W acts: W = weight, say in lbs.: Q = constant Multiplier, which has the same value for all the three forms of section, and is found from Experiment by the Rules:—

Circular sections,

(547.) $$Q = \frac{W \times L \times 2}{3 \cdot 1416 \times R^3};$$

(548.) $$\text{or } Q = \frac{W \times L}{1 \cdot 5708 \times R^3}.$$

Square sections,

(549.) $$Q = \frac{W \times L \times 6}{\sqrt{2} \times S^3};$$

(550.) $$\text{or, } Q = \frac{W \times L}{\cdot 2357 \times S^3}.$$

Rectangular sections,

(551.) $$Q = \frac{W \times L \times 3 \times \sqrt{d^2 + b^2}}{d^2 \times b^2}.$$

(552.) "*Ratio of Round to Square.*"—The Ratio of the Torsional Strength of round and square sections may be found by Rules (543) and (545), for while round bars $= 1 \cdot 5708 \times R^3$, square ones $= \cdot 2357 \times S^3$. Say we take 1-inch bars, then $R = \frac{1}{2}$, and $\frac{1}{2}^3 = \frac{1}{8}$ or $\cdot 125$, hence the ratio of strength will be $(\cdot 2357 \times 1^3) \div (1 \cdot 5708 \times \cdot 125) = 1 \cdot 20$ to $1 \cdot 0$, showing that square bars are 20 per cent. stronger than round ones.

But, when we calculate the values of Q for round and square bars from experiment, as in col. 4 of Table 84, we find that the latter are 34 per cent. greater than the former; thus $35907 \div 26800 = 1 \cdot 34$. So great a difference shows that there is some error in the Rules, although they are based on laws given by the highest authorities: admitting the experiments to be correct, the Rules require modification, and become:—

For Square sections,

(553.) $$W \times L = Q \times S^3 \times \cdot 3158.$$

(554.) $$Q = \frac{W \times L}{\cdot 3158 \times S^3}.$$

Calculating Q by Rule (554) we obtain col. 5 of Table 84: the mean = 26,800, or nearly the same as for the round bars which gave by Rule (542), 26,709. From this it appears that the experimental ratio of the strength of square and round bars is $1 \cdot 6$ to $1 \cdot 0$, the theoretical ratio being $1 \cdot 2$ to $1 \cdot 0$.

(555.) "*Practical Rules.*"—The theoretical rules are inconvenient, although they are fundamental, giving the laws governing the Torsional Strength, and have the advantage of a

value for Q, which is, or should be, constant for all sections. The following Practical Rules are based on the theoretical ones, corrected by experiment (552):—

For Circular Sections,

(556.) $$W = M_t \times D^3 \div L.$$

(557.) $$D = \sqrt[3]{W \times L \div M_t}.$$

(558.) $$M_t = W \times L \div D^3.$$

For Square Sections,

(559.) $$W = M_t \times S^3 \times 1\cdot 6 \div L.$$

(560.) $$S = \sqrt[3]{W \times L \div (M_t \times 1\cdot 6)}.$$

(561.) $$M_t = W \times L \div (S^3 \times 1\cdot 6).$$

For Rectangular Sections,

(562.) $$W = M_t \times d^2 \times b^2 \times 2\cdot 264 \div (\sqrt{d^2 + b^2} \times L).$$

(563.) $$M_t = W \times L \times \sqrt{d^2 + b^2} \div (d^2 \times b^2 \times 2\cdot 264).$$

In which M_t = a constant, having the same value for all the sections: D = diameter: S = side of square: d = depth, and b = breadth of rectangular sections: L = leverage, all in inches: W = weight in lbs. acting with the length of lever, L.

(564.) The values of M_t in col. 6 of Table 84 have been calculated by Rules (558) and (561): thus, the bar 4 inches diameter by Rule (558) gives $M_t = 1938 \times 170 \div 64 = 5148$. Again, the bar 1 inch square, which broke with 231 lbs. and 36 inches leverage, gives by Rule (561), $M_t = 231 \times 36 \div (1^3 \times 1\cdot 6) = 5197$, &c. The mean for 9 bars varying from 2 inches to $4\frac{1}{4}$ inches diameter = 5290: and the mean for 6 square bars = 5290 also, which may therefore be taken as the mean value of M_t for cast iron.

The experimental ratio of square to round bars, namely, 1·6

TABLE 84.—OF EXPERIMENTS ON TORSIONAL STRENGTH.

Material.	Sizes.	Breaking Weight in Lbs.	Leverage.	Rules (547), (549). Q.	Rule (554). Q.	M_t.
	inches.		inches.			
Cast iron.	2 diam.	250	170	..	27,050	5310
"	$2\frac{1}{4}$ "	384	"	..	29,190	5730
"	$2\frac{1}{2}$ "	408	"	..	22,610	4440
"	$2\frac{3}{4}$ "	700	"	..	29,140	5720
"	$3\frac{1}{4}$ "	1170	"	..	29,510	5800
"	$3\frac{1}{2}$ "	1240	"	..	25,030	4920
"	$3\frac{3}{4}$ "	1662	"	..	27,290	5360
"	4 "	1938	"	..	26,220	5150
"	$4\frac{1}{4}$ "	2158	"	..	24,340	5180
				Mean =	26,709	5290
"	$1\frac{1}{16}$ square.	330	36	42,020	31,360	6190
"	$1\frac{1}{16}$ "	310	"	39,474	29,460	5815
"	1 "	237	"	36,198	27,020	5332
"	1 "	218	"	33,296	24,850	4905
"	1 "	191	"	29,172	21,780	4298
"	1 "	231	"	35,282	26,330	5197
			Means=	35,907	26,800	5290
Hornbeam	1 diam.	70	12	..	..	840
Mahogany	1 "	47·3	"	..	..	568
Elm	1 "	43	"	..	..	516
Ash	1 "	42·3	"	..	..	508
Oak, English	1 "	37·3	"	..	..	448
Chestnut ..	1 "	37	"	..	..	444
Pine, Yellow	1 "	27·3	"	..	..	328
	(1)	(2)	(3)	(4)	(5)	(6)

to 1·0, has been adopted in these Rules, instead of the Theoretical Ratio 1·2 to 1·0 (552).

(565.) "*Rectangular Sections.*"—The Rules for Rectangular Sections may be used for square ones, which of course are rectangular with equal sides: say we take a 3-inch square bar with 60 inch leverage: then taking $M_t = 5290$, Rule (559) gives $W = 5290 \times 3^3 \times 1 \cdot 6 \div 60 = 3809$ lbs.: taking it as a rectangular bar, Rule (562) becomes $W = 5290 \times 3^2 \times 3^2 \times 2 \cdot 264 \div (\sqrt{3^2 + 3^2} \times 60) = 3809$ lbs. also.

When the two dimensions of a bar, rectangular in section, are very unequal, or the breadth very great in proportion to the

thickness, the torsional strength is practically as the breadth simply, the thickness being constant. Thus, for cast iron, with $L = 10$, and a thickness of 1 inch and breadths

1	2	4	8	16 inches,

the torsional strength, or W, becomes by Rule (562) =

847	2142	4648	9508	19,127 lbs.

It will be observed that, while the breadths are in the ratio of 2 to 1 throughout, the strength for the greater breadths follow nearly the same ratio.

(566.) "*Hollow Sections.*"—We have seen by Rules (556) and (559) that with square and round bars the torsional strength is directly proportional to D^3 or S^3, so that for diameters in the ratio :—

1	2	3	4	5

the Torsional strength would be in the ratio :—

1	8	27	64	125

By the laws of Torsional elasticity (716) the Stiffness with constant strain is *inversely* proportional to D^4 or S^4, hence in our case D^4 being in the Ratio :—

1	16	81	256	625

the stiffness with constant strain will be in the inverse ratio, or :—

1	$\frac{1}{16}$	$\frac{1}{81}$	$\frac{1}{256}$	$\frac{1}{625}$

But when the strain is proportional to the strength, the Ratio of the torsional angles become :—

	1	$8 \times \frac{1}{16}$	$27 \times \frac{1}{81}$	$64 \times \frac{1}{256}$	$125 \times \frac{1}{625}$
or	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

which is in inverse proportion to the arithmetical ratio of the diameters simply.

(567.) "*Old Rule.*"—The old rule commonly used by practical men is $W = D^3 - d^3$, D being the external, and d = the

internal diameter of a hollow shaft, which is regarded as composed of two shafts having the respective diameters, and it is assumed that, deducting the strength of one from that of the other, we should obtain the strength of the annulus. But a shaft will not yield the full strength due to it, except it be allowed to twist proportionately to its diameter. Say we take a section 2 inches diameter externally and 1 inch internally; then by the old rule the strength will be $2^3 - 1^3 = 7$: but we have seen by the preceding investigation that the diameters, being 2 and 1, the twist should be $\frac{1}{2}$ to 1, that is to say, the internal section should twist twice as much as the external one, which of course is impossible in our case, for obviously both must be twisted to the same extent. As the internal section is allowed to twist to half only of the extent due to its full strength, it will yield half only of that strength; hence we obtain $2^3 - (1^3 \times \frac{1}{2}) = 7 \cdot 5$ for the hollow section, instead of $2^3 - 1^3 = 7$, as by the old Rule.

(568.) Hence the rule for hollow circular sections becomes $W = (D^4 - d^4) \div D$, or in our case $(2^4 - 1^4) \div 2 = 7 \cdot 5$, as before. Taking another case of a hollow shaft with diameters 4 and 3: then by the old rule we should have $4^3 - 3^3 = 37$: but by the preceding analysis, the ultimate twist for diameters in the ratio 3 and 4 = $\frac{1}{3}$ and $\frac{1}{4}$ respectively; hence, instead of the internal diameter yielding $3^3 = 27$, it would give $27 \times \frac{1}{4} \div \frac{1}{3}$, or $27 \times 3 \div 4 = 20 \cdot 25$ only, and the strength of the hollow shaft becomes $4^3 - 20 \cdot 25$, or $64 - 20 \cdot 25 = 43 \cdot 75$, instead of 37. The rule $(D^4 - d^4) \div D$ gives in our case $(4^4 - 3^4) \div 4 = 43 \cdot 75$, as before. These principles apply to the transverse strain also (337).

We have therefore for hollow shafts the rules:—

(569.) For Circular Sections,

$$W = M_t \times \frac{D^4 - d^4}{D} \div L.$$

(570.) For Square Sections,

$$W = M_t \times \frac{S^4 - s^4}{S} \times 1 \cdot 6 \div L.$$

In which D = the external, and d = the internal diameter in inches: S = the external, and s = the internal dimensions of square sections in inches: W = the straining weight in lbs. acting with a leverage of L, in inches: M_t = a constant whose value is given by col. 6 of Table 84, and by (564).

To illustrate the application of these rules: say we have a hollow cast-iron shaft in which D = 6 inches, $d = 4$ inches, L = 84 inches: then Rule (569) becomes $W = 5290 \times \frac{6^4 - 4^4}{6} \div 84 = 10900$ lbs. breaking weight. Again, with a hollow square shaft in which S = 5 inches, $s = 3\frac{1}{2}$ inches, and L = 96 inches: then by Table 85, $5^4 = 625$, and $3\frac{1}{2}^4 = 150$, with which Rule (570) gives $W = 5290 \times \frac{625 - 150}{5} \times 1 \cdot 6 \div 96 = 8376$ lbs.

TABLE 85.—Of the FOURTH POWER of NUMBERS.

N	N^4	N	N^4	N	N^4	N	N^4
1	1·00	$3\frac{1}{4}$	112	$5\frac{1}{2}$	915	$7\frac{3}{4}$	3607
$1\frac{1}{8}$	1·60	$3\frac{3}{8}$	130	$5\frac{5}{8}$	1000	$7\frac{7}{8}$	3846
$1\frac{1}{4}$	2·44	$3\frac{1}{2}$	150	$5\frac{3}{4}$	1093	8	4096
$1\frac{3}{8}$	3·57	$3\frac{5}{8}$	173	$5\frac{7}{8}$	1191	$8\frac{1}{8}$	4358
$1\frac{1}{2}$	5·06	$3\frac{3}{4}$	198	6	1296	$8\frac{1}{4}$	4632
$1\frac{5}{8}$	6·97	$3\frac{7}{8}$	228	$6\frac{1}{8}$	1408	$8\frac{3}{8}$	4920
$1\frac{3}{4}$	9·38	4	256	$6\frac{1}{4}$	1526	$8\frac{1}{2}$	5293
$1\frac{7}{8}$	12·4	$4\frac{1}{8}$	289	$6\frac{3}{8}$	1652	$8\frac{5}{8}$	5534
2	16·0	$4\frac{1}{4}$	326	$6\frac{1}{2}$	1785	$8\frac{3}{4}$	5861
$2\frac{1}{8}$	20·4	$4\frac{3}{8}$	366	$6\frac{5}{8}$	1926	$8\frac{7}{8}$	6204
$2\frac{1}{4}$	25·6	$4\frac{1}{2}$	410	$6\frac{3}{4}$	2076	9	6561
$2\frac{3}{8}$	31·8	$4\frac{5}{8}$	458	$6\frac{7}{8}$	2140	$9\frac{1}{8}$	6933
$2\frac{1}{2}$	39·1	$4\frac{3}{4}$	509	7	2401	$9\frac{1}{4}$	7321
$2\frac{5}{8}$	47·5	$4\frac{7}{8}$	565	$7\frac{1}{8}$	2577	$9\frac{3}{8}$	7725
$2\frac{3}{4}$	57·2	5	625	$7\frac{1}{4}$	2763	$9\frac{1}{2}$	8145
$2\frac{7}{8}$	68·3	$5\frac{1}{8}$	690	$7\frac{3}{8}$	2959	$9\frac{5}{8}$	8582
3	81·0	$5\frac{1}{4}$	760	$7\frac{1}{2}$	3164	$9\frac{3}{4}$	9036
$3\frac{1}{8}$	95·4	$5\frac{3}{8}$	835	$7\frac{5}{8}$	3380	$9\frac{7}{8}$	9509

(571.) "*Wrought Iron and Steel.*"—According to W. and D. Rankine's experiments, the torsional strengths of Cast iron,

Wrought iron, and Steel are in the ratio 1, 2, 3, and these, it will be observed, are nearly the Ratios of the Transverse strengths of the same materials. The values of M_t are therefore 5290, 10,580, and 15,870 lbs. respectively.

It follows from this that for *large* shafts (575) of equal torsional strength in cast iron, wrought iron, and steel, the diameters must be in the ratios 1·0, ·7937, and ·6934 respectively, or, in round numbers, a 10-inch cast iron, 8-inch wrought iron, and 7-inch steel shaft would all have the same torsional *strength*, irrespective of stiffness. The specific strengths being in the ratio 1, 2, 3, we then have for cast iron $10^3 \times 1 = 1000$: wrought iron, $8^3 \times 2 = 1024$: and Steel $= 7^3 \times 3 = 1029$, &c., which are practically equal.

(572.) "*Shafting.*"—The determination of the proper sizes of shafting for driving machinery is pre-eminently a practical question requiring special Rules: instead of straining weights in lbs. acting with certain leverages in inches as in ordinary rules for Torsional Strain, we shall have to deal with Horse-power and Revolutions per minute.

Theoretically, the Horse-power commonly accepted as a Standard is 33,000 foot-pounds, or 33,000 lbs. raised 1 foot high per minute; thus, say we have a shaft with a winding drum at the end of it raising a ton or 2240 lbs. with a velocity of 206 feet per minute, which will be equal to $2240 \times 206 \div 33000 = 14$ Horse-power. The question thus appears to be a very simple one, but in practice there are unavoidable complications due to the friction of the machinery and other conditions, by which the whole subject is very much mystified. For example, say that instead of a simple shaft as we have supposed, the necessities of the case were such that a train of wheels, shafts, &c., intervened between the Engine and the winding drum, such that its friction added say 7 Horse-power to the strain on the Engine: then we should require $14 + 7 = 21$ Horse-power to do 14-Horse useful work. Let us further suppose that the power exerted by the Engine in overcoming its own friction and other sources of loss within itself is equal to 7 Horse-power: then the total *gross* power exerted by the *piston* is $14 + 7 + 7 = 28$-Horse. We have thus obtained three dif-

ferent Horse-powers, 28, 21, and 14 respectively, and the question now is, which of these should be taken as a basis in calculating the strength of the shaft; but we may observe that it is unimportant which is taken, if only the value of the Multiplier is properly adapted to the conditions assumed.

(573.) Engineers usually rate the power of an Engine by the useful work done by it: in our case it would be 14-Horse Reputed or Nominal Power. To give more precision to the matter, say that the Engine was a common High-pressure one, cylinder 12 inches diameter, velocity of piston 220 feet per minute, pressure of steam in Boiler 45 lbs., reduced to 40 lbs. in the cylinder at the commencement of the stroke, cut off by lap of slide at $\frac{7}{10}$ of the stroke, and thereby further reduced by expansion to a *mean* pressure of 37 lbs. throughout the stroke: these conditions prevailing when all the work was being done, and an "Indicator" diagram showing 37 lbs. mean pressure as calculated above. We then obtain for a 12-inch cylinder $= 113$ square inches area, $113 \times 37 \times 220 \div 33000 = 28$ *gross* indicated Horse-power: with the Engine and Gearing alone, no useful work being done, the mean pressure $= 18\frac{1}{2}$ lbs., and we obtain $113 \times 18\frac{1}{2} \times 220 \div 33000 = 14$-Horse: with the Engine alone the mean pressure $= 9\frac{1}{4}$ lbs. $= 113 \times 9\frac{1}{4} \times 220 \div 33000 = 7$-Horse.

Hence we have 28 *gross* indicated Horse-power: $28 - 7 = 21$ *Net* indicated Horse-power, and $21 - 7 = 14$-Horse Nominal Power.

(574.) The nominal Horse-Power is the Standard commonly adopted by Practical men: in the vast majority of cases it is the only one known, for the "Net Indicated" power can only be found by experiment: in all cases it must be in excess of the Nominal in order to cover the friction of the machinery by which the work is done, 50 per cent., as in our illustration, is a fair addition in ordinary cases. Thus, the Ratio of the Nominal, Net Indicated, and Gross indicated Horse-power is 1, $1\frac{1}{2}$, and 2; admitting these Ratios, the value of the Multiplier may be easily adapted to either at pleasure.

(575.) It has been found by experience that the power which a *large* shaft will carry satisfactorily depends on its absolute

torsional *strength*, which, other things being equal, is governed by d^3; but the power of small shafts depends on torsional *stiffness*, which is dominated by D^4 (713).

Taking the Nominal Horse-power as the basis, we have for ordinary wrought-iron shafts of large diameter (or those above $4\frac{5}{8}$ inches) the Rule:—

(576.) $$H = d^3 \times R \div 160.$$

(577.) $$d = \sqrt[3]{(H \times 160 \div R)}.$$

For small shafts (or those under $4\frac{5}{8}$ inches) the rules become:—

(578.) $$H = d^4 \times R \div 740.$$

(579.) $$d = \sqrt[4]{(H \times 740 \div R)}.$$

In which d = diameter of the shaft in inches: R = revolutions per minute: and H = Nominal Horse-power. It will be found that these two sets of Rules coincide in their results when the diameter = $4\frac{5}{8}$ inches: for example, with say $R = 100$, rule (576) gives $H = 4\frac{5}{8}^3 \times 100 \div 160 = 61 \cdot 82$ Horse-power; and Rule (578) becomes $H = 4\frac{5}{8}^4 \times 100 \div 740 = 61 \cdot 82$ Horse-power also.

(580.) The fact that strength and stiffness follow different laws necessitates the use of two sets of Rules for shafts; with diameters above $4\frac{5}{8}$ inches, shafts whose absolute torsional strength is sufficient to carry the power will be stiff enough to do ordinary work properly; but below that size, in order to obtain the proper stiffness, the diameter must be larger than necessary for the mere torsional strength. A shaft may be strong enough to resist the torsional strain upon it without twisting asunder, but may be so elastic because of its great length as to be wholly unfit to drive any machinery in which steadiness of motion is essential. On the other hand, a shaft may be stiff enough to do its work because of its extreme shortness, but its strength may not be sufficient to resist the twisting strain.

(581.) The Torsional strength and stiffness of cast iron are

only about half those of wrought iron (571), (720), but for shafts we shall adopt the lower ratio 9 to 14. The maximum strain on the crank shaft of a single steam Engine is 1·57 times the mean strain, but for a pair of Engines coupled by cranks at a right angle, or 90°, it is 1·11, the mean strain being 1·0, &c. Combining all these results, we have in Table 86 a series of Multipliers for the "Nominal," "Net Indicated," and "Gross Indicated" Horse-power of ordinary and crank shafts for large and small sizes, large shafts being above and small ones below $4\frac{5}{8}$ inches diameter.

TABLE 86.—Of the VALUES of M_S and M_L for SHAFTS.

Kind of Work.	Horse-Power.					
	Nominal.		Net Indicated.		Gross Indicated.	
	Small. M_S.	Large. M_L.	Small. M_S.	Large. M_L.	Small. M_S.	Large. M_L.
Ordinary Wrought-iron Shafts	740	160	493	107	370	80
„ Cast-iron Shafts ..	1150	255	767	170	575	128
Wrought-iron Crank-shafts: Single Engine	1160	257	773	171	580	129
Cast-iron Crank-Shafts: Single Engine	1805	400	1200	267	900	200
Wrought-iron Crank-shafts: Pair of Engines	820	182	547	121	410	91
Cast-iron Crank-Shafts: Pair of Engines	1272	282	848	188	636	141
	(1)	(2)	(3)	(4)	(5)	(6)

NOTE.—Shafts under $4\frac{5}{8}$ inches—Small: above $4\frac{5}{8}$ inches—Large.

We then have for diameters above $4\frac{5}{8}$ inches the Rules:—

(582.) $$H = d^3 \times R \div M_L.$$

(583.) $$d = \sqrt[3]{(H \times M_L \div R)}.$$

For small shafts, or those less than $4\frac{5}{8}$ inches diameter, the Rules become:—

(584.) $$H = d^4 \times R \div M_S.$$

(585.) $$d = \sqrt[4]{(H \times M_S \div R)}.$$

In which M_L and M_S have the values given by Table 86;

H = the Horse-power, which may be Nominal, Net Indicated, or Gross Indicated at discretion: R = revolutions per minute, and d = diameter in inches, &c.

(586.) We may now give examples of the application of these Rules: thus for a 4-inch ordinary wrought-iron shaft, R being say 120, &c.; the diameter being less than $4\frac{5}{8}$ inches, taking M_s from col. 1 of Table 86 at 740, Rule (584) gives $H = 4^4 \times 120 \div 740 = 41 \cdot 5$ Nominal Horse-power; or taking M_s from col. 3 at 493, we obtain $H = 4^4 \times 120 \div 493 = 62 \cdot 3$ Net Indicated Horse-power. As a single engine crank-shaft $M_s = 1160$, hence $H = 4^4 \times 120 \div 1160 = 26 \cdot 5$ Nominal Horse-power; or $4^4 \times 120 \div 773 = 39 \cdot 74$ Net Indicated power, &c.

Again, say we require the diameter for an ordinary cast-iron shaft for a Nominal 50-Horse Engine, R = 40: we know beforehand that the diameter will be above $4\frac{5}{8}$ inches: then with M_L from col. 2 of Table 86 = 255, Rule (583) becomes $(50 \times 255 \div 40)\sqrt[3]{\ } = 6\frac{7}{8}$, or say 7 inches diameter. For a single Engine crank-shaft under the same conditions $M_L = 400$, and we obtain $(50 \times 400 \div 40)\sqrt[3]{\ } = 7\frac{15}{16}$, say 8 inches diameter. But for *a pair* of Engines of equivalent power (25-Horse each) $M_L = 282$, hence $(50 \times 282 \div 40)\sqrt[3]{\ } = 7\frac{1}{8}$ inches diameter in cast iron, &c.

(587.) "*Marine Engine Shafts.*"—The *Gross* Indicated Horse-power is usually the only reliable index of the power of Marine Steam-Engines, being easily obtained by the "Indicator": the ratio which this usually bears to the reputed or Nominal power of the makers is much greater than 2 to 1, which is an ordinary ratio with Land Engines. The ratio with 12 Engines of the largest class from 1350 to 400 Nominal and 6867 to 1400 Gross Indicated Horse-power was found to vary from 5·86 to 3·5, the mean being 4·35 to 1·0. This shows that the Nominal power of Marine engines is utterly unreliable as a basis for calculating the diameter of propeller shafts, and that the Gross Indicated power should always be taken. Table 87 gives a few cases of shafts in practice compared with the sizes calculated by our rule (582). Col. 8 shows that there are extraordinary differences in the practice of even our leading Engineers, some

TABLE 87.—Of MARINE ENGINE SHAFTS in PRACTICE.

Name.	Horse-Power.			Rev.	Diam. in Inches.			Maker.	Value of M_L
	Nominal.	Gross Indicated.	Ratio.		Actual.	Calculated.	Diff. per cent.		
Agincourt	1350	6867	5·1	53	23	22·76	+ 3	Maudslay	95·4
Warrior ..	1250	5470	4·4	53	17	21·1	− 47	Penn	47·6
Serapis ..	700	4100	5·86	58	16	18·6	− 36	Humphreys	55·5
Pera ..	400	1400	3·5	58	$11\frac{3}{8}$	13·0	− 33	Rennie	61·1
Simoom ..	400	1400	3·5	62	$13\frac{3}{8}$	12·71	+ 17	J. Watt & Co.	106
	(1)	(2)	(3)	(4)	(5)	(6)	(7)		(8)

making their propeller shafts about $2\frac{1}{4}$ times the strength of others: thus $106 \div 47 \cdot 6 = 2 \cdot 227$. From this it would appear that there is no Standard Rule, but that each maker follows some rule of his own. In col. 6 we have the calculated sizes by our rule, which for the sake of distinctness we may repeat in more definite form:—

(588.) $$H_G = d^3 \times R \div 91.$$

(589.) $$d = \sqrt[3]{(H_G \times 91 \div R)}.$$

(590.) $$M_L = d^3 \times R \div H_G.$$

In which H_G = the Gross Indicated Horse-power; d = diameter in inches; R = revolutions per minute, and M_L = a constant obtained from a case in practice, which is taken at 91 in the rules and as given by col. 6 of Table 86: col. 8 gives the value for each shaft, thus enabling the engineer to select as a basis the one whose strength commends itself to his judgment. Our Rule (589) gives sizes approximating to the strongest shafts in Table 87, and may be considered perfectly safe.

(591.) There is no important part of Marine Engines which fails so frequently as the propeller shaft: casualties from this cause are constantly occurring, in most cases imperilling the safety of the vessel and the lives of the passengers. In some cases the results have been most disastrous, as with the *Australian*, where the shaft not only failed, but also damaged the vessel so much that she foundered in mid-ocean. These

numerous failures seem to show that propeller shafts are frequently made too weak for their work, which is greatly to be deplored considering the magnitude of the interests involved.

(592.) "*Crane Shafts.*"—The strains on a series of crane shafts being well known, it might be supposed that the diameters might be calculated throughout by the Rule (557), in which the strength is made proportional to the cube of the diameter. But by following that course, the small shafts come out much too small to satisfy practical judgment, and it would appear that the case is governed by stiffness rather than by absolute strength, and that the strains should be in proportion to d^4 instead of d^3.

The short neck of the barrel-shaft, however, may be sized by Rule (557): then taking that diameter as a basis, the strains throughout the series should be as d^4.

(593.) Let Fig. 171 be the outline of the gearing of a large crane to raise 10 tons with a single chain by four men; or 20 tons if a running pulley be used. The strain being 10 tons or 22,400 lbs. on the chain, becomes at c or at the pitch line of the wheel H $= 22{,}400 \times 15 \div 45 = 7467$ lbs.; at b, $7467 \times 7 \cdot 5 \div 27 = 2080$ lbs.; at a, $2080 \times 4 \cdot 5 \div 18 = 520$ lbs.; and at the handles $520 \times 3 \div 16 = 97\frac{1}{2}$ lbs., or say 25 lbs. each man.

We can now determine the diameters of the shafts: the torsional strain on D is 22,400 lbs., but taking the "Factor of Safety" at 10 we have 224,000 lbs. breaking weight: the *acting* radius of the barrel measured at the *centre of the chain* being 15 inches, and the value of M_t for wrought iron $= 10{,}580$ by (571), Rule (557) becomes $d = (224{,}000 \times 15 \div 10{,}580)\sqrt[3]{\ } = 6\frac{7}{8}$, or say 7 inches diameter at the neck of the barrel-shaft D.

Having thus found D, we have to find E, F, G: now $7^4 = 2401$ by Table 85; then for E we obtain $2401 \times 15 \div 90 = 400$ or say $4\frac{1}{2}$ inches diameter by the same Table: F becomes $400 \times 9 \div 54 = 66 \cdot 6$, or say $2\frac{7}{8}$ inches: and lastly the hand-shaft G $= 66 \cdot 6 \times 6 \div 36 = 11 \cdot 1$, or say $1\frac{7}{8}$ inch diameter.

(594.) "*Wheels.*"—Having thus found the sizes of the shafts, we may complete the illustration by calculating the pitch and breadth of the gearing. It is shown in the Author's Treatise

on 'Mill-Gearing' that the absolute strength of a wheel-tooth for a dead load will be given by the Rule :—

(595.) $$W = p \times w \times 350.$$

In which W = the safe load on the tooth in lbs.; p = the pitch, and w = the width on the face, in inches. If we assume for the wheel and pinion H, J, say $p = 2\frac{3}{4}$ inches, and $w = 7\frac{3}{4}$ inches, we obtain $W = 2\frac{3}{4} \times 7\frac{3}{4} \times 350 = 7460$ lbs. at c, or very nearly the actual strain at that point which we found (593) to be 7467 lbs.

If we applied the same method of calculation to K L and M N, it would be found that the pitch and width would come out excessively light, and that it is necessary to use a modification of the Rule for *wheels in motion* in (944), which becomes :—

(596.) $$P = \sqrt{D \times R} \times p^2 \times w.$$

In which D = diameter, say in feet; p = pitch, and w = width, in inches; R = revolutions in *any* given time; P = the power of the wheel, which in this case is a proportional number only, and must be of equal value for all the wheels, for obviously, however the sizes and revolutions may vary, the *mechanical power* exerted, or work done must be the same throughout the series.

(597.) Now, admitting that the sizes found in (595) for the wheel H are correct, namely $7\frac{1}{2}$ feet diameter, $2\frac{3}{4}$ inches pitch, $7\frac{3}{4}$ inches wide, and assuming for the purposes of calculation 1 revolution per minute, Rule (596) gives $P = \sqrt{7 \cdot 5 \times 1} \times 2\frac{3}{4}^2 \times 7\frac{3}{4} = 160$. Then if H = 1 revolution, K = 6, and M = 36 revolutions as in Fig. 171.

Assuming for K, $2\frac{1}{4}$ inches pitch, $6\frac{1}{4}$ inches width, Rule (596) gives $P = \sqrt{4 \cdot 5 \times 6} \times 2\frac{1}{4}^2 \times 6\frac{1}{4} = 164$, which is near enough to 160 as found for H. Again, assuming for M, $1\frac{7}{8}$ inch pitch, $4\frac{1}{2}$ inches wide, we obtain $P = \sqrt{3 \times 36} \times 1\frac{7}{8}^2 \times 4\frac{1}{2} = 164$: we have thus obtained the sizes for all the wheels in the train, giving practically equal strength throughout.

To show the necessity for the method of calculation we have adopted, say we apply Rule (595) by which the proportions of

H were obtained, to the wheel and pinion M, N, where the strain at the pitch line = 520 lbs.: then 1 inch pitch, $1\frac{1}{2}$ inch wide, gives $W = 1 \times 1\frac{1}{2} \times 350 = 525$ lbs. as required, but of course those sizes are obviously, and indeed absurdly too light, when contrasted with $1\frac{7}{8}$ pitch, $4\frac{1}{2}$ wide.

(598.) "10-*ton Crane Gearing.*"—To vary the illustration, let Fig. 172 be the outline of the gearing for a crane to raise 5 tons with a single chain, or 10 tons with an ordinary running pulley, by 4 men. The strain on the chain, or 11,200 lbs., becomes $11{,}200 \times 9 \div 30 = 3360$ lbs. at a; $3360 \times 4 \div 21 = 640$ lbs. at b; and $640 \times 2\frac{1}{2} \div 16 = 100$ lbs. at the handles, giving 25 lbs. to each man.

With 10 for the "Factor of Safety" we have 112,000 lbs. breaking weight at the circumference of the barrel, and by Rule (557) we obtain $d = (112{,}000 \times 9 \div 10{,}580)\sqrt[3]{} = 4\frac{1}{2}$ inches diameter at the neck of the barrel-shaft D. Then following the same course as in (593) we have $4\frac{1}{2}^4 = 410$ for D; E becomes $410 \times 8 \div 60 = 54 \cdot 7$, say $2\frac{3}{4}$ inches; and $F = 54 \cdot 7 \times 5 \div 42 = 6 \cdot 5$, say $1\frac{5}{8}$ inch diameter, all by Table 85.

For the wheels, we will assume for S, 2 inches pitch, $4\frac{3}{4}$ inches wide, for which Rule (595) or $W = p \times w \times 350$, becomes $2 \times 4\frac{3}{4} \times 350 = 3325$ lbs., or very nearly 3360 lbs., the actual strain. Then the Rule (596), or $P = \sqrt{D \times R} \times p^2 \times w$, becomes for this same wheel, $P = \sqrt{5 \times 1} \times 2^2 \times 4\frac{3}{4} = 42 \cdot 5$. Assuming for the wheel U, $1\frac{5}{8}$ inch pitch, $3\frac{1}{8}$ wide, we obtain $P = \sqrt{3 \cdot 5 \times 7 \cdot 5} \times 1\frac{5}{8}^2 \times 3\frac{1}{8} = 42 \cdot 3$, being equal to the wheel S, as required.

(599.) In most cases of heavy cranes, it would be expedient to reduce the strain on the chain by the use of several running pulleys: on the other hand, the strain on the chain and wheel-gearing being the same, the load lifted may be greatly increased by the same means. Thus with 10 tons on the chain, as in (593), we had 20 tons lifted by 1 running pulley: with 2 running pulleys we should have 4 chains, and 40 tons lifted: with 3 running pulleys, 6 chains giving 60 tons, &c. In all those cases the strain on the chain and gearing would be that due to 10 tons only: but of course the strength of the Jib of the crane would require to be adapted to the actual load.

CHAPTER XV.

ON EXTENSION AND COMPRESSION.

(600.) With all materials, a tensile strain has the effect of increasing, and a compressive strain of decreasing the length of a bar subjected to those strains. The variation in length may, with small strains, be infinitesimally small, and quite inappreciable to ordinary observation, but sufficiently refined and delicate measurements show, even with the most rigid materials and the smallest strains, that there is an accompanying alteration in length.

In most cases, this longitudinal change of dimension is accompanied by opposite and contrary changes laterally; that is to say, a tensile strain which increases the length of the bar is accompanied by a corresponding reduction in diameter, &c.; in that case, the density or specific gravity of the body is unchanged. But in other cases there is an obvious disruption or partial dislocation of the component parts of the material, which never return perfectly to their primitive places, the bar remains permanently longer or shorter than before, and takes a "permanent set."

The experimental information on the extension and compression of materials by *direct* strains is very scanty, except for cast and wrought iron; fortunately on these, the most important of all the materials used in the arts, we have in the wonderfully refined and exhaustive labours of Mr. Hodgkinson abundant data, leaving little or nothing to be desired.

For Timber and all other materials we shall have to deduce the longitudinal elasticity from the deflection as a beam by transverse strains, which, as we shall find, is a rather uncertain and unsatisfactory method.

It will be convenient to consider extension and compression together. As they follow similar laws there will be no objection to that course, which will, moreover, have the advantage of giving *comparative* results which will be useful. The resistance to compression where the bar is of considerable length, and is

TABLE 88.—Of the EXTENSION of CAST IRON by TENSILE STRAINS, being the Mean Results of Experiments on four kinds of Iron.

Strain per Square Inch.			Extension in Parts of the Length.						Modulus of Elasticity E_E in Lbs. per Square Inch.	
Experimental Weight, Lbs.	Interpolated.		Total by Experiment.	Reduced Results.					Mean between 0 and the given Weight.	When loaded with the given Weight.
	Lbs.	Tons.		Total.	By Each Successive Half-ton.	Mean Per Ton.	Mean Per Lb.	Per Lb. when Loaded with the given Weight.		
1,054	..	..	·00007500	..	..	..	..	..	..	..
..	1,120	½	·00007991	·00008178	·00008178	·00016356	·00000007302	·00000007433	13,695,200	13,452,750
1,581	..	..	·00011420	..	..	..	..	..	..	..
2,108	..	..	·00015500	..	..	..	..	..	..	..
..	2,240	1	·00016556	·00016655	·00008477	·00016655	·00000007435	·00000007715	13,449,400	12,961,100
3,161	..	..	·00023920	..	..	..	..	..	..	..
..	3,360	1½	·00025555	·00025467	·00008812	·00016978	·00000007580	·00000008034	13,193,500	12,447,900
4,215	..	..	·00032580	..	..	..	..	..	..	..
..	4,480	2	·00034865	·00034658	·00009191	·00017329	·00000007736	·00000008394	12,926,300	11,912,300
5,269	..	..	·00041670	..	..	..	..	..	..	..
..	5,600	2½	·00044625	·00044281	·00009623	·00017712	·00000007907	·00000008919	12,646,500	11,211,550
6,323	..	..	·00051080	..	..	..	..	..	..	..
..	6,720	3	·00054894	·00054430	·00010149	·00018143	·00000008100	·00000009403	12,346,100	10,635,100

7,376	..	..	·00061170	..	..	..	..	..	..	..
..	7,840	3½	·00065753	·00065111	·00010681	·00018603	·00000008305	·00000009851	12,040,900	10,150,950
8,430	..	..	·00071580	..	..	..	..	..	..	..
..	8,960	4	·00077282	·00076521	·00011410	·00019130	·00000008540	·0000001056	11,709,200	9,472,000
9,484	..	..	·00082920	..	..	..	..	..	..	..
..	10,080	4½	·00089564	·00088791	·00012270	·00019731	·00000008809	·0000001142	11,352,500	8,755,000
10,538	..	..	·00094670	..	..	..	..	..	..	..
..	11,200	5	·00102371	·00102153	·00013362	·00020431	·00000009121	·0000001254	10,963,900	7,972,800
11,591	..	..	·00106920	..	..	..	..	..	..	..
..	12,320	5½	·00116430	·00116961	·00014808	·00021266	·00000009494	·0000001408	10,533,400	7,103,250
12,645	..	..	·00120670	..	..	..	..	..	..	..
..	13,440	6	·00134480	·00133821	·00016860	·00022304	·00000009957	·0000001541	10,043,200	6,488,500
13,700	..	..	·00139000	..	..	..	..	..	..	..
..	14,560	6½	·00151530	·00151504	·00017682	·00023308	·00000010405	·0000001614	9,610,300	6,194,680
14,793	..	..	·00154920	..	..	..	..	..	..	..
..	15,680	7	·00167840	·00170000	·00018496	·00024286	·00000010842	·0000001662	9,223,500	6,016,000
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)

left free to deflect sideways, is considered in the Chapter on Pillars, and in treating on elasticity under compression in this chapter, the body will be supposed to be supported laterally, so that it is prevented from yielding by flexure.

(601.) "*Cast Iron.*"—We shall consider first the elasticity of Cast Iron, not only because of the importance of that material, but more particularly because its elasticity being very imperfect, the resulting phenomena are instructive, and will facilitate the study of other more perfect materials, such as wrought iron and steel.

Mr. Hodgkinson made experiments on the extension of cast iron by suspending rods of that material vertically in a lofty building and loading them by direct weights. The bars were about $1\frac{1}{8}$ inch diameter, 1 square inch area, and 50 feet long; they were of four different kinds of Iron, namely, Lowmoor No. 2, Blaenavon No. 2, Gartsherrie No. 3, and a mixture composed of Leeswood No. 3 and Glengarnock No. 3, mixed in equal proportions. The mean ultimate cohesive strength of these four different kinds of iron was 7·014 tons per square inch; they may therefore be regarded as fair samples of British cast iron, the mean ultimate cohesive strength of which, as found by very numerous experiments on 23 kinds of iron, we found (4) to be 7·142 tons per square inch. Table 88 gives in cols. 1 and 4 the mean experimental strains and corresponding extensions of the four different kinds of iron named, the latter being reduced to parts of the length for convenience of reference and general application. The extensions for even tons and half-tons, as given by cols. 2, 3, 4, were obtained by direct interpolation between the next preceding and following experimental numbers, without correction.

It will be observed that the extensions are not simply and directly proportional to the strains, but that they increase in a higher ratio, this fact being due to defective elasticity. Mr. Hodgkinson has given rules which agree moderately well with the experimental results, and may be modified into the following:—

(602.)

$$E = \cdot 00239628 - \left\{ \cdot 00000574215 - (\cdot 00000077044 \times W \right\}^{\frac{1}{2}}.$$

(603.) $$W = (6220 \times E) - (1298000 \times E^2).$$

In which E = Extension in parts of the length.
" W = Tensile Strain in Tons per square inch.

Thus, say we require the extension due to 2 tons per square inch:—we have first ·0000007704 × 2 = ·00000154088. Then (·00000574215 − ·00000154088) $\surd$ = ·00204970, and ·00239628 − ·00204970 = ·00034658, the extension by 2 tons required, which with a length of say 50 feet, or 600 inches = 00034658 × 600 = ·208 inch.

Again, say we require the strain due to an extension of ·00133821; we have first 6220 × ·00133821 = 8·32. Then ·00133821² = ·000001791 and ·000001791 × 1298000 = 2·32; finally, 8·32 − 2·32 = 6 tons per square inch, the strain sought.

The col. 5 in Table 88 has been calculated in this way up to 6 tons per square inch:—for 6½ and 7 tons the rule gives ·00153938 and ·00180546 respectively, which, being considerably in excess of the experimental results as plotted in a diagram, &c., we have adopted the latter for those two strains. Cols. 6 and 7 have been obtained from col. 5.

(604.) "*Defect of Elasticity.*"—With a perfectly elastic material equal increments of weight would produce equal amounts of extension throughout, up to the breaking strain. But col. 6 of Table 88 shows that with cast iron the extensions by each successive half-ton progressively increase from ·00008178 with the first to ·00018496 with the last, the *mean* extension per ton (which is a different thing) being with 1 ton ·00016655, and with 7 tons ·00024286.

With a variable rate of extension as with Cast Iron, it becomes necessary clearly to distinguish the *mean* extension *between* two given strains from that *at* a given strain:—for instance, between 0 and 7 tons per square inch the total extension by col. 5 is ·0017, or at the mean rate of ·0017 ÷ 7 = ·00024286 per ton as given by col. 7; but the first half-ton gives ·00008178, or at the rate of ·00008178 × 2 = ·00016356 per ton, which is considerably less, and the last half-ton gives ·0017 − ·00151504

= ·00018496 as in col. 6, or at the rate of ·00018496 × 2 = ·00036992 per ton, which is considerably greater.

The cols. 8 and 9 were obtained by the same reasoning, and will be understood from the preceding explanations:—thus, with a bar strained with 2 tons or 4480 lbs., each successive pound produces a greater extension than the one preceding, varying by col. 9 from ·00000007433 per lb. when loaded with ½ a ton to ·00000008394 per lb. when loaded with 2 tons. The *mean* rate of extension is not an arithmetical mean of those extremes, but is given by col. 8 at ·00000007736.

The extension of cast iron may be calculated with moderate accuracy by the Rules:—

$$(605.)\qquad E = (\cdot 00015 \times W) + (\cdot 0000122 \times W^2).$$

$$(606.)\qquad W = \left\{\frac{E}{\cdot 0000122} + 37\cdot 8\right\}^{\frac{1}{2}} - 6\cdot 15.$$

By this rule with tensile strains of

1	2	3	4	5	6	7

tons per square inch, the extensions become

·0001622	·0003488	·0005598	·0007952	·0010550	·0013392	·0016478

By Table 88, col. 4, the experimental extensions for the same strains were

·00016556	·00034865	·00054894	·00077282	·00102371	·00134480	·00167840

By Mr. Hodgkinson's rule (602) we found, as in Table 88, &c., the extensions:—

·00016655	·00034658	·00054430	·00076521	·00102153	·00133821	·00180546

The application of Table 88 to practice may be easily illustrated. Say we require the extension of a bar 1000 inches long by a strain of 7000 lbs. per square inch;—then the nearest number in col. 2 is 6720 lbs., when by col. 8 the mean extension is ·000000081 per lb., hence we have ·000000081 × 7000 × 1000 = ·567 inch.

Now, if we required the effect of say 532 lbs. more or less on

the bar already loaded with 7000 lbs., we have by col. 9, $\cdot 00000009403 \times 532 \times 1000 = \cdot 05$, or $\frac{1}{20}$th of an inch. If this bar had been loaded with $\frac{1}{2}$ a ton only, then 532 lbs. would have produced an extension of $\cdot 00000007433 \times 532 \times 1000 = \cdot 0395$, or $\frac{1}{25 \cdot 3}$ inch; but if loaded with 7 tons, then the extension would have been $\cdot 0000001662 \times 532 \times 1000 = \cdot 0884$, or $\frac{1}{11 \cdot 3}$ inch. The effect of a given tensile strain is therefore not uniform, but varies with the load which the bar bears beforehand, in the ratio of $2 \cdot 236$ to 1 by col. 9 with initial strains of 7 tons and $\frac{1}{2}$ ton.

(607.) "*Compression of Cast Iron.*"—Mr. Hodgkinson made experiments on the compression of bars of cast iron about 1 inch square and 10 feet long, which were inclosed in a frame of cast iron, which prevented lateral flexure. They were an easy fit, and the bar was oiled and was struck occasionally in different parts with a hammer to eliminate as much as possible the effects of friction. There were four kinds of iron, being in fact the same as those whose extension under tensile strains had been previously determined (601), namely, Lowmoor No. 2, Blaenavon No. 2, Gartsherrie No. 3, and a mixture of Leeswood No. 3 and Glengarnock No. 3 in equal proportions. There were two experiments on each kind, therefore 8 experiments altogether, and the mean results are given by Table 89, the observed compressions being reduced to parts of the length for convenience of application. It will be observed that, as with the extensions so with the compressions,—they are not simply proportional to the strains, but increase in a higher ratio throughout, this result being due to defect of elasticity.

Mr. Hodgkinson has given rules for the compression of cast iron under crushing strains which agree fairly with the experimental results; the following is a modification of those Rules:—

(608.)

$$C = \cdot 0123634 - \left\{\cdot 000152853 - (\cdot 000004283 \times W_c\right\}^{\frac{1}{2}}.$$

(609.) $$W_c = (5773 \times C) - (233473 \times C^2).$$

In which W_c = the compressive strain in tons per square inch.

" C = the compression in parts of the length.

TABLE 89.—Of the COMPRESSION of CAST IRON by CRUSHING STRAINS, being the Mean Results of Experiments on four kinds of Iron.

Strain per Square Inch.			Compression in Parts of the Length.						Modulus of Elasticity E_c in Lbs. per Square Inch.	
Experimental Weight, Lbs.	Interpolated.		Total, by Experiment.	Reduced Results.						
	Lbs.	Tons.		Total.	By each successive Ton.	Mean, per Ton.	Mean, per Lb.	Per Lb., when loaded with the given Weight.	Mean between 0 and the given Weight.	When loaded with the given Weight.
2,065	..	..	·0001562	..	..	..	..	..	..	..
..	2,240	1	·0001695	·0001744	·0001744	·0001744	·00000007786	·00000007844	12,844.000	12,749,000
4,130	..	..	·0003232	..	..	..	..	..	..	..
..	4,480	2	·0003505	·0003514	·0001770	·0001757	·00000007844	·00000007959	12,748,900	12,563,800
6,194	..	..	·0004982	..	..	..	..	..	..	..
..	6,720	3	·0005405	·0005310	·0001796	·0001770	·00000007902	·00000008082	12,655,300	12,373,100
8,259	..	..	·0006566	..	..	..	..	..	..	..
..	8,960	4	·0007124	·0007135	·0001825	·0001789	·00000007951	·00000008209	12,577,800	12,181,250
10,323	..	..	·0008287	..	..	..	..	..	..	..
..	11,200	5	·0008990	·0008988	·0001853	·0001798	·00000008025	·00000008338	12,461,000	11,992,200
12,388	..	..	·0010025	..	..	..	..	..	..	..
..	13,440	6	·0010876	·0010871	·0001883	·0001812	·00000008088	·00000008477	12,363,100	11,796,500
14,453	..	..	·0011802	..	..	..	..	..	..	..
..	15,680	7	·0012804	·0012786	·0001915	·0001827	·00000008154	·00000008626	12,263,400	11,592,150
16,518	..	..	·0013615	..	..	..	..	..	..	..
..	17,920	8	·0014841	·0014736	·0001950	·0001842	·00000008223	·00000008781	12,160,700	11,388,750

18,583	..	..	·0015421	..	..	..	..	..	..	..
..	20,160	9	·0016770	·0016720	·0001984	·0001858	·00000008294	·00000008941	12,057,400	11,184,200
20,647	..	..	·0017187	..	..	..	..	..	..	..
..	22,400	10	·0018722	·0018742	·0002022	·0001874	·00000008367	·00000009117	11,951,900	10,968,050
..	24,640	11	·0020680	·0020805	·0002063	·0001891	·00000008444	·00000009300	11,843,300	10,752,200
24,777	..	..	·0020801	..	..	..	..	..	..	..
..	26,880	12	·0022812	·0022909	·0002104	·0001909	·00000008523	·00000009493	11,733,400	10,534,950
28,906	..	..	·0024749	..	..	..	..	..	..	..
..	29,120	13	·0024774	·0025058	·0002149	·0001928	·00000008605	·00000009818	11,621,000	10,185,115
..	31,360	14	·0027547	·0027310	·0002252	·0001951	·00000008709	·0000001049	11,482,900	9,535,490
33,030	..	..	·0029451	..	..	..	..	..	..	..
..	33,600	15	·0030118	·0029765	·0002455	·0001984	·00000008859	·0000001154	11,288,400	8,667,700
..	35,840	16	·0032746	·0032493	·0002728	·0002031	·00000009066	·0000001292	11,030,000	7,740,755
37,159	..	..	·0034291	..	..	..	..	..	..	..
..	38,080	17	·0035760	·0035574	·0003081	·0002093	·00000009342	·0000001460	10,704,400	6,849,880
..	40,320	18	·0039333	·0039058	·0003484	·0002170	·00000009687	·0000001641	10,323,100	6,096,100
..	42,560	19	·0043039	·0042945	·0003887	·0002260	·0000001009	·0000001821	9,910,300	5,492,120
42,995	..	..	·0043600	..	..	..	..	..	..	..
..	44,800	20	·0047242	·0047235	·0004290	·0002367	·0000001056	·0000002001	9,463,300	4,997,220
..	47,040	21	·0051762	·0051928	·0004693	·0002473	·0000001104	·0000002194	9,058,700	4,558,178
47,237	..	..	·0052160	..	..	..	..	..	..	..
..	49,280	22	·0057788	·0057024	·0005096	·0002592	·0000001157	·0000002363	8,641,800	4,232,010
51,480	..	..	·0063850	..	..	..	..	..	..	..
..	51,520	23	·0063960	·0062523	·0005499	·0002718	·0000001214	·0000002542	8,240,200	3,934,400
..	53,760	24	..	·0068425	·0005902	·0002851	·0000001273	·0000002722	7,858,800	3,674,030
..	56,000	25	..	·0074730	·0006305	·0002989	·0000001334	·0000002902	7,493,600	3,446,020
..	58,240	26	..	·0081438	·0006708	·0003132	·0000001398	·0000003082	7,151,400	3,244,650
..	60,480	27	..	·0088549	·0007111	·0003279	·0000001464	·0000003262	6,830,100	3,065,550
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)

TABLE 89.—Of the COMPRESSION of CAST IRON by CRUSHING STRAINS, &c.—*continued.*

Strain per Square Inch.			Compression in Parts of the Length.						Modulus of Elasticity E_0 in Lbs. per Square Inch.	
Experimental Weight, Lbs.	Interpolated.		Total, by Experiment.	Reduced Results.						
	Lbs.	Tons.		Total.	By each successive Ton.	Mean, per Ton.	Mean, per Lb.	Per Lb., when Loaded with the given Weight.	Mean between 0 and the given Weight.	When Loaded with the given Weight.
..	62,720	28	..	·0096063	·0007514	·0003431	·0000001532	·0000003441	6,529,000	2,905,900
..	64,960	29	..	·0103980	·0007915	·0003585	·0000001601	·0000003621	6,247,400	2,761,520
..	67,200	30	..	·0112300	·0008320	·0003743	·0000001671	·0000003802	5,983,000	2,630,125
..	69,440	31	..	·0121023	·0008723	·0003904	·0000001743	·0000003982	5,737,700	2,511,220
..	71,680	32	..	·0130149	·0009126	·0004067	·0000001816	·0000004162	5,507,500	2,402,620
..	73,920	33	..	·0139678	·0009529	·0004233	·0000001889	·0000004342	5,292,200	2,303,020
..	76,160	34	..	·0149610	·0009932	·0004400	·0000001964	·0000004522	5,090,600	2,211,370
..	78,400	35	..	·0159945	·0010335	·0004570	·0000002040	·0000004702	4,901,700	2,126,730
..	80,640	36	..	·0170683	·0010738	·0004741	·0000002117	·0000004882	4,724,500	2,048,320
..	82,880	37	..	·0181824	·0011141	·0004914	·0000002194	·0000005062	4,558,200	1,975,495
..	85,120	38	..	·0193368	·0011544	·0005089	·0000002272	·0000005242	4,401,200	1,907,670
..	87,360	39	..	·0205315	·0011947	·0005264	·0000002350	·0000005422	4,254,900	1,844,350
..	89,600	40	..	·0217665	·0012350	·0005442	·0000002429	·0000005602	4,116,400	1,785,110
..	91,840	41	..	·0230418	·0012753	·0005620	·0000002509	·0000005782	3,985,700	1,729,550
..	94,080	42	..	·0243574	·0013156	·0005799	·0000002589	·0000005962	3,862,400	1,677,340
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)

Thus, to find the compression by 12 tons, we have first $\cdot 000004283 \times 12 = \cdot 000051396$; then $(\cdot 000152853 - \cdot 000051396)\sqrt{} = \cdot 0100725$, and $\cdot 0123634 - \cdot 0100725 = \cdot 0022909$, the compression sought, which, with a length of say 30 feet, or 360 inches, becomes $\cdot 0022909 \times 360 = \cdot 0825$ inch.

Again,—to find the strain due to a compression of ·00187422 in a bar whose length = 1·0; we have first $\cdot 00187422^2 = \cdot 000003513$, then

$$\begin{aligned} 5733 \times \cdot 00187422 &= 10\cdot 82 \\ 233473 \times \cdot 000003513 &= \cdot 82 \\ \hline &\ 10\cdot 00 \text{ tons compressive strain.} \end{aligned}$$

The col. 5 in Table 89 has been calculated by this rule up to 13 tons, beyond which the experimental results, as plotted in a diagram, seem to show that the rule gives the compressions too small. In the experiments the maximum strain was 51,480 lbs., or nearly 23 tons per square inch; continuing the curve by judgment, we have obtained the compressions up to 42 tons per square inch, which is nearly the mean ultimate crushing strain for British Cast iron, as determined by numerous experiments on 23 different kinds of iron; see Table 31 and (132). Beyond the experimental strain of 23 tons, therefore, the compressions given are of doubtful accuracy.

(610.) In order to determine the compressions with heavier strains up to the crushing load, Mr. Hodgkinson made another series of experiments on short cylinders of various kinds of cast iron ¾ inch diameter, and 1½ inch high, except one specimen, which was 1¼ inch diameter and 2½ inches high, the results of which are given by Table 90. For comparison we have given in the first line the mean compressions of long bars in a guide-frame, as found by the former experiments, and given in Table 89.

(611.) It will be observed that the experiments on short cylinders gave very anomalous results, exceeding greatly those obtained from the long bars. Confining ourselves to 20 tons per square inch, which is well within the limits of the *observed*

TABLE 90.—Of EXPERIMENTS on the COMPRESSION of SHORT CYLINDERS of CAST IRON under CRUSHING STRAINS.

Kind of Iron.	Tons per Square Inch.								
	7	10	15	20	25	30	35	40	45
	Compression in Parts of the Length.								
Mean of 4 (in frame)	·0012786	·0018742	·002976	·0047242	·007473	·01123	·0160	·021767	..
Lowmoor, No. 1 ..	·0029	·0044	·0061	·0161	·0398	..	..	..	..
Ynescedwyn	..	..	..	·0115	·0250	·0523	..	..	..
Brymbo, Calder ..	..	..	..	·0128	·0242	·0464	..	..	..
Bowling	..	..	..	·0126	·0239	·0475	..	..	..
Brymbo	..	..	..	..	·0221	·0412	..	..	..
Lowmoor, No. 2 ..	..	..	..	·0121	·0175	·0267	·0458	..	..
Blaenavon	..	..	..	·0089	·0160	·0248	·0367	·0548	·0950
Coltness	..	..	..	·0086	·0140	·0217	·0308	·0458	..
Mixed Irons	..	..	..	·0083	·0126	·0192	·0318	·0561	..
Ystalyfera	..	..	..	·0077	·0116	·0179	·0280	·0452	..
Clyde..	..	..	..	·0083	·0122	·0183	·0260	·0384	·0573

compressions with the long bars, we find that the short cylinder of Ystalyfera iron gave $\cdot 0083 \div \cdot 0047242 = 1 \cdot 76$, and the Lowmoor $\cdot 0161 \div \cdot 004742 = 3 \cdot 4$ times the compression given by the long bars. With 42 tons, which is nearly the mean crushing strain for British iron, the short cylinders of Clyde iron gave $\cdot 0446 \div \cdot 0243574 = 1 \cdot 83$, and the mixed irons $\cdot 07 \div \cdot 0243574 = 2 \cdot 87$ times the *estimated* (609) compression of the long bars.

In searching for a reason for this great discrepancy, we might at first be led to suppose that it was due to the fact that the short cylinders were free to expand laterally in proportion to the longitudinal compression, while the long bars, being confined in a guide-frame, were prevented from doing so; but the fit of the latter was presumably so slack that this reason seems inadequate to account for the differences observed.

Mr. Hodgkinson himself seems to have suspected that there were considerable errors in the observed compressions of the short cylinders due to the method by which they were taken; he says "they were crushed between two discs of steel $\frac{3}{4}$ inch thick, which were parallel to each other. Between the disc and the specimen, both at top and bottom, a very thin piece of lead was interposed to prevent irregular action against each other; but, notwithstanding the care taken, it is probable that the results of these experiments are not free from considerable errors arising from the following causes: the great weights applied, 20 or 30 tons per square inch of section, caused the ends of the cylinders to be driven into the surface of the discs to such a degree that the surface of the steel sometimes remained irregular and broken after the experiments, showing the form of the ends of the cylinder. From the same cause the discs of steel would become slightly incurvated, and their distances asunder would be decreased more than was due to the shortening of the cylinder by the quantity of its penetration into the discs, added to their approach through flexure."

(612.) The experiments on short cylinders must therefore be regarded as of doubtful accuracy, and in the present state of our knowledge the compressions given by Table 89 from experi-

ments on long bars may be taken as approximately correct for all strains up to the crushing load. The shortening of a pillar under heavy strains is very considerable; thus, by Table 89, with 42 tons per square inch, a pillar 10 feet, or 120 inches, long is shortened $\cdot 0243574 \times 120 = 2 \cdot 92$, or nearly 3 inches. If we admitted the results given by Table 90 for mixed iron, we obtain $\cdot 07 \times 120 = 8 \cdot 4$ inches compression; even under ordinary strains, the amount of compression is much greater than most practical men are aware of; for instance, with the ordinary safe strain of one-third of the ultimate crushing load, or 14 tons per square inch, a pillar or series of pillars joined end to end, 50 feet, or 600 inches long, by col. 5 of Table 89 is shortened $\cdot 002731 \times 600 = 1 \cdot 6386$, or $1\frac{5}{8}$ inch, and when thus loaded, 1000 lbs., more or less, will, by col. 9, cause a further change of length $= \cdot 0000001049 \times 1000 \times 600 = \cdot 06294$, or $\frac{1}{16}$ inch.

(613.) In Table 89, col. 6 shows that the compression is not simply proportional to the weight, but, on the contrary, is progressively increased from $\cdot 0001744$ with the first ton to $\cdot 0013156$ with the last, or 42nd. The ratio of the compressions with equal weights is therefore $\cdot 0013156 \div \cdot 0001744 = 7 \cdot 53$ to $1 \cdot 0$; this being due to defect of Elasticity (604).

The rate of compression being thus variable, it becomes necessary, as we found to be the case with extension (604), to distinguish the *mean* compression between two given strains from that produced by a certain weight on a bar already loaded. This has been explained and illustrated for variable extension, and need not be repeated here. Cols. 8 and 9 have been obtained by analysis with the numbers given by the experiments: an illustration of their application will suffice; a pillar 1000 inches long, loaded with 20,000 lbs. per square inch, will, by col. 8, be shortened $\cdot 00000008294 \times 20000 \times 1000 = 1 \cdot 659$ inch, and when thus loaded, an extra strain of 1 cwt., or 112 lbs., will, by col. 9, give $\cdot 00000008941 \times 112 \times 1000 = \cdot 01$, or $\frac{1}{100}$th inch compression, &c.

For moderate compressive strains, say under 15 tons per square inch, the compression of cast iron will be given with considerable accuracy by the Rules:—

(614.) $C = (\cdot 00017 \times W_c) + (\cdot 0000018 \times W_c^2)$.

(615.) $W_c = \left\{\frac{C}{\cdot 0000018} + 2228\right\}^{\frac{1}{2}} - 47 \cdot 2$.

In which C = Compression in parts of the length.
W_c = Crushing weight in Tons per square inch.

Thus, to find the compression due to 15 tons per square inch, 15^2 being = 225, we have $\cdot 00017 \times 15 = \cdot 00255$; and $\cdot 0000018 \times 225 = \cdot 000405$. Then $\cdot 00255 + \cdot 000405 = \cdot 002955$, the compression sought.

Again; to find the compressive strain due to a compression $= \cdot 002955$, we have $\cdot 002955 \div \cdot 0000018 = 1641$; then $(1641 + 2228) \sqrt{\ } = 62 \cdot 2$, and $62 \cdot 2 - 47 \cdot 2 = 15$ tons, the strain required, &c. Calculating in this way, we obtain the following results:—

Tons.	Compression.	Tons.	Compression.	Tons.	Compression.
1 =	·0001718	6 =	·0010848	11 =	·0020878
2 =	·0003472	7 =	·0012782	12 =	·0022692
3 =	·0005262	8 =	·0014752	13 =	·0025142
4 =	·0007088	9 =	·0016758	14 =	·0027328
5 =	·0008950	10 =	·0018800	15 =	·0029550

These compressions differ very little from those given by Table 89; with great strains it would seem that the compressions become too anomalous and irregular to be expressed by any ordinary practical rule. Thus, for 35 tons, Mr. Hodgkinson's rule in (608) gives ·01064785, but the diagram, based on the experiments on long bars, and col. 5 of Table 89, gives ·0159945, and the direct experiments on short cylinders, Table 90, from ·026 to ·0458. Except for the purposes of scientific research, cast iron is never strained beyond one-third of the ultimate crushing load, or beyond 14 tons per square inch; hence the uncertainty as to its compression under excessive strains is of little practical importance.

(616.) "*Comparative Extension and Compression of Cast Iron.*"—We should have expected instinctively that all materials would resist compression with greater energy than extension,

but experiment has shown that with cast iron, and still more with wrought iron (628), the material yields more to a small compressive strain than to a tensile. Confining ourselves to the direct and unreduced experiments as given by cols. 4, 4, in Tables 88, 89, we obtain the results given by Table 91, which show that for strains below 2 tons per square inch the compressions exceed the extensions; with 2·355 tons they are equal, as shown by Table 92; with greater strains the extensions exceed the compressions, a fact which is due to defect of elasticity. The ultimate resistance of cast iron to compressive strains being six times that for tensile ones, and defect of elasticity increasing rapidly as the ultimate strain is approached, this fact tells more influentially on the extensions than on the compressions.

(617.) It should be observed that 2·355 tons is very nearly $\frac{1}{3}$rd of 7·142 tons, the mean ultimate Tensile strength of cast iron, and $\frac{1}{18}$th of 43 tons, the mean Crushing strength. Let Fig. 206$\frac{1}{2}$ be the section of a bar 1 inch square and 1 foot long, loaded transversely until the tensile strain at B and crushing strain at C are both = 2·355 tons per square inch. The extension and compression with that strain being, as we have seen, equal to one another, it will follow that the neutral axis N. A. will be in the centre of the section. By Rule (639), the transverse load will in our case be $\frac{1}{18}$th of the maximum strain at B or C, hence we have $M_T = 2 \cdot 355 \div 18 = \cdot 1314$ ton, which is $\frac{1}{7}$th only of ·921 tons, its mean value for breaking load, as given by col. 6 of Table 66 (523).

We now have this remarkable fact: that the "Factor of Safety" varies greatly with the three great strains involved in the case; for the Tensile it is = 3, for the Crushing = 18, and for the resulting Transverse strain = 7.

It will be evident from this that, if a flanged girder were loaded to $\frac{1}{7}$th only of its Breaking weight, the proper form of section would be one with equal top and bottom flange. As the strain increases beyond $\frac{1}{7}$th up to the breaking weight the neutral axis rises from the position in Fig. 206 until it becomes as in Fig. 168, when, as shown by Mr. Hodgkinson's experiments, the most economical form of section is with flanges

TABLE 91.—Of the COMPARATIVE EXTENSION and COMPRESSION of CAST and WROUGHT IRON: from Direct Experiment.

Tons per Square Inch.	Cast Iron. Extension. E.	Cast Iron. Compression. C.	Cast Iron. Ratio. C ÷ E.	Wrought Iron. Extension. E.	Wrought Iron. Compression. C.	Wrought Iron. Ratio. C ÷ E.
½	·00007991	·0000847	1·0600	..	..	..
1	·00016556	·0001695	1·0232	·000080121	·00010138	1·265
1½	·00025555	·0002600	1·0174	..	..	..
2	·00034865	·0003505	1·0053	·000159253	·00020278	1·273
2½	·00044625	·0004455	·9983	..	..	..
3	·00054894	·0005405	·9846	·000241136	·00030778	1·276
3½	·00065753	·0006264	·9527	..	..	..
4	·00077282	·0007124	·9218	·000319204	·00040362	1·264
4½	·00089564	·0008057	·8995	..	..	..
5	·00102371	·0008990	·8684	·000398725	·00049340	1·237
5½	·00116430	·0009933	·8531	..	..	..
6	·00134480	·0010876	·8088	·000479029	·00058314	1·217
6½	·00151530	·0011840	·7814	..	..	..
7	·00167840	·0012804	·7629	·000558628	·00068197	1·220
8	..	..	..	·000638774	·00077791	1·218
9	..	..	..	·000724527	·00087432	1·207
10	..	..	..	·000811457	·00098353	1·212
11	..	..	..	·000908845	·00109665	1·207
12	..	..	..	·001039175	·00121975	1·174
13	..	..	..	·001244605	·00137519	1·105
14	..	..	..	·002013840	·00159414	0·791
(1)	(2)	(3)	(4)	(5)	(6)	(7)

having areas of 6 to 1. His girders were all of the same depth, namely, $5\frac{1}{8}$ inches; the ratio of areas were 1 to

1	2·04	3·75	4·4	5·74	6·732

The mean Breaking transverse loads per square inch of whole sectional areas were—

2368	2567	2710	3198	3332	3729

lbs.; hence, taking the equal-flanged girder as = 1·0, the mean increase in strength per cent. becomes

0	8·4	14·4	35·0	40·7	57·5

TABLE 92.—Of the COINCIDENCE of EXTENSION and COMPRESSION of CAST IRON.

Tons per Square Inch.	Extension.	Compression.	Difference.
2·0	·00034865	·00035050	+·00000185
2·1	·00036817	·00036950	+·00000133
2·2	·00038769	·00038850	+·00000081
2·3	·00040721	·00040750	+·00000029
2·33	·00041307	·00041320	+·00000013
2·355	·00041795	·00041795	·00000000
2·36	·00041892	·00041890	—·00000002
2·4	·00042673	·00042650	—·00000023
2·5	·00044625	·00044550	—·00000075

(618.) It would appear, however, that the ratio of the areas of the flanges is not a fixed one for all strains, but should vary with the ratio of the working load to the breaking weight, rising from 1 to 1 with $\frac{1}{7}$th to 6 to 1 with the Breaking weight.

It has been gratuitously assumed that the best form of section for the Breaking Weight must of necessity be the best for lower strains also, say $\frac{1}{3}$rd, which is the ratio commonly used, and thus Mr. Hodgkinson's form, with flanges in the ratio 6 to 1, has been almost universally adopted in supposed deference to authority, although that proportion would not commend itself to the unbiassed mechanical instinct of practical men: Mr. Hodgkinson, however, is not responsible for the erroneous deduction from his experimental results.

Reasoning seems to show that with Factor 3 the ratio of the flanges should be somewhere between 1 to 1 as for $\frac{1}{7}$th, and 6 to 1 as for the Breaking weight. Fig. 207 gives a section which fulfils the condition that the maximum tensile strain shall not exceed $\frac{1}{3}$rd of the Breaking weight. Thus the area of bottom flange = 10 square inches, the strain at T = 2·355 tons per square inch, and the distance from N. A. = 4; hence we have 10 × 2·355 × 4 = 94·2. Then, the area of top flange = 4·5 square inches, the strain at C = 3·5 tons per square inch, and the leverage = 6, giving 4·5 × 3·5 × 6 = 94·5, or practically the same as the resistance to tension, which is a necessity: in this case the ratio of the flanges = 10 ÷ 4·5 = 2·22 to 1·0.

But we must see to it, that extension being to the compression in Fig. 207 as 4 to 6, corresponds to 2·355, and 3·5 tons, the respective strains. Table 92 shows that for Tensile strain of 2·355 tons, the extension = ·00042; then the compression must be ·00042 × 6 ÷ 4 = ·00063, which is due to 3·5 tons per square inch by Table 89. Fig. 208 is a "UNIT" girder having the same proportions as Fig. 207, from which the dimensions for any load or span may be found, as explained in (485).

Fig. 209 is a Diagram which shows that, admitting equal flanges as the best for $\frac{1}{7}$th of the Breaking weight, and 6 to 1 for the Breaking weight, we have 2·08 to 1·0 for $\frac{1}{3}$rd, which is the ordinary working load, and agrees moderately with 2·22 as found by analysis: 3·08 to 1·0 for $\frac{1}{2}$; and 4·08 to 1·0 for $\frac{2}{3}$ of the Breaking weight. It is probable from (617) that these ratios of *strains* apply principally to the bottom flange: in Fig. 207 the Factor is 7·142 ÷ 2·355 = 3 for the Tensile, and 43 ÷ 3·5 = 12·3 for the Crushing Strain. See (354) and the Diagram, Fig. 81, &c.

(619.) The actual and comparative lengthening and shortening of cast iron under different tensile and crushing strains may be shown distinctly by calculating the lengths of bars that would be altered in length 1 inch by different strains. Table 93 gives that length for cast and wrought iron.

(620.) "*Extension of Wrought Iron.*" — The elasticity of

TABLE 93.—Of the LENGTH of BARS which will be STRETCHED or SHORTENED 1 INCH by GIVEN STRAINS.

Tons per Square Inch.	Cast Iron.		Wrought Iron.		Tons per Square Inch.	Cast Iron.	Wrought Iron.		Tons per Square Inch.	Cast Iron.
	Tensile.	Compressive.	Tensile.	Compressive.		Compressive.	Tensile.	Compressive.		Compressive.
	feet.	feet.	feet.	feet.		feet.	feet.	feet.		feet.
1	500	478	1040	822	8	56·5	130	107	21	..
1½	327	..	..	..	9	50·0	115	95·3	22	14·6
2	240	237	523	411	10	44·5	103	84·7	23	..
2½	188	..	..	..	11	40·0	91·7	76·0	24	12·2
3	153	157	345	274	12	36·4	80·2	68·3	26	10·2
3½	128	..	..	..	13	33·3	66·9	60·6	28	8·68
4	109	117	261	210	14	30·5	..	..	30	7·42
4½	94	..	..	..	15	28·0	..	..	32	6·40
5	82	93	209	169	16	25 6	..	..	34	5·57
5½	71	..	..	..	17	23·4	..	..	36	4·88
6	62	77	174	143	18	21·4	..	..	38	4·31
6½	55	..	..	..	19	19·4	..	..	40	3·82
7	49	65	149	122	20	17·6	..	..	42	3·42

wrought iron is very nearly perfect with moderate strains, say up to 8 tons per square inch, differing entirely in this respect from cast iron, as we have seen (604). Mr. Hodgkinson made experiments on two bars 50 feet long of the respective diameters of $\frac{1}{2}$ and $\frac{3}{4}$ inch, the results of which reduced to parts of the length are given by cols. 3, 3 in Tables 94, 95. For the weights in even tons, cols. 1, 1, the extensions were obtained by interpolating between the next greater and lesser experimental numbers as given by cols. 3, 3, without correction, that is to say, without attempting to equalize or eliminate the unavoidable errors and anomalies of experiment; the mean result of the two experiments is given by col. 5 of Table 91. It will be observed that the Modulus of Elasticity, cols. 4, 4 in Tables 94, 95, with which perfect elasticity would have been the same throughout, is practically uniform up to about 8 tons with the $\frac{3}{4}$-inch bar, and about 9 tons with the $\frac{1}{2}$-inch bar, the departures from uniformity being no doubt due to errors of observation. Defect of elasticity would have been manifested by a regularly progressive reduction of the Modulus; if there were such a reduction it must have been exceedingly small, and being obscured by errors of observation, it does not appear in the experiments.

(621.) "*Effect of Time.*"—One very instructive and important point brought out very clearly by these two experiments is that with heavy loads the extension is not governed by the strain alone, but becomes also a question of *time*. The falling off in the Modulus with strains greater than 8 or 9 tons, seems to show that the bars were overloaded, and in all probability observations to that end would then have begun to show the effect of time, but such observations were not made until the $\frac{3}{4}$-inch bar was loaded with about 13 tons, and the $\frac{1}{2}$-inch bar with 14·3 tons per square inch, and then, even five minutes of time had a great effect on the result.

The Table alone gives a very imperfect idea of the relative effects of the gradually increasing strains on the extensions and of the influence of time on the results; the Diagram, Fig. 215, shows both graphically. A careful comparison of the results of the two experiments in Tables 94, 95, will show a

TABLE 94.—Of the EXTENSION and PERMANENT "SET" of a BAR of WROUGHT IRON ¾ inch Diameter.

Tensile Strain per Square Inch.		Extension in Parts of the Length.	Modulus of Elasticity E_s, in Lbs. per Square Inch.	Permanent "Set" in Parts of the Length.	Ratio of Set to Extension per Cent.
Tons.	Lbs.				
..	1,262	·00004333	29,125,300	..	..
1	2,240	·00008402	26,660,000	..	..
..	2,524	·00009583	26,328,000	..	..
..	3,786	·00014083	26,885,000	·00000417	3·4
2	4,480	·00016606	26,371,000	..	..
..	5,047	·00018667	27,000,000	·00000500	2·6
..	6,309	·00023100	27,311,000	·00000417	1·8
3	6,720	·00024560	27,361,000	..	..
..	7,571	·00027483	27,539,000	·00000375	1·4
..	8,833	·00031583	27,967,000	·00000417	1·3
4	8,960	·00032011	27,990,000	..	..
..	10,095	·00035833	28,172,000	·00000417	1·1
5	11,200	·00039786	28,150,000	·00000489	..
..	11,357	·00040450	28,083,000	..	..
..	12,618	·00044750	28,196,000	·00000583	1·3
6	13,440	·00047898	28,060,000	·00000664	..
..	13,880	·00049583	27,993,000	..	..
..	15,142	·00054000	28,041,000	..	..
7	15,680	·00055776	28,112,000	·00000882	..
..	16,404	·00058166	28,220,000	..	..
..	17,666	·00062750	28,153,000	·0000109	1·7
8	17,920	·00063823	28,078,000	·0000120	..
..	18,928	·00068083	27,801,000	..	..
9	20,160	·00072720	27,723,000	·0000225	..
..	20,190	·00072833	27,721,000	·0000225	3·1
..	21,451	·00077583	27,649,000	..	..
10	22,400	·00081405	27,517,000	·0000327	..
..	22,713	·00082666	27,475,000	·0000342	4·1
..	23,975	·00088083	27,186,000	..	..
11	24,640	00091070	27,056,000	·0000516	..
..	25,237	·00093750	26,919,000	·0000567	6·1
..	26,499	·00103330	25,645,000	..	..
12	26,880	·00104538	25,713,000	·0000848	..
..	27,761	·00107333	25,864,000	·0001000	9·3
..	29,022	·00120830	24,011,000	..	..
13	29,120	·00124333	23,421,000	·0003760	..
13·52	30,284	·00165920	18,252,000	..	36·5*
			Increase per cent.		
"	Do.	·00167250	5 min. 0 02	..	..
"	Do.	·00168167	10 do. 1·35	·000613	..
"	Do.	·00171167	15 do. 3·1	·000645	..
(1)	(2)	(3)	(4)	(5)	(6)

TABLE 94.—Of the EXTENSION and PERMANENT "SET" of a BAR of WROUGHT IRON ¾ inch Diameter—*continued.*

Tensile Strain per Square Inch.		Extension in Parts of the Length.	Effect of Time on the Extension per Cent.	Permanent "Set" in Parts of the Length.	Ratio of Set to Extension per Cent.
Tons.	Lbs.				
13·52	30,284	·00173333	20 min. 4·5	·000664	..
"	Do.	·00174667	1 hour 5·3	·000678	..
"	Do.	·00197167	17 do. 19·0	·000902	..
14·08	31,546	·00201670	5 min. ..	·000903	44·8
14·65	32,808	·0045900	.. 0·0	·003450	75·2
"	Do.	·0058533	5 do. 27·5	..	..
"	Do.	·0066383	10 do. 44·5	·005460	..
"	Do.	·0084500	15 do. 83·7	·007220	..
15·21	34,070	·0112170	1 do. 0·0	..	90·0
"	Do.	·0116667	2 do. 0·04	·0120	..
"	Do.	·0148833	1 hour 32·0	·0136	..
15·77	35,332	·0170000	5 min. 0·0	·0156	92·0
"	Do.	·0181667	10 do. 6·9	·0168	..
16·34	36,594	·0212000	6 do. ..	..	..
16·90	37,856	·0241000	.. ..	..	..
(1)	(2)	(3)	(4)	(5)	(6)

TABLE 95.—Of the EXTENSION and PERMANENT "SET" of a BAR of WROUGHT IRON ½ inch Diameter.

Tensile Strain per Square Inch.		Extension in Parts of the Length.	Modulus of Elasticity E_E, in Lbs. per Square Inch.	Permanent "Set" in Parts of the Length.	Ratio of Set to Extension per Cent.
Tons.	Lbs.				
1	2,240	·000076223	29,387,000	..	..
..	2,667	·000082167	32,458,000	..	..
2	4,480	·000152447	29,387,000	..	..
..	5,335	·000185583	28,747,000	..	..
3	6,720	·000236673	28,365,000	..	..
..	8,003	·000283917	28,184,000	·00000254	..
4	8,960	·000318297	28,150,005	·00000285	..
..	10,670	·000379670	28,103,000	·00000339	0·89
5	11,200	·000398690	28,092,000	·00000356	..
..	13,338	·000475417	28,055,000	·00000424	0·89
6	13,440	·000479078	28,053,000	·00000427	..
7	15,680	·000559497	28,025,000	·00000499	..
..	16,005	·000571167	28,022,000	·00000508	0·89
8	17,920	·000639317	28,030,000	·00000631	..
..	18,672	·000666083	28,034,000	·00000678	1·0
(1)	(2)	(3)	(4)	(5)	(6)

TABLE 95.—Of the EXTENSION and PERMANENT "SET" of a BAR of WROUGHT IRON ½ inch Diameter—*continued.*

Tensile Strain per Square Inch.		Extension in Parts of the Length.	Modulus of Elasticity E_s, in Lbs. per Square Inch.	Permanent "Set" in Parts of the Length.	Ratio of Set to Extension per Cent.
Tons.	Lbs.				
9	20,160	·000721853	27,928,000	·00001010	..
..	21,340	·000766083	27,856,000	·0000127	1·6
10	22,400	·000808863	27,693,000	·0000208	..
..	24,008	·00087375	27,477,000	·0000330	3·8
11	24,640	·00090699	27,167,000	·0000455	..
..	26,675	·00101358	26,318,000	·0000830	8·2
12	26,880	·00103297	26,022,000	·0000968	..
13	29,120	·00124588	23,373,000	·000247	..
..	29,343	·00128817	22,779,000	·000263	20·0*
14	31,360	·00200000	15,680,000	·000919	..
14·29	32,011	·00222867	14,363,000	·001130	48·0
			Increase per cent.		
..	Do.	·00235592	5 min. 6·0	..	..
15·48	34,678	·0042900	..	·00307	71·4
16·16	37,346	·0091625	..	·00847	92·0
17·86	40,013	·0102667	5 min. 0·0	·00911	..
"	Do.	·0117583	1 hour 14·5	..	..
"	Do.	·0118667	2 do. 15·5	..	..
"	Do.	·0119417	3 do. 16·3	..	..
"	Do.	·0119500	4 do. 16·4	..	..
"	Do.	·0119667	5 do. 16·6	..	..
"	Do.	·0119750	6 do. 16·7	..	..
"	Do.	·0120250	7 do. 17·1	..	..
"	Do.	Do.	8 do. 17·1	..	..
"	Do.	Do.	9 do. 17·1	..	..
"	Do.	Do.	10 do. 17·1	..	..
19·05	42,681	·0179000	5 min. 0·0	·01650	92
"	Do.	·0194917	10 do. 9·0	..	..
"	Do.	·0198583	46 hours 10·9	·01864	..
20·24	45,348	·0215000	5 min. 0·0	·01980	92
"	Do.	·0217083	1 hour 1·0	..	..
"	Do.	Do.	2 do. 1·0	..	..
"	Do.	Do.	19 do. 1·0	·02000	..
21·43	48,016	·0247917	5 min. 0·0	·02280	92
"	Do.	·0251583	1 hour 1·4	..	..
"	Do.	·0252417	11 do. 1·8	..	..
22·63	50,683	·0349583	10 min. 0·0	·0329	94
"	Do.	·0352250	7 hours 0·08	..	..
"	Do.	Do.	12 do. 0·08	..	..
(1)	(2)	(3)	(4)	(5)	(6)

tolerably close agreement in the extensions up to 13 tons per square inch, but beyond that point the $\frac{3}{4}$-inch bar yields more both to time and extra strain than the $\frac{1}{2}$-inch one; it will therefore be expedient to consider them separately.

(622.) With the $\frac{1}{2}$-inch bar, the most remarkable fact is, that the effect of time with each load is at first very great, but continually decreases until it becomes nil, time having no further effect. Thus with 40,013 lbs., or 17·86 tons, per square inch, the first hour produces by col. 4 an increase of 14·5 per cent., but the next hour only 1 per cent.; the next only 0·8; the next 0·1 per cent., &c., until the 7th hour, after which, time ceases to have any effect. Similarly, with 42,681 lbs., or 19·05 tons, the effect of $10 - 5 = 5$ minutes, is to increase the extension 9 per cent., and then 46 hours gives a further increase of only $10 \cdot 9 - 9 = 1 \cdot 9$ per cent. With 45,348 lbs., or 20·24 tons, 1 hour gave an increase of 1 per cent., 2 and 19 hours having no further effect. Finally, with so great a strain as 50,683 lbs., or 22·63 tons, per square inch, 7 hours gave only the small increase of 0·08 per cent., 12 hours having no further effect on the extension.

(623.) The $\frac{3}{4}$-inch bar seems to have been a softer and more ductile iron than the other; the effect of time on the extension was much greater and much more persistent; although the first increment was the most influential, the effect was reduced less rapidly, as shown by col. 4. Thus, with a strain of 30,284 lbs., or 13·52 tons, per square inch, the extension increases regularly (although after 1 hour at a diminishing rate) up to 17 hours, when it became 19 per cent. With 32,808 lbs., or 14·65 tons, per square inch, which is not a very heavy strain, the effect of time was enormous, 5, 10, and 15 minutes giving an increase on the extension of 27·5, 44·5, and 83·7 per cent. respectively; at this last rate the extension of the bar would in 18 minutes have been double that produced immediately by 14·65 tons on this particular bar. With 34,070 lbs., or 15·21 tons, $2 - 1 =$ 1 minute, gave ·04, and 1 hour $=$ 32 per cent. increase. Finally, with 35,332 lbs., or 15·77 tons, $10 - 5 = 5$ minutes, gave 6·9 per cent. increase, &c. The combined results of *both* sets of experiments are given by Table 96.

TABLE 96.—Of the EXTENSION of WROUGHT IRON by TENSILE STRAINS: by Experiments.

Strain per Square Inch.				Extension in Parts of the Length.	Modulus of Elasticity in Lbs. per Square Inch.	Reduced Results.	
Experiment 1, Lbs.	Experiment 2, Lbs.	Interpolated. Lbs.	Interpolated. Tons.			Extension in Parts of the Length.	Modulus of Elasticity in Lbs. per Square Inch.
1,262	..	..	..	·000043330	29,125,300	..	..
..	..	2,240	1	·000080121	27,957,000	·00008	28,000,000
2,524	..	..	..	·000095830	26,338,000	..	..
..	2,667	..	..	·000082167	32,458,000	..	..
3,786	..	..	..	·000140820	26,885,000	..	..
..	..	4,480	2	·000159252	28,131,000	·00016	28,000,000
5,047	..	..	..	·000186670	27,000,000	..	..
..	5,335	..	..	·000185583	28,747,000	..	..
6,309	..	..	..	·000231000	27,311,000	..	..
..	..	6,720	3	·000241136	27,868,000	·00024	28,000,000
7,571	..	..	..	·000274830	27,539,000	..	..
..	8,002	..	..	·000283917	28,143,000	..	..
8,833	..	..	..	·000315830	27,967,000	..	..
..	..	8,960	4	·000319204	28,070,000	·00032	28,000,000
10,095	..	..	..	·000358300	28,172,000	..	..
..	10,670	..	..	·000379670	28,103,000	..	..
..	..	11,200	5	·000398725	28,090,000	·00040	28,000,000
11,357	..	..	..	·000404500	28,083,000	..	..
12,618	..	..	..	·000447500	28,196,000	..	..
..	13,338	..	..	·000475417	28,055,000	..	..
..	..	13,440	6	·000479029	28,057,000	·00048	28,000,000

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
13,880	..	..	..	·000495830	27,993,000	..	..
15,142	..	..	..	·000540000	28,041,000	..	..
..	..	15,680	7	·000558628	28,069,000	·00056	28,000,000
..	16,005	..	..	·000571167	28,022,000	..	..
16,404	..	..	..	·000581660	28,220,000	..	..
17,666	..	..	..	·000627500	28,153,000	..	..
..	..	17,920	8	·000638774	28,054,000	·00064	28,000,000
..	18,672	..	..	·000666083	28,034,000	..	..
18,928	..	..	..	·000680830	27,801,000	..	..
..	..	20,160	9	·000724527	27,825,000	·000725	27,806,900
20,190	..	..	..	·000728330	27,721,000	..	..
..	21,340	..	..	·000766083	27,856,000	..	..
21,451	..	..	..	·000775830	27,649,000	..	..
..	..	22,400	10	·000811457	27,606,000	·00081	27,654,000
22,713	..	..	..	·000826660	27,475,000	..	..
23,975	..	..	..	·000880830	27,186,000	..	..
..	24,008	..	..	·000873750	27,477,000	..	..
..	..	24,640	11	·000908845	27,111,000	·00091	27,077,000
25,237	..	..	..	·000937500	26,919,000	..	..
26,499	..	..	..	·001033300	25,645,000	..	..
..	26,675	..	..	·001013580	26,318,000	..	..
..	..	26,880	12	·001039175	25,867,000	·00104	25,846,000
27,761	..	..	..	·001073330	25,864,000	..	..
29,022	..	..	..	·001208300	24,011,000	..	..
..	..	29,120	13	·001244605	23,397,000	·00124	23,484,000
..	29,343	..	..	·001288170	22,779,000	..	..
30,284	..	..	..	·001659200	18,252,000	..	..
..	..	31,360	14	·002013840	15,572,000	·00200	15,680,000
..	32,011	..	..	·002228670	14,363,000	..	..

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(624.) The general conclusion from these experiments is, that for permanent structures the maximum working load on rolled bars of wrought iron should not exceed 8 tons per square inch, which is nearly one-third of the mean ultimate cohesive strength of British iron, namely, 25·7 tons per square inch, as shown by Table 1. Within the limit of 8 or 9 tons, the extensions are practically simply proportional to the strains, and may be taken approximately at ·00008 of the length per ton. With heavier strains, the extensions increase more rapidly than the load, and with all such strains are still further extended by time to an extent varying considerably with the ductility of the particular specimen of iron.

(625.) Another important practical lesson may be drawn from this investigation, namely, that *where the length is fixed*, whatever *initial* strain may be put on wrought iron, the *permanent* strain after a certain time will not exceed 8 or 9 tons per square inch. For instance, say that a wrought-iron hoop or wheel tire is shrunk hot on a cast-iron wheel, &c., as in Fig. 25 and (90), so that when cold it shrinks to such an extent as to yield at first say 20 tons per square inch; but that strain would not be permanent; the bar would immediately begin to stretch, and by so doing relieve the pressure upon it, at first very rapidly, afterwards, as the strain was reduced, more and more slowly, until after a long period it became 8 or 9 tons only, although the extent to which it was originally stretched was that due to 20 tons, and that strain was really obtained for a few seconds (90).

(626.) "*Compression of Wrought Iron.*"—Experiments were made by Mr. Hodgkinson on two bars of wrought iron, each about 1 inch square and 10 feet long:—they were enclosed in a frame to prevent lateral flexure, in the same way as with the cast-iron bars in (607). The results are given in Table 97, but the experimental strains on the bars (which were rather more than one inch square) are reduced to equivalent strains per square inch, and the observed compressions are reduced to parts of the length of the bar.

The compressions for even tons were obtained by interpolating between the greater and lesser experimental numbers.

The *mean* compression of the two bars is given by Table 98, and in cols. 4, 5, reduced results are given, which may be taken as sufficiently correct for ordinary purposes.

(627.) "*Effect of Time.*"—The effect of time in increasing the amount of compression was observed with one of the bars only, and that not before a strain of 30,858 lbs., or 13·77 tons, was applied: the influence of time was then found to be very great, the compression being increased $\cdot 002175 \div \cdot 00158333 = 1\cdot 37$, or 37 per cent. in $\frac{1}{2}$ an hour; 48 per cent. in $\frac{3}{4}$ hour; and 73 per cent. in $1\frac{1}{4}$ hour. In all probability, time would have been found influential with compressive strains of 8 or 10 tons per inch if observations had been made, and we may infer that for permanent structures the maximum working compressive strain should not exceed 8 tons per square inch, being the same as we found for the Tensile strain (625).

(628.) "*Comparative Extension and Compression of Wrought Iron.*"—We found (616) that with small strains, when defect of elasticity was uninfluential, cast iron yielded more to compressive than to equivalent tensile strains. A similar comparison of the elasticity of wrought iron leads yet more clearly to the same result, as shown by Table 91; the elasticity of wrought iron is so nearly perfect that the ratio is nearly the same with all strains up to 10 or 11 tons, the mean ratio in col. 7 from 1 to 11 tons is 1·236.

The length of rods and pillars of wrought iron that would be shortened 1 inch by different tensile and compressive strains is given by Table 93, and will suffice to give a general idea where it is desired to avoid the trouble of exact calculation.

(629.) "*Comparative Extension of Cast and Wrought Iron.*"—These two important materials are frequently combined in structures, and the differences in their elasticity cause unequal strains under circumstances where perfect equality might have been expected.

We will first consider the relative extensions of cast and wrought iron under the same tensile strain :—here the maximum strain must of course be limited by the strength of the weaker material of the two, namely, cast iron, which may be taken at

TABLE 97.—Of the COMPRESSION of WROUGHT IRON by CRUSHING STRAINS: from Direct Experiments on two Bars 1 Inch Square.

Compressive Strain per Square Inch.		Shortening in Parts of the Length.	Modulus of Elasticity E_c in Lbs. per Square Inch.
Tons.	Lbs.		
1	2,240	·00010463	21,409,000
2	4,480	·00020927	21,409,000
	4,852	·00023333	20,795,000
3	6,720	·00032112	20,926,000
4	8,960	·00042641	21,013,000
	9,116	·00043333	21,037,000
5	11,200	·00051967	21,552,000
	13,380	·00061000	21,934,000
6	13,440	·00061277	21,934,000
	15,513	·00070833	21,898,000
7	15,680	·00071551	21,914,000
	17,645	·00080000	22,062,000
8	17,920	·00081182	22,074,000
	19,777	·00089167	22,183,000
9	20,160	·00090981	22,158,000
	21,909	·00099267	22,071,000
10	22,400	·00101445	22,081,000
	24,049	·00108333	(22,191,000)
11	24,640	·00111115	22,168,000
	26,173	·00118333	22,115,000
12	26,880	·00122117	22,012,000
	28,305	·00128333	22,056,000
13	29,120	·00134698	21,619,000
	30,439	·00145000	20,993,000
14	31,360	·00159414	19,666,000
	32,569	·00178333	18,263,000
(1)	(2)	(3)	(4)

SECOND BAR.

1	2,240	·00009814	22,824,000
2	4,480	·00019629	22,824,000
	4,919	·00022500	21,886,000
3	6,720	·00029444	22,823,000
4	8,960	·00038082	23,528,000
	9,242	·00039167	23,596,000
5	11,200	·00046714	23,975,000
6	13,440	·00055352	24,281,000
	13,565	·00055833	(24,295,000)
(1)	(2)	(3)	(4)

TABLE 97.—Of the COMPRESSION of WROUGHT IRON by CRUSHING STRAINS—*continued.*

Compressive Strain per Square Inch.		Shortening in Parts of the Length.	Modulus of Elasticity E_c in Lbs. per Square Inch.
Tons.	Lbs.		
7	15,680	·00064843	24,181,000
	17,888	·00074266	24,089,000
8	17,920	·00074400	24,086,000
	20,050	·00083333	24,060,000
9	20,160	·00083884	24,033,000
	22,211	·00094167	23,586,000
10	22,400	·00095261	23,514,000
	24,372	·00106667	22,850,000
11	24,640	·00108216	22,769,000
	26,534	·00119167	22,265,000
12	26,880	·00121834	22,063,000
	28,696	·00135834	21,126,000
13	28,130	·00140340	20,756,000
	30,858	·00158333	19,490,000
	In ½ hour	·00217500	..
	" ¾ "	·00235000	..
	" 1¼ "	·00273334	..
(1)	(2)	(3)	(4)

7 tons per square inch. In Table 99 the cols. 2 and 3 have been taken from cols. 4, 4, of Tables 88, 96, and col. 4 gives the ratio which increases regularly throughout, this fact being due to defect of elasticity in the cast iron as opposed to the almost perfect elasticity of wrought iron, the ratio rising from 2·066 with 1 ton to 3·008 with 7 tons per square inch. Thus, even with so low a strain as 1 ton, the extension of cast iron is more than double that of wrought iron.

(630.) But this statement does not give a clear idea of the effect of the unequal resistance to the *same extension* which happens in those numerous cases where the two materials are so combined that a given load must of necessity stretch them both to the same extent. Thus, in Fig. 123, let A be a rod of cast iron one square inch in area, and B a similar one of wrought iron stretched simultaneously and of necessity to the same extent by the weight W. Now, with say 4 tons per square inch on A the extension by col. 4 of Table 88 would be ·00077282,

TABLE 98.—Of the MEAN COMPRESSION of WROUGHT IRON by CRUSHING STRAINS: from Direct Experiment.

Compressive Strain in Tons per Square Inch.	Mean Compression in Parts of the Length.	Modulus of Elasticity E_c in Lbs. per Square Inch.	Reduced Results.	
			Compression in Parts of the Length.	Modulus of Elasticity E_c in Lbs. per Square Inch.
1	·00010138	22,095,000	·0001	22,400,000
2	·00020278	22,093,000	·0002	22,400,000
3	·00030778	21,834,000	·0003	22,400,000
4	·00040362	22,199,000	·0004	22,400,000
5	·00049340	22,700,000	·0005	22,400,000
6	·00058314	23,047,000	·0006	22,400,000
7	·00068197	23,000,000	·0007	22,400,000
8	·00077791	23,036,000	·0008	22,400,000
9	·00087432	23,059,000	·0009	22,400,000
10	·00098353	22,775,000	·0010	22,400,000
11	·00109665	22,468,000	·0011	22,400,000
12	·00121975	22,037,000	·00122	22,033,000
13	·00137519	21,174,000	·00138	21,102,000
14	·00159414	19,666,000	·00164	19,122,000
(1)	(2)	(3)	(4)	(5)

TABLE 99.—Of the COMPARATIVE EXTENSION of CAST and WROUGHT IRON under the same TENSILE STRAINS.

Tons per Square Inch.	Extension in Parts of the Length.		Ratio. $\frac{c}{w}$
	Cast Iron. c.	Wrought Iron. w.	
1	·00016556	·000080121	2·066
2	·00034865	·000159253	2·189
3	·00054894	·000241136	2·276
4	·00077282	·000319204	2·421
5	·00102371	·000398725	2·567
6	·00134480	·000479029	2·807
7	·00167840	·000558628	3·008
(1)	(2)	(3)	(4)

and as B must also be stretched to that same extent, the question is, What will be the tensile strain due to that extension with wrought iron? By col. 4 of Table 96, the strain is some-

where between 9 and 10 tons per square inch, and by interpolation we shall find the exact strain to be 9·555 tons; the weight W in Fig. 123 is therefore 4 + 9·555 = 13·555 tons. Instead, therefore, of the two bars A and B dividing the load equally between themselves, we find that it is very unequally divided: Table 100 has been calculated in this way; col. 4 shows that the ratio of the strains is variable, attaining a maximum with 4 tons per square inch.

TABLE 100.—Of the COMPARATIVE RESISTANCE of CAST and WROUGHT IRON to the same amount of EXTENSION by TENSILE STRAINS.

Extension in Parts of the Length.	Tons per Square Inch.		Ratio. $\frac{w}{c}$
	Cast Iron. c.	Wrought Iron. w.	
·00016556	1	2·077	2·077
·00034865	2	4·370	2·185
·00054894	3	6·878	2·293
·00077282	4	9·555	(2·389)
·00102371	5	11·881	2·376
·00134480	6	13·132	2·189
·00167840	7	13·564	1·938
(1)	(2)	(3)	(4)

(631.) We found in (617) that the working safe tensile strain on cast iron was about $2\frac{1}{3}$ tons, with which the extension was ·000413; now with that extension the resistance of wrought iron would, by col. 5 of Table 91, be somewhere between 5 and 6 tons per square inch, and by interpolation we find the exact strain to be 5·31 tons, from which we find the weight W = 2·33 + 5·31 = 7·64 tons.

(632.) We have so far supposed that the bars A, B, were of the same area for the sake of illustration; obviously the areas might be adjusted so that the weight borne by the two bars would be equalized, but the strains *per square inch* could not be altered without a violation of correct principles. Thus, if the cross-sectional area of A were 1 square inch, then by making that of B 2·33 ÷ 5·31 = ·44 square inch, the weight borne by B would be the same as that borne by A, namely, ·44 × 5·31

= 2·33 tons, and we thus obtain W = 4·66 tons, the strain per square inch on the wrought-iron bar being still 5·31 tons.

(633.) It will be evident from this that with any ordinary combination of cast and wrought iron the full value of the working strength of the latter, namely, 8 tons, cannot be realised; for this and other reasons such combinations are not expedient, at least in cases where considerable strains have to be borne. With 8 tons per square inch on the wrought iron the extension by col. 4 of Table 96 would be ·000638774, and with that extension the strain on the cast iron by col. 4 of Table 88 would be nearly 3½ tons, or about *half* the breaking weight, which would not be safe.

(634.) "*Comparative Compression of Cast and Wrought Iron.*"—The relative ultimate cohesive strength of cast and wrought iron being 7·142 and 25·7 tons per square inch, we found in (629) that the strength of a combination of the two metals in resisting a *tensile* strain was governed potentially by that of the weak cast iron. But in resisting compressive strains this order is reversed, the ultimate strength of cast iron being 43 tons per square inch, while that of wrought iron does not practically exceed 13 or 14 tons (627) for even a temporary load, the metal yielding so much that it becomes valueless: for permanent loads the strain should not exceed 8 or 9 tons, as we have seen (624), while that for cast iron may be as much as 14 tons (618). Table 101 gives the compressions of cast and wrought iron under the same crushing strains.

(635.) In Fig. 124 let C be a cast-iron and D a wrought-iron pillar one square inch in area, so close to one another, &c., that they must of necessity be shortened equally by the weight W, and let the strain on C be 5 tons per square inch, with which by col. 4 of Table 89, the compression of the cast iron will be ·000899: with that compression the resistance of wrought iron by col. 2 of Table 98 would be between 9 and 10 tons per square inch; by interpolation we find the exact strain to be 9·226 tons per square inch, the weight W will therefore be 5 + 9·226 = 14·226 tons. Calculating in this way we obtain Table 102. Beyond 14 tons in col. 3, the

TABLE 101.—Of the COMPRESSION of CAST and WROUGHT IRON under the same CRUSHING STRAINS.

Tons per Square Inch.	Compression in Parts of the Length.		Ratio. $\frac{c}{w}$
	Cast Iron. c.	Wrought Iron. w.	
1	·0001695	·00010138	1·672
2	·0003505	·00020278	1·728
3	·0005405	·00030778	1·756
4	·0007124	·00040362	1·765
5	·0008990	·00049340	1·822
6	·0010876	·00058314	1·865
7	·0012804	·00068197	1·878
8	·0014841	·00077791	1·908
9	·0016770	·00087432	(1·918)
10	·0018722	·00098353	1·903
11	·0020680	·00109665	1·886
12	·0022812	·00121975	1·870
13	·0024774	·00137519	1·801
14	·0027547	·00159414	1·721
(1)	(2)	(3)	(4)

resistance of the wrought iron has been taken as constant, which is practically true even with strains of a very temporary character (634), to which alone, in fact, this part of the table strictly applies. With strains not exceeding 5 tons on the cast-iron and 9·226 tons per square inch on the wrought-iron bars, the Table may be regarded as accurate for permanent loads.

(636.) It appears from this investigation that in combinations of cast and wrought iron under compressive strains, the working load on the cast iron must not exceed 5 tons per square inch, although the ordinary safe strain is 14 tons, and the ultimate, or crushing strain 43 tons per square inch. This shows clearly the inexpediency of such combinations in ordinary cases and for heavy loads.

(637.) "*Extension and Compression of Timber, &c.*"—We have unfortunately no direct experiments on the extension, &c., of most materials, and shall be obliged to refer to experiments on the transverse strength and stiffness, and to calculate the longitudinal strains and elasticities from the transverse ones. There

TABLE 102.—Of the COMPARATIVE RESISTANCE of CAST and WROUGHT IRON to the same AMOUNT of COMPRESSION by CRUSHING STRAINS.

Compression in Parts of the Length.	Tons per Square Inch.		Ratio. $\frac{w}{c}$
	Cast Iron. c.	Wrought Iron. w.	
·0001695	1	1·672	1·672
·0003505	2	3·446	1·723
·0005405	3	5·525	1·842
·0007124	4	7·317	1·829
·0008990	5	9·226	(1·845)
·0010876	6	10·920	1·820
·0012804	7	12·390	1·770
·0014841	8	13·497	1·687
·0016770	9	14·372	1·597
·0018722	10	14·4	1·44
·0020680	11	14·4	1·31
·0022812	12	14·4	1·20
·0024774	13	14·4	1·11
·0027547	14	14·4	1·03
(1)	(2)	(3)	(4)

is considerable uncertainty in this method, which has also the further disadvantage of giving inseparably the result of the compressive and tensile strains combined, so that we cannot determine the value of either alone. It will, therefore, be well to see how far this method agrees with direct experiments on such materials as cast and wrought iron, whose strength and elasticity are known with certainty (600).

(638.) We require 1st from a known weight in the centre of a rectangular beam of given dimensions, to find the maximum longitudinal strains, or those at the upper and lower edges of the section. Obviously the strain varies from nothing at the neutral axis to a maximum at the upper and lower edges (494). Then 2nd, from the observed deflection we have to determine the extension and compression produced by those longitudinal strains.

To find the maximum longitudinal strains from the transverse load, we have the Rule (513), or $f = \frac{3 \times W \times l}{2 \times D^2 \times B}$. For a bar

1 inch square and 1 foot, or 12 inches, between supports, the rule evidently becomes $f = \frac{3 \times W \times 12}{2 \times 1^2 \times 1}$, or $\frac{36 \times W}{2}$, or $18 \times W$, from which we have the simple law for a bar of those sizes, &c., that the maximum longitudinal strain is 18 times the transverse load: hence we have the Rule:—

(639.) $$f = W \times 18.$$

We may now apply this Rule to Timber whose tensile and crushing strength is known by direct experiment: for the former we shall take Mr. Barlow's results in Table 1, and for the latter, Mr. Hodgkinson's in Table 32: the Transverse strengths will be given by Table 65.

(640.) A bar of ash 1 inch square and 1 foot long, breaks with a load of 681 lbs. in the centre, the maximum longitudinal strain $f = 681 \times 18 = 12258$ lbs. per square inch; by direct experiments, the tensile strength $T = 17077$ lbs.; and the crushing strength $C = 9023$ lbs. per square inch: the mean of the two is 13,050 lbs., or nearly as given by the Rule.

English oak breaks transversely with 509 lbs.; hence $f = 509 \times 18 = 9162$ lbs. per square inch: by direct experiments $T = 10389$ lbs., and $C = 8271$ lbs. per square inch; the mean of the two $= 9330$ lbs.

Beech breaks transversely with 558 lbs.: hence $f = 558 \times 18 = 10044$ lbs. per square inch; by direct experiments $T = 11467$ lbs., and $C = 8548$ lbs.; the mean being 10,007 lbs. per square inch.

Teak breaks transversely with 724 lbs.: hence $f = 724 \times 18 = 13032$ lbs. per square inch; by direct experiments $T = 15090$ and $C = 10706$, the mean being 12,898 lbs. per square inch, &c.

The application of the rule to cast and wrought iron is given in (520).

(641.) Having found the value of f, or the maximum strain at the upper and lower edges of the section, we have now to find the extension and compression there from the deflection of the beam with a given load in the centre, for which we have the Rule:—

(642.) $$E_z = \frac{3 \times D \times \delta}{2 \times (l \div 2)^2},$$

in which D = depth of rectangular bar in inches, l = length between supports in inches; δ = deflection at the centre in inches: and E_x = extension in parts of the length: it will be observed that the central load, and the breadth have nothing to do with the question in this case. The rule also supposes that the compression C is equal to E_x, which perhaps is nearly true for light strains (617) with most materials.

(643.) For a bar 1 inch square, and 12 inches long, $l \div 2 = 6$, and the rule becomes $E_x = \frac{3 \times 1 \times \delta}{2 \times 6^2}$, or $\frac{3 \times \delta}{72}$, or $\frac{\delta}{24}$, that is to say, the maximum extension and compression is $\frac{1}{24}$th of the transverse deflection.

Table 105 gives in col. 4 the mean deflection of bars of many materials, 1 inch square, 1 foot or 12 inches long, by 1 lb. in the centre, which by (638) is equivalent to 18 lbs. longitudinally. Thus with cast iron the rule $\frac{\delta}{24}$ becomes $\cdot00002886 \div 24 = \cdot0000012$ for 18 lbs., therefore $\cdot0000012 \div 18 = \cdot0000000667$ for 1 lb., or $\cdot0000000667 \times 2240 = \cdot0001494$ per ton, the length strained being $1 \cdot 0$.

The rule $\frac{\delta}{24}$ giving in this case the extension for 18 lbs. per square inch longitudinally, is equivalent to $\frac{\delta}{24 \times 18}$ or $\frac{\delta}{432}$ for 1 lb., and to $\frac{\delta \times 2240}{432}$ or $\delta \times 5 \cdot 18$ for 1 ton strain per square inch.

For a standard bar 1 inch square, and 1 foot long, we have the Rules:—

(644.) $$E_p = \frac{\delta}{432}.$$

(645.) $$E_t = \delta \times 5 \cdot 18,$$

in which E_p = the extension or compression in parts of the length per lb. per square inch: E_t = the extension or compression per ton per square inch: δ = the deflection in inches by 1 lb. in centre: thus for cast iron $\delta = \cdot00002886$ by col. 4

of Table 105; hence, Rule (645) becomes ·00002886 × 5·18 = ·0001494 as before (643).

(646.) Calculating in this way we have obtained col. 8 of Table 105. Comparing these results for cast and wrought iron, with those obtained by direct and exact experiment in Table 91, we have for cast iron ·0001494 by Table 105, while Table 91 gives for extension ·00016556, and for compression ·0001695 parts of the length, by 1 ton. For wrought iron, Table 105 gives ·00008106, while Table 91 gives for extension ·000080121, and for compression, ·00010138 parts of the length, by 1 ton. These results agree fairly well together; hence we may have confidence in the method we have followed, and by which col. 8 of Table 105 has been obtained.

We may now illustrate the application of these rules to practice: say we have a wrought-iron bar 50 feet, or 600 inches long, with a tensile strain of 8 tons per inch: then the extension = ·00008106 × 600 × 8 = 0·389 inch. Direct experiment gives by col. 5 of Table 91, ·000638774 × 600 = ·383 inch.

Again, a bar of Riga fir, with say 2 tons per inch, and a length of 30 feet, or 360 inches, will stretch ·00197 × 360 × 2 = 1·42 inch.

Again, a pillar of English oak 12 feet or 144 inches long, with a compressive strain of $\frac{3}{4}$ ton per square inch will be shortened by the pressure ·002042 × 144 × $\frac{3}{4}$ = 0·22 inch: this being the *mean*, col. 8.

CHAPTER XVI.

ON THE DEFLECTION OF BEAMS.

(647.) "*Form of Curve of Flexure.*"—With a parallel beam, or one having uniform depth and breadth throughout the length, resting on bearings at each end, and loaded with a central weight, the strain on the material is a maximum at the centre, and is progressively reduced toward the ends, where it becomes

nothing. In that case the elastic curve has its shortest radius at the centre, the curve becoming progressively flatter toward the ends, where it is a straight line.

When the strength at every point is proportional to the strain there, for example when the depth is uniform, and the breadth is reduced toward the ends in arithmetical ratio as in Fig. 116, the elastic curve is uniform in its radius from end-to-end, that is to say, it is a simple spherical curve.

"*Curve with Central Load.*"—Let Fig. 174 be a parallel beam resting on two bearings and loaded in the centre: then we have the Rule:—

(648.) $$y = \frac{3 \times \delta}{(\frac{1}{2}\,\mathrm{L})^3} \times \left(\frac{(\frac{1}{2}\,\mathrm{L}) \times x^2}{2} - \frac{x^3}{6}\right).$$

In which L = length of the beam between bearings.
δ = deflection in centre by central strain.
x = distance from centre to a point whose deflection is required.
y = co-ordinate of the curve at that point.

Of course all the dimensions must be in the same terms, feet, inches, &c.

(649.) Thus, with a beam 10 feet long, say $\delta = 0 \cdot 25$ foot; $x = 3$ feet: then by the Rule $y = \frac{3 \times \cdot 25}{125} \times \left(\frac{5 \times 9}{2} - \frac{27}{6}\right)$ $= 0 \cdot 108$ foot, or $1 \cdot 296$ inch: hence the deflection at that point is $3 - 1 \cdot 296 = 1 \cdot 704$ inch. Calculating in this way we may obtain any number of points through which the entire curve of flexure may be drawn. Table 103 has been calculated by the rule, the *half*-length of the beam being divided into 10 parts, and the central deflection $= 1 \cdot 0$, from which we may easily find the deflection at any point in a beam whose central deflection is known:—thus in our case, the point x being $3 \div 5$ $= 0 \cdot 6$ of the half-length distant from the centre, we find the deflection at that point $= 0 \cdot 568 \times 3 = 1 \cdot 704$ inch as before. The curve B in Diagram, Fig. 213, has been obtained from Table 103.

"*Load out of Centre.*"—Let Fig. 175 be a parallel beam with a load W out of the centre: knowing the deflection which any

TABLE 103.—Of the FORM of the CURVE of FLEXURE in a PARALLEL BEAM DEFLECTED by a CENTRAL WEIGHT, Fig. 212.

Distance from Centre.	Deflection at that Distance.	Value of y.	Distance from Centre.	Deflection at that Distance.	Value of y.
·0	1·0000	·0000	·6	·5680	·4320
·1	·9855	·0145	·7	·4365	·5635
·2	·9439	·0560	·8	·2960	·7040
·3	·8785	·1215	·9	·1495	·8505
·4	·7920	·2080	1·0	·0000	1·0000
·5	·6875	·3125	..	..	..
(1)	(2)	(3)	(1)	(2)	(3)

weight would produce at the centre, we may find the deflection at any other point by the same weight applied at that point, by the Rule:—

$$(650.) \qquad d = \frac{\delta}{(\frac{1}{2}\,L)^4} \times \left\{L \times l) - l^2\right\}^2.$$

In which l = the distance from the weight to the nearest bearing in the same terms as L; d = the deflection at the point of application of the weight W, and the rest as before. Thus taking the beam in (649) whose central deflection = 3 inches, and say we require the deflection at a point 3 feet from the end, therefore 2 feet from the centre:—then the Rule gives $d = \frac{0\cdot25}{5^4} \times \left\{10 \times 3) - 3^2\right\}^2 = 0\cdot1764$ foot, or $2\cdot1168$ inches. Table 104 has been calculated by the Rule, taking the central deflection = 1·0, and dividing the half-length into 20 parts, col. 2 gives the deflection at each point in the length by that same constant weight.

(651.) For example, in our case, the point is $2 \div 5 = 0\cdot40$ from the centre, hence the deflection there = $\cdot7056 \times 3 = 2\cdot1168$ inches as before. The curve A in Diagram, Fig. 213, has been obtained from col. 2 of Table 104, and gives the deflection from centre to end for a rolling constant load. For example, say we have a beam 24 feet long, and we require the deflection at 6 feet from the centre by a certain weight applied at that point. We have first to find what the central deflection

TABLE 104.—Of the RATIOS of DEFLECTION in BEAMS VARIOUSLY LOADED.

Distance of Weight from Centre. Z.	Ratio of Deflection with same Weight.	Product of Part-lengths.	Ratio of Safe Loads. W.	Ratio of Deflection with Safe Loads. D.	W × D.
·00	1·0000	20 × 20 = 400	1·000	1·0000	1·000
·05	·9950	21 × 19 = 399	1·003	·9975	"
·10	·9801	22 × 18 = 396	1·012	·9900	"
·15	·9555	23 × 17 = 391	1·023	·9775	"
·20	·9216	24 × 16 = 384	1·042	·9600	"
·25	·8789	25 × 15 = 375	1·067	·9375	"
·30	·8281	26 × 14 = 364	1·099	·9100	"
·35	·7700	27 × 13 = 351	1·140	·8775	"
·40	·7056	28 × 12 = 336	1·191	·8400	"
·45	·6360	29 × 11 = 319	1·254	·7975	"
·50	·5625	30 × 10 = 300	1·333	·7500	"
·55	·4865	31 × 9 = 279	1·434	·6975	"
·60	·4096	32 × 8 = 256	1·564	·6400	"
·65	·3335	33 × 7 = 231	1·732	·5775	"
·70	·2601	34 × 6 = 204	1·961	·5100	"
·75	·1914	35 × 5 = 175	2·286	·4375	"
·80	·1296	36 × 4 = 144	2·778	·3600	"
·85	·0770	37 × 3 = 111	3·604	·2775	"
·90	·0361	38 × 2 = 76	5·263	·1900	"
·95	·0095	39 × 1 = 39	10·260	·0975	"
1·00	·0000	40 × 0 = 0	infinite	·0000	"
(1)	(2)	(3)	(4)	(5)	(6)

would be with that same weight in the centre: say we find by calculation or experiment it was $1\frac{1}{4}$ inch: then the half-length being 12 feet, our point is obviously $6 \div 12 = 0 \cdot 5$, for which col. 2 gives 0·5625: hence the deflection at that point = $\cdot 5625 \times 1\frac{1}{4} = 0 \cdot 7$ inch.

(652.) "*Safe Load.*"—We have here taken the load as constant, at whatever point in the length it might be placed, but the *safe* load increases as it moves from the centre toward the end in inverse ratio of the product of the two parts into which the length is divided by the weight (420), for example with a beam 20 feet long, a weight in the centre divides it into two lengths, each 10 feet, and we have $10 \times 10 = 100$: now say

that the weight is 3 feet from one end, therefore 17 feet from the other: then, $3 \times 17 = 51$, or about half that with central load, showing that the *equivalent* load there is double the central load, the beam being equally strained in both cases, although the actual weights are in the ratio of 2 to 1, &c.

In Table 104 the whole length is divided into 40, and the half-length into 20 parts; col. 4 gives the ratio of the equivalent or safe load at each point, and col. 5 the ratio of the deflection at that point due to that load. Thus: the deflection with any central load being $1 \cdot 0$, at a point midway between the centre and the end, or $\cdot 5$ of the half-length, it would become $\cdot 5625$ with that same weight at that point by col. 2: but the *safe* load would then become $1 \cdot 0 \times 400 \div 300 = 1 \cdot 333$, as in col. 4, with which the deflection would be increased to $\cdot 5625 \times 1 \cdot 333 = \cdot 75$, as in col. 5. The curve C in Diagram, Fig. 213, has been obtained from col. 5, and this curve, it may be observed, is a parabola, but differs very slightly indeed from a simple spherical curve.

"*Curve of Flexure for Un-central Load.*"—When the load is out of the centre, as in Fig. 175, the elastic curve may be found by the Rule:—

$$(653.) \quad y = 3\,\delta \times \frac{(\frac{1}{2}\,\mathrm{L}) + 2}{(\frac{1}{2}\,\mathrm{L})^4} \times \left\{ \frac{(\frac{1}{2}\,\mathrm{L}] - z) \times 2}{3} \times z \times x \right) + \left(\frac{(\frac{1}{2}\,\mathrm{L}] - z) \times x^2}{2} - \frac{x^3}{6} \right\}.$$

In which L = length between bearings: δ = deflection in centre which a given weight would produce if placed there: z = the distance of the same weight from the centre of the beam: x = the distance from the weight towards the nearest support, of a point in the curve of flexure where the deflection is required: y = co-ordinate of the curve at that point.

(654.) The deflection produced by the weight at the point of application may be found by the Rule (650), &c.: for instance we have there calculated that with a certain beam 10 feet long, a weight which at the centre gave $\delta = 3$ inches, or $\frac{1}{4}$ foot, produced a deflection of $2 \cdot 1168$ inches at 2 feet from the centre when placed there. Now, if we make x = to the whole

distance from the weight to the support, y would obviously be $2 \cdot 1168$, or equal to the deflection: then

$$y = 3 \times \tfrac{1}{4} \times \frac{5+2}{625} \times \left\{\frac{5-2) \times 2}{3} \times 2 \times 3\right) + \left(\frac{5-2) \times 9}{2} - \frac{27}{6}\right\} = \cdot 1764 \text{ foot,}$$

or $2 \cdot 1168$ inches, as before. For the point B we have:—

$$y = 3 \times \tfrac{1}{4} \times \frac{5+2}{625} \times \left\{\frac{5-2) \times 2}{3} \times 2 \times 1\right) + \left(\frac{5-2) \times 1}{2} - \frac{1}{6}\right\} = \cdot 0448 \text{ foot,}$$

or $\cdot 5366$ inch: hence the deflection at B $= 2 \cdot 1168 - \cdot 5366 = 1 \cdot 5802$ inch. Similarly for the point C we have:—

$$y = 3 \times \tfrac{1}{4} \times \frac{5+2}{625} \times \left\{\frac{5-2) \times 2}{3} \times 2 \times 2\right) + \left(\frac{5-2) \times 4}{2} - \frac{8}{6}\right\} = \cdot 1064 \text{ foot,}$$

or $1 \cdot 2531$ inch: hence the deflection at C $= 2 \cdot 1168 - 1 \cdot 2531 = \cdot 8637$ inch, &c.

Thus the flexure at any number of points between the weight and 0 may be found, and by making z negative, the other part of the curve between the weight and n may be found also.

(655.) The curve of flexure has in all cases the shortest radius at the point where the weight is applied, showing that the strain is the greatest at that point, and that the beam will break there, but the deflection of the beam is not a maximum at that point except when the weight is at the centre. The deflection is a maximum between the weight and the centre of the beam, but much nearer the latter than the former: its position may be found by the Rule:—

$$(656.) \quad m = (\tfrac{1}{2}\,L] + z) - \left((\tfrac{1}{2}\,L)^2 + \frac{(\frac{1}{2}\,L) \times z \times 2}{3} - \frac{z^2}{3}\right)^{\frac{1}{2}}.$$

In which m = the distance of the weight from the point of

maximum deflection and the rest as before: see Fig. 175. Thus in the case of the beam in (654) we obtain $m = (5 + 2) - \left\{25 + \frac{5 \times 2 \times 2}{3} - \frac{4}{3}\right\}^{\frac{1}{2}} = 1 \cdot 5$ foot from the weight, therefore $2 - 1 \cdot 5 = 0 \cdot 5$ foot, or 6 inches from the centre.

As the distance of the weight from the centre increases, so does m increase, and it becomes a maximum when the weight is at the support, when $z = (\frac{1}{2}\,L)$, and it then becomes $m = (5 + 5) - \left\{25 + \frac{5 \times 5 \times 2}{3} - \frac{25}{3}\right\}^{\frac{1}{2}} = 4 \cdot 227$ feet from the weight (and the support), or $5 - 4 \cdot 227 = 0 \cdot 773$ foot from the centre, which is equal to $\cdot 773 \div 5 = \cdot 1546$ of the half-length of the beam. From this it appears that wherever a weight is placed, the point of maximum deflection can never be more than $0 \cdot 1546$ of the half-length distant from the centre.

(657.) Comparing the curves A and B in the Diagram, Fig. 213, we observe this remarkable fact; that the deflection at any and every point of a beam with a central load is greater than would be produced by that same weight at any other point: for example, the curve B = the deflection at every point throughout the length by a given central weight, and A = the deflection at the same point by the self-same weight placed there. This of course is due to the fact that the beam is less strained by a weight out of the centre than by the same weight in the centre. When strained at every point to the same extent, or in proportion to the strength, the deflections are given by the line C.

(658.) "*Laws of Deflection.*"—The Deflection of beams varies very much with the methods of fixing and loading (667): to simplify the matter we may take as a Standard case, that of a horizontal beam, supported at each end and loaded in the centre: other conditions may be considered afterwards. We then have the Rules—

(659.) $$\delta = \frac{L^3 \times W \times C}{d^3 \times b}.$$

(660.) $$d = \sqrt[3]{\left(\frac{L^3 \times W \times C}{\delta \times b}\right)}.$$

TABLE 105.—Of the DEFLECTION of BEAMS 1 foot long between Supports, 1 inch square, with 1 lb. in the Centre, being the Mean per lb. up to about One-third the Breaking Weight.

Material.	Deflection in Inches.			No. of Experiment.	Authorities.	Modulus of Elasticity E_D in Lbs. per Square Inch.	Extension and Compression per Ton per Square Inch.
	Maximum.	Minimum.	Mean.				
Cast Iron	·00003744	·00001866	·00002886	221	Hodgkinson	14,968,800	·00014940
Wrought Iron	..	..	·00001565	1	„	27,603,900	·00008106
Cast Steel	·00001514	·00001336	·00001433	4	Fairbairn	30,146,600	·00007423
Shear Steel	..	..	·00001386	1	Hodgkinson	31,169,000	·00007179
Bell-metal	..	..	·00003893	1	„	11,096,800	·00020166
Soft Brass	..	..	·00004123	1	„	10,477,900	·0002133
Slate (Bangor Split) ..	·0000445	·0000258	·0000364	7	Box	11,868,100	·0001885
Alder	..	..	·0003975	1	Ebbels	1,086,800	·002059
Ash	·0002695	·000257	·0002625	3	Barlow	1,645,720	·001360
„	·0002832	·000227	·0002551	2	Tredgold	1,693,450	·001321
„	..	..	·0002600	5	Mean	1,661,390	·001347
Beech	·0003343	·000311	·0003200	3	Barlow	1,350,000	·001658
„	..	..	·0003282	1	Ebbels	1,316,270	·001700
Birch	..	..	·0003536	1	„	1,221,720	·001832
Cedar of Lebanon	..	..	·0008889	1	Tredgold	488,237	·004604
Cherry-tree	..	..	·0003460	1	Ebbels	1,248,550	·001792
Chestnut, Horse	..	..	·0004051	1	Tredgold	1,066,410	·002098
„ Spanish	..	..	·0003800	2	„	1,136,850	·001968
„ „	..	..	·0004670	1	Ebbels	925,053	·002419
Elm, common	·0007725	·00053	·0006288	3	Barlow	687,022	·003257
„ Wych	..	..	·0003478	1	Ebbels	1,242,090	·001802
Fir, Riga	·000469	·000303	·0003804	6	Barlow	1,135,650	·001970
„ Yellow	..	..	·0002602	1	Tredgold	1,660,270	·001348
„ Memel	·0002338	·0002207	·0002223	2	„	1,943,330	·001152

„ Scotch	..	..	·0003596	1	„	1,201,340	·001863
„ Christiania, Spruce ..	..	..	·0002392	1	„	1,806,000	·001239
„ Quebec	..	..	·0003472	1	„	1,244,240	·001798
„ New England	·0003017	·0002681	·0002895	3	Barlow	1,492,230	·001500
„ Norway	..	..	·0002395	1	Tredgold	1,803,760	·001241
Larch	·0007463	·0003704	·0004944	13	Barlow	873,800	·002561
„	·0007078	·0003168	·0004562	3	Tredgold	946,950	·002363
„	..	..	·0004873	16	Mean	886,500	·002524
Lime-tree	..	..	·0003810	1	Tredgold	1,133,850	·001973
Mahogany, Honduras ..	..	..	·0002712	1	„	1,592,920	·001405
„ Spanish	..	..	·0003441	1	„	1,255,450	·001782
Maple, common	..	..	·0004923	1	„	877,514	·002550
Oak, English	·0005287	·0002939	·0003966	6	Barlow	1,089,260	·002070
„ „	·0006068	·0002520	·0004050	5	Tredgold	1,066,670	·002098
„ „	..	..	·0003330	1	Ebbels	1,297,300	·001725
„ „	..	..	·0003942	12	Mean	1,095,900	·002042
„ Canadian	·0002384	·0002092	·0002239	3	Barlow	1,929,440	·001160
„ Adriatic	·0005349	·0003327	·0004447	3	„	971,440	·002304
„ Dantzic	·0004327	·0002937	·0003708	3	„	1,165,050	·001921
Pear-tree	..	..	·0005378	1	Ebbels	803,273	·002786
Pine, Pitch	·0003626	·0003390	·0003527	3	Barlow	1,224,840	·001827
„ Red	·0002566	·0002177	·0002348	3	„	1,838,600	·001216
„ American	·0002684	·0002637	·0002661	2	Tredgold	1,623,450	·001378
Plane-tree	..	..	·0003216	1	Ebbels	1,343,280	·001660
Poplar, Lombardy	..	..	·0005663	1	„	762,850	·002933
Sycamore	..	..	·0004211	1	„	1,025,880	·002182
Teak	·0001984	·0001618	·0001790	3	Barlow	2,413,410	·000927
Walnut	..	..	·0005161	1	Ebbels	837,050	·002673
Willow	..	..	·0007627	1	Tredgold	566,409	·003951
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

TABLE 106.—Of the DEFLECTION of

Weight in Centre.	Length between supports, 13·5 ft. Depth .. .. 1·515 inch. Breadth .. .. 5·523 inches.			Permanent Set.	Lengths between supports, 13·5 ft. Depth .. .. .. 1·027 inch. Breadth .. .. .. 5·510 inches.		
	Deflection.		Modulus of Elasticity in Lbs. per Square Inch.		Deflection.		Modulus of Elasticity in Lbs. per Square Inch.
	By Experiment.	Calculated.			By Experiment.	Calculated.	
lbs.				inches.			
28	·051	·0561	31,092,700	..	..	..	..
56	·112	·1123	27,672,000	..	·3585	·3613	27,817,800
112	·232	·2246	26,717,000	..	·7200	·7226	27,701,900
168	·344	·3368	27,028,500	·001	1·0700	1·084	27,960,700
224	·458	·4491	27,067,900	·002	1·3940	1·445	28,616,000
280	·571	·5614	27,139,000	·003	1·7410	1·804	27,477,800
336	·684	·6737	27,186,600	·003	2·1590	2·168	27,778,500
392	·800	·7860	27,118,600	·004	2·5065	2·529	27,851,000
448	·916	·8982	27,067,900	·006	2·8015	2·890	28,478,100
504	1·005	1·010	27,754,700	·007	3·1635	3·252	28,371,800
560	1·124	1·223	27,573,600	·008	3·6095	3·613	27,629,000
616	1·222	1·235	27,895,000	·010	3·9595	3·974	27,705,300
672	1·332	1·374	27,921,400	·011	4·3370	4·336	27,593,300
728	1·434	1·459	28,096,500	·017	..	..	..
784	1·547	1·572	28,047,700	·019	..	..	..
840	1·693	1·684	27,459,600	·019	..	..	..
896	1·824	1·797	27,264,200	·019	..	..	..
952	1·933	1·909	27,256,900	·020	..	..	..
1008	2·044	2·021	27,293,000	·021	..	..	..
1064	2·165	2·133	27,199,100	·022	..	..	..
1120	2·256	2·246	27,470,000	..	..	..	..
1232	2·461	2·470	27,705,800	..	..	..	..
1288	2·546	2·582	27,998,100	..	..	..	..
1344	2·659	2·695	27,978,100	..	..	..	..
1568	3·057	3·144	28,387,100	·024	..	..	..
1792	3·567	3·592	27,803,300	..	..	..	..
2016	4·160	4·042	26,820,600	·268	..	..	..
Min.							
2240 = 5	5·930	4·491	20,905,000	..	..	..	..
" 10	6·51	"	"	2·233	..	..	..
" 15	6·65	"	"	..	..	..	..
	"Limit of Elasticity" = 1878 lbs.				"Limit of Elasticity" = 861 lbs.		
(1)	(2)	(3)	(4)	(5)	(2)	(3)	(4)

BARS OF ROLLED WROUGHT IRON.

ANNEALED IRON. Length between supports .. 6·75 feet. Depth .. .. 1·027 inch. Breadth .. .. 5·510 inches.			Length between supports, 13·5 feet. Depth .. .. 1·026 inch. Breadth .. .. 5·050 inches.		
Deflection.		Modulus of Elasticity in Lbs. per Square Inch.	Deflection.		Modulus of Elasticity in Lbs. per Square Inch.
By Experiment.	Calculated.		By Experiment.	Calculated.	
..	..	..	·1900	·1976	28,718,200
..	..	..	·3855	·3953	28,000,000
·0865	·09032	28,822,700	·7690	·7727	28,382,200
..	..	..	..	..	..
·1800	·18064	27,765,700	1·564	1·581	27,910,000
..	..	..	..	..	..
..	..	..	2·345	2·372	27,921,600
..	..	..	..	..	..
·3595	·36128	27,734,000	3·075	3·163	28,382,200
..	..	..	..	..	..
..	..	..	3·833	3·953	28,471,000
..	..	..	..	..	..
·5397	·5419	27,717,200	4·775	4·744	27,425,200
..	..	..	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
·7190	·7226	27,740,400	7·095	6·325	24,610,000
..	..	..	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
·9040	·9032	27,579,300	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
1·093	1·084	27,384,900	..	..	..
1·299	1·264	26,870,000	..	..	..
1·536	1·445	25,970,500	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
"Limit of Elasticity" = 1722 lbs.			"Limit of Elasticity" = 788 lbs.		
(2)	(3)	(4)	(2)	(3)	(4)

TABLE 107.—Of the TRANSVERSE STRENGTH and

Weight in Centre. Lbs.	Cammel and Co.'s "Diamond" Steel: Bar 1·054 Inch Square.			Cammel and Co.'s Chisel Steel: Bar 0·994 Inch Square.		
	By Experiment.	Calculated.	Modulus of Elasticity: Lbs. E_D.	By Experiment.	Calculated.	Modulus of Elasticity: Lbs. E_D.
100	·117	·1058	27,262,900	·141	·1338	28,599,600
200	·225	·2116	28,353,400	·268	·2676	30,093,500
300	·333	·3174	28,736,600	·398	·4014	30,396,000
400	·427	·4232	29,880,600	·522	·5352	30,900,650
500	·534	·5290	29,866,700	·653	·6690	30,877,000
600	·632	·6348	30,187,000	·804	·8028	30,093,500
700	·741	·7416	30,132,700	·924	·9366	30,549,500
800	·860	·8464	29,672,200	1·064	1·0704	30,300,000
900	·982	·9522	29,234,100	1·194	1·204	30,396,000
1000	1·092	1·058	29,210,000	1·274	1·338	31,652,500
1100	1·192	1·164	29,435,800	1·454	1·472	30,507,500
1150	1·242	1·217	29,534,800	1·504	1·539	30,833,800
1200	1·302	1·270	29,398,600	1·594	1·606	30,357,800
1300	1·372	1·375	30,223,700	1·924	1·739	27,246,800
1400	1·512	1·481	29,534,800	2·884	1·873	19,597,400
1450	1·562	1·534	29,610,000	3·294	1·940	17,759,800
1500	1·662	1·587	28,788,400	..	..	..
1600	1·832	1·693	27,858,200	..	..	..
1710	2·062	1·809	26,452,400	..	..	..
1766	2·302	1·868	24,470,000	..	..	..
1822	2·662	1·928	21,832,200	..	..	..
1878	3·042	1·987	19,692,200	..	..	..
1934	3·732	2·046	15,530,000	..	..	..
(1)	(2)	(3)	(4)	(2)	(3)	(4)

Sunk with 1934 lbs.: M_T = 7432 lbs.
Calculated "Limit of Elasticity" = 1457 lbs.

Sunk with 1450 lbs.: M_T = 6644 lbs.
"Limit of "Elasticity" = 1222 lbs.

DEFLECTION of STEEL BARS, 4½ feet long.

Brown and Co.'s Best Cast Steel for Turning Tools: Bar 0·97 Inch Square.			Brown and Co.'s Best Cast Steel for Chisels: Bar 0·97 Inch Square.		
By Experiment.	Calculated.	Modulus of Elasticity: Lbs. E_D.	By Experiment.	Calculated.	Modulus of Elasticity: Lbs. E_D.
·148	·1475	30,045,000	·166	·1475	26,787,100
·283	·2950	31,415,200	·310	·2950	28,688,100
·415	·4425	32,144,500	·462	·4425	28,874,400
·555	·5900	32,047,900	·614	·5900	28,968,300
·690	·7375	32,222,100	·772	·7375	28,799,600
·837	·8850	31,875,700	·932	·8850	28,626,500
·977	1·0325	31,859,400	1·082	1·0325	28,767,700
1·117	1·180	31,847,200	1·242	1·180	28,648,500
1·237	1·327	32,352,500	1·402	1·327	28,544,900
..	..	..	..	..	..
..	..	..	..	..	..
1·747	1·696	29,291,100	2·642	1·696	19,355,300
..	..	..	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
..	..	..	..	..	..
(2)	(3)	(4)	(2)	(3)	(4)

Sunk with 1400 lbs.: M_T = 6903 lbs.
"Limit of Elasticity" = 1136 lbs.

Sunk with 1150 lbs.: M_T = 5671 lbs.
"Limit of Elasticity" = 1136 lbs.

(661.) $$b = \frac{L^3 \times W \times C}{\delta \times d^3}.$$

(662.) $$W = \frac{d^3 \times b \times \delta}{L^3 \times C}.$$

(663.) $$L = \sqrt[3]{\left(\frac{d^3 \times b \times \delta}{W \times C}\right)}.$$

(664.) $$C = \frac{d^3 \times b \times \delta}{L^3 \times W}.$$

In which W = weight or load in lbs., tons, &c., dependent on the value of C.
d = depth, in inches.
b = breadth, in inches.
L = length between bearings, in Feet.
δ = deflection, in inches.
C = Constant derived from experiment, in lbs., tons, &c.

(665.) The mean value of C for most ordinary Materials is given by col. 4, &c., of Table 105. Table 64 gives in col. 6 the mean value for 54 different kinds of Cast iron = ·00002886. Table 106 gives the result of experiments on the Deflection of wrought-iron bars by Mr. Hodgkinson; cols. 3, 3, 3, 3 having been calculated by Rule (659), the value of C being taken at ·00001565 from col. 4 of Table 105: thus for the bar in which L = 13·5 feet, d = 1·515; b = 5·523, and say W = 112 lbs., we obtain $\delta = \frac{13\cdot5^3 \times 112 \times \cdot00001565}{1\cdot515^3 \times 5\cdot523}$ = 0·2246 inch deflection, as in col. 3 of Table 106. Col. 3 of Table 70 has also been calculated by this rule: col. 4 shows very clearly the effect of defect of Elasticity, the Ratio rising from 1·0 with light loads to 13·13 with Breaking-down load.

Table 107 gives the result of Mr. Fairbairn's experiments on the deflection of Steel; cols. 3, 3, 3, 3 have been calculated by Rule (659), the value of C being taken at ·00001433 from col. 4

of Table 105; thus for the bar in which $L = 4 \cdot 5$ feet; $d = 1 \cdot 054$; $b = 1 \cdot 054$, and say $W = 1000$ lbs., we obtain $\delta = \frac{4 \cdot 5^3 \times 1000 \times \cdot 00001433}{1 \cdot 054^3 \times 1 \cdot 054} = 1 \cdot 058$ inch deflection, as in col. 3 of Table 107.

(666.) Table 108 gives the result of special experiments on Blaenavon Cast iron; it will be found that with small loads, say up to ⅓rd of the breaking weight, the deflections will be given by Rule (659) with moderate accuracy, as shown by col. 7, but as the load increases the experimental deflections exceed the calculated ones more and more. This fact is due to *defect of elasticity*, leading to the necessity for special Rules for Cast iron under heavy strains: this matter is fully considered in (688). Col. 7 has been calculated by Rule (659), taking the value of C for Blaenavon iron, at ·00003133 from col. 6 of Table 64. Thus for the bar in Table 108, in which $L = 13 \cdot 5$; $d = 1 \cdot 522$; $b = 3 \cdot 066$; and say $W = 112$ lbs., we obtain $\delta = \frac{13 \cdot 5^3 \times 112 \times \cdot 00003133}{1 \cdot 522^3 \times 3 \cdot 066} = 0 \cdot 7987$, or say $0 \cdot 8$ inch, &c., as in col. 7 of Table 108.

Table 112 gives in cols. 2, 6 the deflections of two large beams of American Pine from the experiments of Mr. Edwin Clark; cols. 3 and 7 give the calculated deflections by Rule (659). Taking the value of C by Tredgold's experiments in col. 4 of Table 105, at ·0002661 inch, we obtain for say 3653 lbs. $\delta = \frac{15^3 \times 3653 \times \cdot 0002661}{12^3 \times 12} = 0 \cdot 1582$ inch deflection, as in col. 3: experiment gave $0 \cdot 15$ in col. 2. It will be observed that up to the safe load, say ⅓th of the breaking weight (888), the deflections as calculated agree fairly with experiment, but as the load is increased, the actual deflections are more and more in excess of those given by the rule, this being due to defect of Elasticity (692).

(667.) "*Effect of Modes of Fixing and Loading.*"—When the deflection for the Standard case of a beam, having the load in centre and supported at the ends, is known by calculation or

TABLE 108.—Of the DEFLECTION, &c., of BARS of BLAENAVON IRON, 13½ feet long, 1·522 inch deep, 3·066 inches wide, &c.

Weight in Centre.		Deflection.					Permanent Set.			Modulus of Elasticity. E_D.
		By Experiment.		By Calculation.			By Experiment.		By Rule (762).	
Lbs.	Ratio to Breaking Weight.	Total Inches.	By each successive 56 Lbs.	By Rule (690).	By each successive 56 Lbs.	By the ordinary Rule.	Total Inches.	By each successive 56 Lbs.		
28	·033	·1810	·3620	·1976	·3952	0·2	·0016	..	·0012	15,216,300
56	·067	·3574	·3754	·4063	·4063	0·4	·0068	·0068	·0049	14,663,900
112	·133	·7686	·3932	·8574	·4511	0·8	·0192	·0124	·0196	14,328,000
168	·200	1·184	·4154	1·353	·4956	1·2	·0468	·0276	·0441	13,951,600
224	·267	1·632	·4480	1·894	·5410	1·6	·0914	·0446	·0784	13,495,700
280	·333	2·105	·4730	2·479	·5850	2·0	·1486	·0572	·1225	13,079,000
336	·400	2·604	·4990	3·109	·6300	2·4	·2266	·0780	·176	12,687,200
392	·467	3·169	·5650	3·784	·6750	2·8	·3292	·1026	·240	12,162,700
448	·533	3·756	·5870	4·503	·7190	3·2	·4574	·1282	·314	11,727,900
504	·600	4·402	·6460	5·268	·7650	3·6	·6078	·1504	·397	11,257,700
560	·667	5·035	·6330	6·077	·8090	4·0	·7854	·1776	·490	10,936,000
616	·733	5·777	·7420	6·930	·8530	4·4	1·038	·2526	·593	10,484,500
672	·800	6·565	·7880	7·829	·9000	4·8	1·287	·2490	·706	10,064,800
728	·867	7·610	1·045	8·772	·9430	5·2	1·707	·4200	·828	9,406,200
784	·933	8·730	1·120	9·760	·9880	5·6	2·186	·4790	·960	8,830,180
840	1·000	9·887	1·157	10·790	1·0300	6·0	2·691	·5050	1·103	8,353,770
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)

experiment, the effect of other conditions may be most readily found by the use of Constants: we then have the ratios—

Beam supported at ends and weight in centre	deflection	=	1·0
Beam supported at ends, load equally distributed all over	"	=	$\frac{5}{8}$
Beam built into walls, &c., at each end, load in centre	"	=	$\frac{2}{3}$
Beam built into walls, &c., load equally distributed all over	"	=	$\frac{5}{12}$
Beam fixed at one end and loaded at the other	"	=	32
Beam fixed at one end, load equally distributed all over	"	=	12

In all these cases, the weight is supposed to be constant. There is considerable uncertainty in the deflection of beams fixed at one end, arising from irregularities in fixing. This is shown by Mr. Fincham's experiments, who found the ratio to vary from 18·6 to 44·5, the mean of 14 experiments being 28, whereas, the theoretical ratio, as we have shown, is 32.

(668.) "*Ratio of Round and Square Sections.*"—Theoretically a round bar deflects more than a square one in the ratio of 1·7 to 1·0, the weight, &c., being the same in both cases, and this ratio should be the same for all materials. It is probable that this ratio is correct for light strains, but when the breaking weight is approached the conditions are changed, and the ratio of stiffness seems to change also: from the inadequate experiments we have the experimental ratio is for wrought iron 1·6 to 1·0, and for cast iron 1·5 to 1·0.

"*Cast-iron* ⌶ *Sections.*"—When the top and bottom flanges of a girder are equal to one another the theoretical Rule for deflection is

$$(669.)\quad \delta = L^3 \times W \times C \div \left\{\frac{D^4 \times B) - (d^4 \times b}{D}\right\}.$$

In which D = the total depth, B = breadth of flanges, and d = the depth between top and bottom flanges, b = breadth of flange

minus the thickness of the vertical web, all in inches; L = the length of the beam between supports in feet; W = central load in tons, lbs., &c., dependent on C, the value of which is given by col. 4 of Table 105: col. 6 of Table 64, &c.

"*Old Rule.*"—The Rule commonly used, although not so correct in principle, will give results which agree better with experiment: this rule becomes

(670.) $\delta = L^3 \times W \times C \div \left\{ D^3 \times B) - (d^3 \times b \right\}$.

"*Unequal-flanged Sections.*"—In ordinary cases, the flanges of cast-iron girders are unequal, as in Fig. 79, which is the section of large girders experimented upon by Mr. Owen, whose results are given by Table 68. For such sections we have the Rule:—

(671.)

$$\delta = L^3 \times W \times C \div \left\{ D^3 \times B) - (d^3 \times b] + [d_0^3 \times b_0 \right\}.$$

In which D = total depth, d = total depth minus that of the bottom flange, d_0 = the depth between top and bottom flanges: B = breadth of bottom flange, b = the breadth of bottom flange minus that of the top one, b_0 = breadth of top flange, minus the thickness of the web. Thus in Fig. 79, D = 14, $d = 14 - 1\frac{3}{4} = 12\frac{1}{4}$, $d_0 = 11\frac{1}{4}$, B = 12, $b = 12 - 3\frac{1}{2} = 8\frac{1}{2}$, $b_0 = 3\frac{1}{2} - 1 = 2\frac{1}{2}$: then taking C from col. 6 of Table 64 = ·00002886, and W = say 7 tons, or 15,680 lbs., and L = 16 feet, we obtain $\delta = 16^3 \times 15680 \times \cdot 00002886 \div \left\{ 14^3 \times 12) - (12\frac{1}{4}^3 \times 8\frac{1}{2}] + [11\frac{1}{4}^3 \times 2\frac{1}{2} \right\}$ = ·1349 inch deflection of a *parallel* beam, but in our case the flanges were bellied, as in Fig. 131, when the deflections are greater in the ratio 1·44 to 1·0 (701), hence ·1349 × 1·44 = ·1942 inch deflection. The experimental deflections were very variable, as shown by Table 68, ranging in 11 specimens from ·14 inch to ·42 inch, or in the ratio 1 to 3.

"*Cast Iron* ⊥ *and* ⊤ *Sections.*"—We found in (344) that the transverse *strength* of these sections depends on their position, being greater in ⊥ than in ⊤ in the Ratio 3·08 to 1·0 in that particular case. But the *stiffness* of such beams is the same in either position, as shown by Mr. Hodgkinson's experiments: thus in Fig. 72, G and H were practically identical, and with a length of $6\frac{1}{2}$ feet the results were —

		⊤	⊥
With 14 lbs.,	the deflections were	·032 inch, and	·025 inch.
21 „	„	·046 „	·045 „
28 „	„	·064 „	·065 „
56 „	„	·130 „	·134 „
112 „	„	·273 „	·270 „

Showing almost perfect equality up to 112 lbs., which is about $\frac{1}{3}$rd of the breaking weight in position ⊤ (364 lbs.), but $\frac{1}{10}$th only in position ⊥ (1120 lbs.). The old Rule (670) will give the same deflection in either position: thus with Fig. 72, and W = say 112 lbs. we obtain $\delta = 6\frac{1}{2}^3 \times 112 \times {\cdot}00002886 \div \left\{1{\cdot}55^3 \times 5) - (1{\cdot}25^3 \times 4{\cdot}64\right\} = {\cdot}1853$ inch deflection: experiment gave ·273 inch.

(672.) "*Wrought-iron ⌶ Sections.*"—The Rules we have given for cast-iron ⊥, ⊤, and ⌶ sections will apply equally to wrought-iron ones, with the proper value of C, which for lbs. = ·00001565 by col. 4 of Table 105. Thus, with Fig. 154, by Rule (669) $D^4 = 10000$, d^4 or $8\frac{3}{4}^4 = 5862$, $b = 4\frac{3}{4} - \frac{1}{2} = 4\frac{1}{4}$, &c., L^3 or $18^3 = 5832$, and with W = say $30\frac{1}{4}$ cwt. or 33,880 lbs. we obtain $\delta = 5832 \times 33880 \times {\cdot}00001565 \div \left\{\frac{10000 \times 4\frac{3}{4}) - (5862 \times 4\frac{1}{4}}{10}\right\} =$ ·1369 inch deflection: experiment gave ·16 inch. Table 73 gives in col. 8 the experimental deflection of a series of rolled beams of ordinary equal-flanged sections: col. 9 has been calculated by Rule (669) and shows an error of − 14 per cent. The old Rule (670), although not so correct in principle, will give results which agree better with experiment; col. 10 has been calculated by that rule; the mean error of the whole is + 5 per cent. only.

"*Unequal Sections.*"—When the top and bottom flanges are unequal, as in Fig. 80, the most correct method of calculation will be to estimate from the bottom or the line N. A., as we found to be necessary in calculating the *strength* in (378): we then have the rule:—

$$(673.) \quad \delta = L^3 \times W \times {\cdot}00001565 \div \Big\{D^3 - d^3] \times B\Big) + \Big(d^3 - d_1^3] \times b\Big) + \Big(d_1^3 \times C\Big\}.$$

TABLE 109.—Of the DEFLECTION of ROLLED IRON I BEAMS.

Fig. 90: 10 Feet Long.			Fig. 89: 11 Feet Long.			
Weight.	By Experiment.	By Rule (673).	Weight.	By Experiment.	By Rule (673).	By Rule (671).
lbs.			lbs.			
885	..	·0237	885	·04	·0475	·0323
2,631	·04	·0704	2,581	·12	·1385	·0942
4,358	·12	·1165	4,317	·20	·2317	·1575
6,098	·15	·1630	6,050	·26	·3248	·2207
7,827	·19	·2093	7,743	·35	·4157	·2460
9,585	·21	·2563	9,493	·46	·5096	·3511
11,278	·26	·3086	11,253	·60	·6041	·4105
12,980	·30	·3471	12,955 =	failed	..	..
14,693	·35	·3930	..	..	..	..
16,373	·45	·4378	..	..	..	..
18,115	·68	·4845	..	..	..	..
18,962 =	failed	..	..	..	..	..
(1)	(2)	(3)	(4)	(5)	(6)	(7)

The values of D, d, d_1, B, b, and C, are given by Fig. 80, and the rest as in (664). Figs. 89, 90 are sections of beams experimented upon by Mr. Fairbairn, the deflections being given in cols. 2, 5 of Table 109. Thus in Fig. 89, with 885 lbs., the Rule gives

$$\delta = 11^3 \times 885 \times \cdot 00001565 \div \left\{ 7^3 - 6^3] \times 2\tfrac{1}{2} \right)$$
$$+ \left(6^3 - \cdot 38^3] \times \cdot 325 \right) + \left(\cdot 38^3 \times 4 \right\} = \cdot 04751 \text{ inch}$$

deflection: experiment gave ·04 inch, &c. Cols. 3, 6 of Table 109 have been calculated by this rule, and show a fair agreement with experiment: the calculated deflections with Fig. 89 show an error of + 11·9 per cent., and of Fig. 90, + 2·3 per cent.: the mean of the whole being + 6·4 per cent.

It is remarkable that this rule applied to sections with equal flanges does not give satisfactory results: col. 11 of Table 73 has been calculated by it and shows a mean error of + 42 per cent.; while Rule (669) gave − 14, and Rule (670), + 5 per cent. In order to render Table 73 directly available for practical purposes, we have given in col. 13 the experimental

deflection for each section by 1 cwt. in the centre of a beam 1 foot between supports, and as the deflections are simply proportional to the cube of the length and the weight, we have the Rule:—

(674.) $$\delta = T_D \times L^3 \times W.$$

In which T_D = the Tabular number in col. 13; L = length in feet; W = weight in cwts.; and δ = deflection in inches. Thus with No. 6, say L = 20 feet; W = 35 cwt.: then we obtain $\cdot 00000239 \times 8000 \times 35 = 0\cdot 6692$ inch deflection: experiment gave $\frac{5}{8}$ inch, col. 8.

(675.) "*Wrought-iron* T *Sections.*"—This form of section in wrought iron should always be loaded with the top flange uppermost in the ordinary case of a beam supported at both ends, for reasons given in (377), we must then calculate the deflections by measuring the depths from the bottom or from the line N. A. in Fig. 132, and we have the Rule:—

(676.)
$$\delta = L^3 \times W \times \cdot 00001565 \div \left\{ D^3 - d^3 \right] \times B \Big) + \Big(d^3 \times b \right\}.$$

In which the values of D, d, B, b, are given by Fig. 132 and the rest as in (664). Thus, Fig. 87 is the section of a bar, which with a length of 10 feet, deflected $\frac{1}{4}$ inch with 2 cwt. in the centre: then the Rule gives $\delta = 10^3 \times 224 \times \cdot 00001565 \div \left\{ 2\tfrac{1}{2}^3 - 2\tfrac{1}{4}^3 \right] \times 2\tfrac{1}{2} \Big) + \Big(2\tfrac{1}{4}^3 \times \tfrac{3}{8} \right\} = 0\cdot 2359$ inch deflection: experiment gave $0\cdot 25$ inch, &c. Table 71 gives in col. 11 the experimental deflections of a series of T iron bars: col. 12 has been calculated by the Rule.

When the depth is equal to the breadth and the thickness is the same all over, the rule becomes

(677.) $$\delta = L^3 \times W \times 00001565 \div (D^4 - d^4).$$

In which the section A is regarded as composed of two bars B, C, as in Fig. 70, which is not strictly correct, as explained in (337), a more correct rule would be:—

(678.) $$\delta = L^3 \times W \times 00001565 \div \left(\frac{D^5 - d^5}{D} \right).$$

The effect of the two rules may be shown if we take the section Fig. 132, and calculate the deflection by both, for say 12 cwt., or 1344 lbs., with a length of 10 feet. Then Rule (677) becomes $\delta = 10^3 \times 1344 \times \cdot00001565 \div (3^4 - 2\frac{1}{2}^4) = 0 \cdot 5008$ inch deflection. By Rule (678) we obtain $\delta = 10^3 \times 1344 \times \cdot00001565 \div \left(\frac{3^5 - 2\frac{1}{2}^5}{3}\right) = 0 \cdot 4341$ inch deflection.

TABLE 110.—Of the DEFLECTION of ROLLED WROUGHT-IRON T BEAMS, 1 foot long, with a weight of 1 cwt. in the Centre, the flange being uppermost.

Depth of Beam and Width of Flange.	Thickness all over.					
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$
	Deflection in Inches.					
$1\frac{1}{2}$	·000677	..	..	..	..	..
2	·000268	·0002247	..	..	..	..
$2\frac{1}{2}$	·000132	·0001097	..	..	..	..
3	..	·0000615	·0000529	·00004227	..	..
$3\frac{1}{2}$	..	·0000379	·0000325	·00002041	..	..
4	..	..	·0000213	·00001673	·00001404	..
5	..	..	·0000106	·00000825	·00000685	·00000593
6	..	..	..	·00000465	·00000384	·00000331

Table 110 gives the deflection of Standard sizes of T iron bars 1 foot long, with a load of 1 cwt. in the centre, calculated by the Rule :—

(679.) $$\delta = \cdot001773 \div (D^4 - d^4).$$

The deflection for any load and length may be easily found from Table 110 by the Rule (674): thus, a T bar $4 \times 4 \times \frac{1}{2}$ inch thick, 20 feet long, with 10 cwt. in the centre, will deflect $\cdot00001673 \times 8000 \times 10 = 1 \cdot 34$ inch, &c.

(680.) *Deflection of Wrought-iron Lattice Girders.*"—The deflection of lattice girders may be calculated from elementary principles. To do this with scientific accuracy is a difficult mathematical problem, but we may obtain approximately correct results by ordinary reasoning and common arithmetic.

In a lattice beam the strength is almost entirely in the top and bottom members, and the deflection of the beam arises from the alteration in length which those members suffer by their respective strains:—in the case of a beam supported at the ends and loaded in the centre, the top suffers a crushing strain and becomes shorter, while the bottom bears a tensile strain and becomes longer. If we know the respective strains we may calculate the corresponding extension and compression—and knowing these we can calculate the deflection of the beam. Let A in Fig. 188 be a beam deflected by a transverse strain to the form shown:—for ordinary cases in practice in which the deflections are very small compared to the length of the beam, we may admit that the difference in length of the top and bottom members arising from the deflection is equal to the sum of B and C, and knowing one, say C, we may calculate G, F, which is greater than C in the ratio of D, G to D, E, or in other words the ratio of half the length of the beam to its depth, D, F being perpendicular to D, E and a *tangent* to the curve D, H at the point D. Having thus found G, F, we may easily calculate G, H, or the deflection required, for the curve of the beam approximates to a parabola, and it is a principle that the height of a parabola J, K in Fig. 189, is always half the distance J, L, L being the point in the axis where a tangent to the base of the parabola at M, cuts the axis. Returning to Fig. 188 we thus find that the deflection sought, G, H, is equal to half the distance G, F.

(681.) The application of all this will be best illustrated by a case in practice worked out in detail. We will take the case in (446) of a lattice girder, Fig. 155, 32 feet between supports, loaded in the centre with 4 tons, the section being as shown by Fig. 141, the top is formed by two angle-irons $4 \times 2\frac{1}{2} \times \frac{1}{2}$, whose united area = 6 square inches, the rail R gives $1 \cdot 6$ square inches more, making the total area of the top = $7 \cdot 6$ square inches. The bottom, formed of two angle-irons $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{2}$, has an area of $4 \cdot 5$ square inches.

(682.) In calculating the deflection from these data, we have first to find the strains on the top and bottom members of the girder; these are equal to one another in all cases, but are not

uniform from end to end. Fig. 155 shows that the strain is a maximum at the centre and diminishes to nothing at the supports, in an arithmetical ratio. The *sum* of all the strains in the top or bottom is 128 tons, and the number of bays being 16, we obtain $128 \div 16 = 8$ tons as the *mean* strain from end to end. The same result may be attained thus:—4 tons in the centre is equal to 2 tons on each support, the half-length of the beam being 16 feet and the depth 2 feet, the maximum central strain is $2 \times 16 \div 2 = 16$ tons, and the mean strain from end to end $16 \div 2 = 8$ tons, as before.

(683.) Now, this strain has in the case of the top to be borne by $7 \cdot 6$ square inches, hence it is equal to $8 \div 7 \cdot 6 = 1 \cdot 05$ ton per square inch compressive strain, and the length of the bar being 384 inches, and the compression $\cdot 0001$ per ton by col. 4 of Table 98, we have $0001 \times 1 \cdot 05 \times 384 = \cdot 04032$ inch as the reduction in length of the top due to the compressive strain. Then, in the bottom flange, $4 \cdot 5$ square inches bear 8 tons, or $8 \div 4 \cdot 5 = 1 \cdot 78$ ton per square inch; by col. 6 of Table 96 the extension is $\cdot 00008$ per ton, hence we get $\cdot 00008 \times 1 \cdot 78 \times 384 = \cdot 05468$ inch as the extension of the bottom by tensile strain. Adding these together, we obtain $\cdot 04032 + \cdot 05468 = \cdot 095$ inch for the difference in the length of the top and bottom arising from the strains on them, or the sum of B and C in Fig. 188, and as these are equal to one another, and we require only one (say C), we have $\cdot 095 \div 2 = \cdot 0475 = C$, from which we get G, F $= \cdot 0475 \times 192 \div 23 = \cdot 392$ inch, and hence G, H, or the deflection sought, will be $\cdot 392 \div 2 = \cdot 196$ inch. The experimental deflection of a girder with these proportions and load was roughly measured as ¼ inch bare. It will be observed that the *effective* depth of the girder is the distance between the centres of gravity of the top and bottom angle-irons (449), and is taken in the above at 23 inches, the half-length of the girder being 192 inches.

(684.) "*Deflection of Tubular Bridges.*"—The approximate method we have explained and illustrated is not intended to supersede more precise modes of calculation, such as large and important works may demand, still we may obtain by it moderately correct results. We will take the case of the well-known

Conway Tube, 400 feet between supports, the experimental deflection of which was $3 \cdot 05$ inches with 301 tons near the centre. The depth at the *centre of effort* or the centre of gravity of the cells (449) was $22\frac{1}{4}$ feet at the middle, and $20\frac{3}{4}$ feet at the ends, the mean being $21\frac{1}{2}$ feet. The area at the top was 614, and at the bottom 460 square inches (taking a mean between the central and end areas). Then 301 tons in the centre $= 150 \cdot 5$ tons on each support, hence we have $150 \cdot 5 \times 200 \div 21 \cdot 5 = 1400$ tons central maximum strain, or $1400 \div 2 = 700$ tons mean strain from end to end on both top and bottom. This is equal to $700 \div 460 = 1 \cdot 525$ ton per square inch tensile, and $700 \div 614 = 1 \cdot 14$ ton per square inch compressive strain. The extension will then be $\cdot 00008 \times 1 \cdot 525 \times 4800 = \cdot 5846$ inch, and the compression $\cdot 0001 \times 1 \cdot 14 \times 4800 = \cdot 5472$ inch. The sum of the two ($B + C$ in Fig. 188) is $\cdot 5846 + \cdot 5472 = 1 \cdot 1318$ inch, hence $C = 1 \cdot 1318 \div 2 = \cdot 5659$ inch, G, $F = \cdot 5659 \times 200 \div 20 \cdot 75 = 5 \cdot 454$ inches, and G, H, or the deflection sought, $5 \cdot 454 \div 2 = 2 \cdot 727$ inches, which is $3 \cdot 05 - 2 \cdot 727 = \cdot 323$ inch less than by experiment.

(685.) "*Deflection of Plate-iron Girders.*"—The deflections of plate-iron beams may be calculated on the same principles as those of lattice girders. We will take the case of a beam experimented upon by Mr. Fairbairn, and shown in section by Fig. 133, the length between supports was 20 feet, and the deflection with 3 tons in the centre was $0 \cdot 17$ inch. With 3 tons in the centre we have $1 \cdot 5$ ton on each support, and the *effective* depth (449), or the distance between the centres of gravity of its top and bottom members being $14 \cdot 5$ inches, the maximum central strain becomes $1 \cdot 5 \times 120 \div 14 \cdot 5 = 12 \cdot 4$ tons, or $6 \cdot 2$ tons mean strain from end to end. The area of the top was $4 \cdot 55$ square inches, hence we have $6 \cdot 2 \div 4 \cdot 55 = 1 \cdot 362$ ton per square inch, the compression due to which is $\cdot 0001 \times 1 \cdot 362 \times 240 = \cdot 0327$ inch. The area of the bottom flange being $2 \cdot 4$ square inches, we get $6 \cdot 2 \div 2 \cdot 4 = 2 \cdot 584$ tons per square inch, and the extension $\cdot 00008 \times 2 \cdot 584 \times 240 = \cdot 0496$ inch. The sum of the two is $\cdot 0327 + \cdot 0496 = \cdot 0823$ inch, and the deflection $(\cdot 0823 \times 120) \div (14 \cdot 5 \times 4) = \cdot 17$ inch, or precisely as by experiment.

In order to facilitate calculation, we may put the preceding analytical method into the form of a Rule, which becomes:—

$$(686.)\quad \delta = \frac{W \times l^3 \times \cdot 000001563}{d^2 \times A} + \frac{W \times l^3 \times \cdot 00000125}{d^2 \times B}.$$

In which A = gross area of the top in square inches.
B = „ area of the bottom in „
l = length of the beam between supports in inches.
d = effective depth (between centres of gravity) in inches.
W = Weight in centre in tons.
δ = Deflection in inches.

Thus, taking the case of the girder in the last example, we have

$$\frac{3 \times 240^3 \times \cdot 000001563}{14 \cdot 5^2 \times 4 \cdot 55} + \frac{3 \times 240^3 \times \cdot 00000125}{14 \cdot 5^2 \times 4 \cdot 55}$$

$= \cdot 17$ inch, as before.

(687.) By this rule col. 6 in Table 76 has been calculated. The deflection of any of the girders in that Table, with any weight less than ⅓rd of the breaking weight, may be found by Rule (674), namely, by multiplying col. 6 or 7 by the cube of the length in feet between supports, and by the given weight in tons. Thus, for Fig. 105, say 20 feet long, with 20 tons spread all over, will deflect $\cdot 000001931 \times 8000 \times 20 = \cdot 309$ inch. With the same weight in the centre the deflection would be $\cdot 00000309 \times 8000 \times 20 = \cdot 4944$ inch, &c. It should be observed that this rule supposes the girder to be of uniform sectional area and depth from end to end, and any departure from those conditions must be allowed for.

DEFECT OF ELASTICITY IN BEAMS.

(688.) We have so far assumed that beams are perfectly elastic, that is to say that the deflection is simply and exactly proportional to the weights. But if the successive deflections of a bar, say of cast iron, with equal increments of weight, be very carefully observed, it will be found that every successive

weight produces a greater deflection than the one preceding, and that the departures from uniformity increase nearly as the *squares* of the weight applied. This is shown clearly by Table 111, where the load is divided into 20 parts, and the *Ratio* of the deflections of Cast-iron beams is given by col. 2, while those of Timber are given in col. 6. These Ratios were obtained by a Diagram (Fig. 216), in which the experimental deflections were plotted, and the irregularities equalized by a curve. In cols. 3 and 7 the deflections are assumed as supposed to be due with perfect elasticity, and in cols. 4 and 8 the defect of elasticity is given on the hypothesis that it varies as W^2. We have thus obtained cols. 5 and 9, comparing which with cols. 2 and 6 they will be found to agree very well up to about half the breaking load, beyond which they become irregular. This, however, is unimportant, as in practice beams of Cast iron are seldom loaded above $\frac{1}{3}$ and Timber ones $\frac{1}{4}$ or $\frac{1}{5}$th of the breaking weight.

The effect of defect of elasticity is shown by Table 108 also; with perfect elasticity the deflections would have been simply proportional to the load, as in col. 7, but col. 4 shows that they increase more rapidly than the weights throughout. But, with loads not exceeding $\frac{1}{3}$rd of the breaking weight, the departures from uniformity are not great, and within that limit, the ordinary rules are correct enough for practical purposes; where, however, great exactness is necessary, they require correction. For a bar 1 inch square and 1 foot long we have the Rule:—

$$(689.)\quad \delta = (\cdot 00002397 \times W) + (\cdot 000000006827 \times W^2).$$

In which W = the weight in centre in lbs., and δ = deflection in inches.

Thus, the mean strength of British Cast iron by col. 7 of Table 64 is 2063 lbs. breaking weight, and by col. 8 the mean deflection = ·0785 inch. With $\frac{1}{3}$ of that weight, or 687·6 lbs., the deflection by col. 11 = ·01971 inch.

By rule (689) we get with 2063 lbs., $\delta = (00002397 \times 2063) + (\cdot 000000006827 \times 2063^2) = \cdot 0785$ inch, and with 687·6 lbs., $\delta = (\cdot 00002397 \times 687\cdot 6) + (\cdot 000000006827 \times 687\cdot 6^2) =$

TABLE 111.—Of the RATIOS of DEFLECTION in BEAMS of CAST IRON and TIMBER.

Ratio of Load. W.	Cast Iron.				Timber.			
	Ratio of Deflection.	Perfect Elasticity.	Defect of Elasticity as W².	Calculated Deflection.	Ratio of Deflection.	Perfect Elasticity.	Defect of Elasticity as W².	Calculated Deflection.
·05	·0278	·0270 +	·0008 =	·0278	·0248	·0247 +	·0004 =	·0251
·1	·0578	·0540 +	·0032 =	·0572	·0502	·0494 +	·0018 =	·0512
·15	·0888	·0810 +	·0072 =	·0882	·0766	·0741 +	·0040 =	·0781
·2	·1210	·1080 +	·0128 =	·1208	·1038	·0988 +	·0071 =	·1059
·25	·1546	·1350 +	·0200 =	·1550	·1320	·1235 +	·0111 =	·1346
·3	·1898	·1620 +	·0288 =	·1908	·1613	·1482 +	·0160 =	·1642
·35	·2268	·1890 +	·0392 =	·2282	·1918	·1729 +	·0218 =	·1947
·4	·2658	·2160 +	·0512 =	·2672	·2236	·1976 +	·0285 =	·2261
·45	·3070	·2430 +	·0648 =	·3078	·2568	·2223 +	·0360 =	·2583
·5	·3506	·2700 +	·0800 =	·3500	·2915	·2470 +	·0445 =	·2915
·55	·3968	·2970 +	·0968 =	·3938	·3278	·2717 +	·0538 =	·3255
·6	·4458	·3240 +	·1152 =	·4392	·3658	·2964 +	·0641 =	·3605
·65	·4978	·3510 +	·1352 =	·4862	·4053	·3211 +	·0752 =	·3963
·7	·5530	·3780 +	·1568 =	·5348	·4470	·3458 +	·0872 =	·4330
·75	·6117	·4050 +	·1800 =	·5850	·4907	·3705 +	·1001 =	·4706
·8	·6744	·4320 +	·2048 =	·6368	·5365	·3952 +	·1140 =	·5092
·85	·7421	·4590 +	·2312 =	·6902	·5845	·4199 +	·1286 =	·5485
·9	·8163	·4860 +	·2592 =	·7452	·6450	·4446 +	·1442 =	·5888
·95	·9000	·5130 +	·2888 =	·8018	·7360	·4693 +	·1606 =	·6299
1·00	1·0000	·5400 +	·3200 =	·8600	1·000	·4940 +	·1780 =	·6720
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

·01971 inch, or precisely the same as the experimental results. Thus, while the loads are in the ratio 1 to 3, the deflections are in the ratio 1 to $\cdot 0785 \div \cdot 01971 = 3\cdot 983$, or nearly 1 to 4.

With any other dimensions for rectangular bars we have the Rule:—

(690.)

$$\delta = \left(\frac{W \div z \times L^3 \times \cdot 00002397}{d^3 \times b}\right) + \left(\frac{(W \div z)^2 \times L^4 \times \cdot 000000006827}{d^4 \times b^2}\right).$$

In which L = length between supports, in feet; d = depth in inches; b = breadth in inches; and z = the constant for the thickness of metal, as in (934) and col. 7 of Table 18.

By this rule col. 5 of Table 108 has been calculated. Taking the weight W in tons, and the rest as before, the Rule becomes:—

(691.)

$$\delta = \left(\frac{W \div z \times L^3 \times \cdot 05369}{d^3 \times b}\right) + \left(\frac{(W \div z)^2 \times L^4 \times \cdot 03126}{d^4 \times b^2}\right).$$

(692.) "*Wrought Iron.*"—The elasticity of wrought iron is very much more perfect than that of cast iron, as we found by its behaviour under tensile (620) and compressive strains (626). Table 106 shows the same result under transverse strains, the deflection in cols. 2, 2, 2, 2 being nearly in the simple ratio of the weights up to the "limit of Elasticity," or half the breaking-down load (see Table 67). We may therefore admit that certainly within that limit the deflections are given accurately by the Rules.

TABLE 112.—Of the DEFLECTION, &c., of Two BEAMS of AMERICAN RED PINE, 12 inches square, 15 feet long.

Central Weight.	Deflection.		Modulus of Elasticity: Lbs. E_D.	Central Weight.	Deflection.		Modulus of Elasticity: Lbs. E_D.
	By Experiment.	By Calculation.			By Experiment.	By Calculation.	
lbs.				lbs.			
3,653	·15	·1582	1,712,350	5,436	·25	·2354	1,528,880
5,815	·25	·2518	1,615,480	8,428	·50	·3650	1,185,200
8,374	·40	·3627	1,472,000	9,604	·57	·4160	1,184,710
10,213	·50	·4419	1,434,800	17,785	1·10	·7703	1,136,830
17,472	·90	·7567	1,365,000	22,848	1·52	·9896	1,056,910
22,483	1·20	·9737	1,317,370	24,080	1·67	1·043	1,013,740
24,394	1·37	1·057	1,252,000	26,600	2·00	1·152	935,100
26,527	1·50	1·149	1,243,470	28,058	2·25	1·215	876,700
28,370	1·70	1·229	1,173,400	28,812†	2·50	1·248	810,340
30,610	2·20	1·326	978,300	29,646*	2·97	1·284	701,700
31,651	2·70	1·371	821,620	(5)	(6)	(7)	(8)
32,800	3·30	1·421	698,868				
33,000	3·45	1·430	672,558				
33,186*	4·00	1·437	583,350				
(1)	(2)	(3)	(4)				

** Breaking Weights.
† Gradually Sinking.

But with strains beyond that limit defect of elasticity manifests itself very clearly, as shown by col. 4 of Table 70. Thus,

taking M_T = 2000 lbs. from col. 5 of Table 66, Rule (324) gives W = $1\frac{1}{2}^2 \times 1\frac{1}{2} \times 2000 \div (3 \times 112) = 20$ cwt. as the "limit of Elasticity," up to which point, by col. 4 of Table 70, the deflections are nearly as calculated for perfect Elasticity; but with heavier strains the Ratio progressively rises, becoming finally as much as 13·13 with the breaking-down load.

"*Steel.*"—The transverse elasticity of Steel is more perfect than even that of wrought iron, as shown by Table 107, the deflections being simply proportional to the load, and therefore the Modulus of Elasticity constant up to the "limit of Elasticity," or $\frac{5}{6}$ths of the Breaking-down load.

"*Timber.*"—Timber beams have very imperfect elasticities, as shown by cols. 4, 8, in Table 112, the value of the Modulus of Elasticity falling off regularly as the load is increased. The constants for the deflection of Timber in col. 4 of Table 105 were for the most part obtained with $\frac{1}{3}$rd to $\frac{1}{5}$th of the breaking weights, and the Rules (658), &c., will be correct enough for practice within those limits.

DEFLECTION WITH SAFE LOAD.

(693.) It is usual in practice to make the Working or Safe load on beams a certain Standard fraction of the breaking weight by the use of a "Factor of Safety" (880): in that case, the ordinary Rules (658), &c., admit of certain modifications by which calculations of the deflection may be simplified very considerably.

By Rule (659) it is shown that the deflection of a rectangular beam of any material is proportional to $\frac{W \times L^3}{d^3 \times b}$: now if with the same beam we take lengths in the ratio 1, 2, 3, &c., obviously the transverse strengths, or the load W, would vary in inverse ratio to those lengths, becoming 1, $\frac{1}{2}$, $\frac{1}{3}$, &c., and with those weights the deflections (being = to $W \times L^3$) become $1 \times 1^3 = 1 \cdot 0$; $\frac{1}{2} \times 2^3 = 4$; $\frac{1}{3} \times 3^3 = 9$, &c., which are in the direct ratio of the *square* of the lengths.

Then, with depths d in the ratio 1, 2, 3, &c., W would be in the ratio of the square of d, and become 1^2, 2^2, 3^2, or 1, 4, 9;

the deflections being proportional to $\frac{W}{d^3}$, become $\frac{1}{1^3} = 1 \cdot 0$; $\frac{4}{2^3} = \frac{1}{2}$; $\frac{9}{3^3} = \frac{1}{3}$, &c.; or *inversely* as the depths simply.

Then for breadths in the ratio 1, 2, 3, &c., W would vary in the ratio 1, 2, 3 also, and the deflections being proportional to $\frac{W}{b}$ become $\frac{1}{1} = 1 \cdot 0$; $\frac{2}{2} = 1 \cdot 0$; $\frac{3}{3} = 1 \cdot 0$; being the same in all cases, showing that the deflection *with safe load* is independent of the breadth.

(694.) From all this we get the general law that with *similar* beams, all *loaded in proportion to their strength*, the deflections are proportional to the square of the length, divided by the depth, and are independent of the breadth. This is true for all sections, whether circular, square, girder-sections, &c., so long as the beams compared are *similar* and are loaded to the same extent in proportion to their strength. We then have the Rules:—

(695.)
$$\delta = L^2 \times C \div d.$$
$$d = L^2 \times C \div \delta.$$
$$L = \sqrt{\delta \times d \div C}.$$

In which L = length of beam between supports in feet: d = the depth in inches: δ = deflection in inches: and C = a constant for the material, mode of fixing, loading, &c.

These Rules may be applied correctly to beams or girders of all sections, whether parallel from end to end, or with bellied flanges, also with any mode of fixing, any method of distributing the load, and any material, so long only as the Constant C is adapted to *all* the circumstances of the case. The effect of defect of Elasticity (688) is also covered by these rules, because the value of C is supposed to have been found by experiment from beams loaded to a certain degree in proportion to their ultimate strength, and will therefore apply without correction to all beams similarly loaded, &c., &c.

(696.) The most useful values of C are C_S for the Safe Load, and C_B for the Breaking Weight, giving δ_S and δ_B or the deflections with Safe and Breaking loads respectively. Table 64 gives these values for 54 varieties of British Cast iron, the mean for C_S being $\cdot 01971$, or say $\cdot 02$, by col. 11; and for $C_B = \cdot 0785$

by col. 8. Table 67 gives, in cols. 2 and 4, the values of C_S and C_B for many materials, these being in fact the deflections of a bar 1 inch square and 1 foot long with the Safe and Breaking Loads respectively: in that particular case, C_S and C_B are identical with δ_S and δ_B.

It will be observed that the values of C_S and C_B for the two Standard cases, are not simply proportional to the load, or "Factor of Safety." In Table 67, col. 7 gives the Factor of Safety, and col. 8 the ratios of the deflections with Safe and Breaking loads respectively: thus with Cast iron the ratio of the loads is 3 to 1 by col. 7, but the ratio of the corresponding deflections is 4 to 1 by col. 8: again with Ash, the ratio of the Loads = 5 to 1, but the ratio of the deflections = 10·7 to 1·0.

Thus, a beam of Ash, say 15 feet long, 7 inches deep, 3 inches wide, would give for the breaking weight by Rule (324), the Value of M_T for safe load being 136 lbs. by col. 3 of Table 67, $W = 7^2 \times 3 \times 136 \div 15 = 1333$ lbs., the deflection with which by rule (659) is $\delta = \dfrac{15^3 \times 1333 \times \cdot 00026}{7^3 \times 3} = 1 \cdot 13$ inch deflection with safe load. Now applying Rule (695) and taking C_S from col. 4 of Table 67 at ·0354, we obtain $\delta_S = 15^2 \times \cdot 0354 \div 7 = 1 \cdot 13$ inch deflection, as before: but with the breaking weight $\delta_B = 15^2 \times \cdot 375 \div 7 = 12$ inches deflection, &c.

Again, with a bar of Cast iron 3 inches deep, 4 inches wide, and 11 feet long, taking M_T for safe load = 688 lbs. from col. 10 of Table 64, Rule (324) gives $W = 3^2 \times 4 \times 688 \div 11 = 2251$ lbs. Safe load, with which Rule (659) gives $\delta = \dfrac{11^3 \times 2251 \times \cdot 00002886}{3^3 \times 4} = \cdot 8$ inch deflection. By Rule (695) we obtain much more easily $\delta_S = 11^2 \times \cdot 02 \div 3 = \cdot 8$ inch also. We have here taken the deflection per lb. = ·00002886 from col. 6 of Table 64, or col. 4 of Table 105. The deflection of the same bar with the Breaking weight becomes $\delta_B = 11^2 \times \cdot 0785 \div 3 = 3 \cdot 17$ inches.

But these Rules need not be restricted to δ_S and δ_B, but will apply equally well to any standard fraction of the breaking weight, so long as the great principle is maintained, that the beams compared shall be loaded always in some given and constant proportion to their strength. Thus for Wrought iron

and Steel, we found (374) (376) it convenient to take the "limit of Elasticity" and the "Working Safe Load" as data: then putting δ_E and δ_S for the deflections with those strains, we have for Wrought iron, the Rules:—

(697.)
$$\delta_E = L^2 \times \cdot 035 \div d.$$
$$\delta_S = L^2 \times \cdot 0235 \div d.$$

For Steel these Rules become:—

(698.)
$$\delta_E = L^2 \times \cdot 0802 \div d.$$
$$\delta_S = L^2 \times \cdot 0481 \div d.$$

Thus, with a wrought-iron bar 2 inches deep, 4 inches wide, and 12 feet long, taking from col. 5 of Table 66, $M_T = 1500$ lbs. for the safe working load, we have by Rule (324) $W = 2^2 \times 4 \times 1500 \div 12 = 2000$ lbs., with which the deflection by Rule (659) becomes $\delta = \dfrac{12^3 \times 2000 \times \cdot 00001565}{2^3 \times 4} = 1 \cdot 69$ inch. By Rule (697) we have $\delta_s = 12^2 \times \cdot 0235 \div 2 = 1 \cdot 69$ inch also.

Again: with a Steel bar $1\frac{1}{2}$ inch deep, 5 inches wide, and 10 feet long, the load by which the bar will be strained to the "limit of elasticity" will be $W = 1\frac{1}{2}^2 \times 5 \times 5600 \div 10 = 6300$ lbs., with which the deflection by Rule (659) becomes $\delta = \dfrac{10^3 \times 6300 \times \cdot 00001433}{1\frac{1}{2}^3 \times 5} = 5 \cdot 35$ inches. By Rule (698) we obtain $\delta_E = 10^2 \times \cdot 0802 \div 1 \cdot 5 = 5 \cdot 35$ inches also.

(699.) When the load is not in the centre, and when the beam is not supported at both ends as is assumed in the ordinary rules, the case is complicated, but by combining the data given in (431) and (667) the matter may be simplified.

	Constant Load.	Ratio of Deflection.	Ratio of Safe Load.	Ratio of Deflection.
Supported at ends, load in the centre ..	1·0	1·0	1·0	1·0
" " equally distributed	1·0	$\frac{5}{8}$	2·0	1·25
Built into walls, &c., at both ends, load in the centre	1·0	$\frac{2}{3}$	1·5	1·0
Built into walls, &c., at both ends, load distributed	1·0	$\frac{5}{12}$	3·0	1·25
Fixed at one end, loaded at the other ..	1·0	32	0·25	8·0
" " load equally distributed	1·0	12	0·5	6·0

Thus, for example, if the beam of ash in (696) had been built into walls at both ends, the safe central load would have been 1·50, or 50 per cent. greater, but the deflection with that increased load would have been 1·0, or precisely as before.

Again: if the bar of cast iron in (696) had been fixed at one end, and the *safe* load had been equally distributed, that load would be 0·5, or half only of its former amount, but the deflection with that reduced load would be 6 times greater than before, and becomes ·8 × 6 = 4·8 inches, &c.

(700.) Again: say we require an Oak Bressummer to carry the front of a house, the estimated distributed load being 19 tons and the span 12 feet, the ends being built into the walls in the usual way. Say we try 12 inches square, then by col. 3 of Table 67, M_T = 102 lbs. for Safe load: then Rule (324) becomes $W = 12^2 \times 12 \times 102 \div 12 = 14688$ lbs., or 6·56 tons for the ordinary case of a beam merely supported at ends and loaded in the centre; but by (431) in our case, 6·56 × 3 = 19·68 tons safe distributed load, or very nearly the actual load.

Then, for the deflection, Rule (695) gives $\delta = 12^2 \times \cdot 04 \div 12 = 0 \cdot 48$ inches for the deflection with safe load in centre, which by (699) becomes $\cdot 48 \times 1 \cdot 25 = 0 \cdot 6$ inch deflection, when the load is distributed and the ends built in, as in our case. We have taken the value of C_S = ·04 from col. 4 of Table 67.

(701.) Although, as stated in (695), the value of C_S should strictly be adapted to the special section of the beam, we may with moderate accuracy apply the value for simple rectangular beams to ordinary girders, at least where the beam is parallel. But when the flanges are bellied, as in Fig. 131, the deflections are about 40 per cent. greater than with parallel beams, as shown by Mr. Hodgkinson's experiments. For ordinary parallel girders of cast iron loaded to $\frac{1}{3}$ of the Breaking Weight, we have the Rule:—

(702.) $$\delta_s = L^2 \times \cdot 02 \div d.$$

For cast-iron girders reduced in section progressively towards the ends, or having bellied flanges, as in Fig. 131, the Rule becomes:—

(703.) $$\delta_s = L \times \cdot 027 \div d.$$

Thus, for the large girders in Table 68 and Fig. 79, the length being 16 feet and the depth 14 inches, with bellied flanges, we obtain $\delta_s = 16^2 \times \cdot 027 \div 14 = 0 \cdot 4937$ inch, the deflection with central safe load. The mean breaking weight was 38·3 tons, or 38·3 ÷ 3 = 12·8 tons Safe load, the nearest load to which in Table 68 is 14 tons, with which the mean deflection = ·0525 inch; therefore ·0525 × 12·8 ÷ 14 = 0·48 inch with safe load of 12·8 tons, &c., or nearly as calculated by the Rule.

CHAPTER XVII.

ON TORSIONAL ELASTICITY.

(704.) "*Methods of Estimating.*"—There are two ways of measuring torsional elasticity. 1st, by the *angle* of torsion or the number of degrees of twist, that is to say, if the whole circle = 360°, the amount of torsion would be expressed by the number of degrees of that circle produced in a long bar by a torsional strain. The other and more convenient method is to express the torsional elasticity by the descent of the end of the lever by which the twisting weight is applied; this method is applicable to those cases only where the angle of torsion is small, but as this is always the fact in practice, this is no objection.

(705). It is necessary to observe that in this method of estimating torsional elasticity, the descent of the straining weight is proportional to the *square* of the length of the lever; thus in Fig. 173, we have a constant weight = 1, acting with leverages in the ratio 1, 2, 3, it will therefore strain the bar D in those same ratios, and the descent of the lever *measured at E* will be 1, 2, 3 also, but measured at the points of application of the weight, as in our case, we obtain 1, 4, 9 for the ratios of the descent of the weight as in the figure, or in the ratios of the

square of the length of lever for 1^2, 2^2, 3^2, $= 1$, 4, and 9 respectively.

"*Laws of Torsional Elasticity.*"—The fundamental laws as determined by mathematicians may be expressed by the Rules:—

For Circular Sections:—

$$(706.)\qquad T = \frac{L^2 \times l \times W \times 2}{3 \cdot 1416 \times R^4 \times M_t}.$$

$$(707.)\qquad M_t = \frac{W \times L^2 \times l \times 2}{3 \cdot 1416 \times R^4 \times T}.$$

For Square Sections,

$$(708.)\qquad T = \frac{L^2 \times l \times W \times 6}{S^4 \times M_t}.$$

$$(709.)\qquad M_t = \frac{W \times L^2 \times l \times 6}{S^4 \times T}.$$

For Rectangular Sections,

$$(710.)\qquad T = \frac{L^2 \times l \times (d^2 + b^2) \times 3}{d^3 \times b^3 \times M_t}.$$

$$(711.)\qquad M_t = \frac{W \times L^2 \times 3 \times (d^2 + b^2) \times l}{d^3 \times b^3 \times T}.$$

In which R = radius of circular sections in inches: S = side of square: d and b = depth and breadth of rectangular sections: L = leverage in inches by which the weight W in pounds acts in twisting the bar: l = the length of bar twisted in inches: M_t = Multiplier for torsional elasticity derived from experiment: and T = the elastic torsion measured by the descent of the weight in inches.

By Mr. Bevan's experiments, the mean value of M_t for cast iron = 5,709,600 lbs.: wrought iron and steel were nearly equal to each other; a mean from eight experiments on wrought iron and three on steel gave M_t = 10,674,540 lbs.

To determine the value of M_t, say we find by experiment with a 4-inch round bar of cast iron 60 inches long, with

W = 5000 lbs, and L = 24 inches, that T, or the descent of the weight = 1·206 inch, then by Rule (707),

$$M_t = \frac{5000 \times 24^2 \times 60 \times 2}{3 \cdot 1416 \times 2^4 \times 1 \cdot 206} \text{ or } \frac{5000 \times 576 \times 60 \times 2}{3 \cdot 1416 \times 16 \times 1 \cdot 206}$$

= 5,701,000, the value of M_t in that case.

(712.) "*Ratio of Stiffness of Round and Square Bars.*"—Comparing rules (707) and (709) we can find a general ratio for the torsional stiffness of round and square bars: thus, for 4 inches diameter, R = 2, and by Rule (707) we obtain $\frac{2}{3 \cdot 1416 \times 2^4} = \cdot 03979$. For a 4-inch square bar, Rule (709) gives $\frac{6}{4^4}$ or $\frac{6}{256} = \cdot 02344$; hence we have ·03979 ÷ ·02344 = 1·7 to 1·0 = the ratio of the stiffness of square and round

TABLE 113.—Of TORSIONAL ELASTICITY.

Material.	Value of C_T. Lbs.	Value of C_T. Cwts.	Material.	Value of C_T. Lbs.	Value of C_T. Cwts.
Wrought Iron and Steel	606	5·41	Hornbeam	9·00	·081
			Lancewood	8·60	·077
Cast Iron	324	2·90	Larch	6·46	·058
Alder	5·53	·049	Lime-tree	6·23	·056
Ash	6·91	·062	Oak, English ..	6·81	·061
Apple-tree	6·95	·062	Oak, Dantzic ..	5·62	·050
Beech	7·23	·065	Pear-tree	6·18	·055
Birch	5·88	·052	Pine, Memel ..	5·11	·046
Boxwood..	10·00	·090	Pine, American ..	5·02	·045
Brazil-wood	12·50	·112	Plane-tree	6·00	·054
Chestnut (Horse) ..	7·56	·067	Sycamore	7·80	·070
Deal..	3·82	·034	Teak	9·30	·083
Elm	4·60	·041	Walnut	6·72	·060
Fir, Scotch	4·68	·042			

bars. It is shown in (552) that the ratio of torsional *strength* of square and round bars is theoretically 1·2 to 1·0; but by experiment 1·6 to 1·0.

"*Practical Rules.*"—The theoretical Rules may be put in

more convenient form for practical use on the large scale; they then become:—

(713.) For Circular sections:—

$$T = \frac{L^2 \times l \times W}{D^4 \times C_T}.$$

(714.) For Square sections:—

$$T = \frac{L^2 \times l \times W}{S^4 \times C_T \times 1 \cdot 7}.$$

(715.) For Rectangular sections:—

$$T = \frac{L^2 \times l \times (d^2 + b^2) \times W}{d^3 \times b^3 \times C_T \times 3 \cdot 4}.$$

In which D = diameter in circular sections in inches: S = side of square in inches: d and b = depth and breadth of rectangular sections in inches: l = length of bar twisted in feet: L = leverage in feet with which the weight W, in lbs. or cwts., &c., acts in twisting the bar: T = the descent of the weight in inches, due to the twisting of the bar: and C_T = a Constant from experiment, the value of which is given by Table 113 as reduced from the results obtained by Mr. Bevan.

To find the value of C_T from experiment, the Rules become:—

(716.) For Circular sections:—

$$C_T = \frac{L^2 \times l \times W}{D^4 \times T}.$$

(717.) For Square sections:—

$$C_T = \frac{L^2 \times l \times W}{S^4 \times T \times 1 \cdot 7}.$$

(718.) For Rectangular sections:—

$$C_T = \frac{L^2 \times l \times (d^2 + b^2) \times W}{d^3 \times b^3 \times T \times 3 \cdot 4}.$$

(719.) We may now give some illustrations of the application of the Rules: say we take the bar considered before (711), in which D = 4; l = 5 feet; L = 2 feet; and W = 5000 lbs.

Taking the value of C_T at 324 from Table 113, Rule (713) becomes, $T = \frac{2^4 \times 5 \times 5000}{4^4 \times 324}$ or $\frac{4 \times 5 \times 5000}{256 \times 324} = 1 \cdot 206$ inch, the descent of the lever and weight due to the twist.

For a square bar of the same dimensions, Rule (714) becomes, $T = \frac{4 \times 5 \times 5000}{256 \times 324 \times 1 \cdot 7} = \cdot 709$ inch. We should obtain the same result if we applied to this bar the rule for rectangular sections, for obviously a square bar may be regarded as a rectangular one with equal sides: then Rule (715) gives $T = \frac{4 \times 5 \times (16 + 16) \times 5000}{64 \times 64 \times 324 \times 3 \cdot 4} = \cdot 709$ inch, as before.

Again: say we have a Deal Plank 3×11 inches, 12 feet long, twisted by 2 cwt., with a lever 3 feet long. Then taking C_T from Table 113 at ·034, Rule (715) becomes

$$T = \frac{3^2 \times 12 \times (11^2 + 3^2) \times 2}{11^3 \times 3^3 \times \cdot 034 \times 3 \cdot 4} = 6 \cdot 76 \text{ inches,}$$

the descent of the lever and weight.

(720.) Table 113 shows that there are very great differences in the Torsional stiffness of Materials: thus wrought iron has no less than $606 \div 3 \cdot 82 = 158$ times the stiffness of Deal. This is the more remarkable because by (571) and Table 84, the Torsional *strength* of say Yellow Pine and Wrought iron are in the ratio $10580 \div 328 = 32 \cdot 25$ to $1 \cdot 0$ only. Again: the *Transverse* stiffness of say Memel Fir and wrought iron are by col. 4 of Table 105, in the ratio $\cdot 0002223 \div \cdot 00001565 = 13 \cdot 5$ to $1 \cdot 0$ only; whereas, as we have seen, the Torsional stiffness is 158 to 1·0. This shows that wood is not adapted for torsional strains, at least for cases where stiffness is required.

The Table 113 shows, also, that the torsional stiffness of Wrought iron and Steel are equal to one another, although, as shown in (571), the torsional strengths are in the ratio of 2 to 3. The stiffness of wrought and cast iron is about 2 to 1.

CHAPTER XVIII.

ON THE MODULUS OF ELASTICITY.

(721.) "*General Principles.*"—The Modulus of Elasticity is the tensile force that would stretch a bar to double its primitive length, and is usually expressed in pounds per square inch of area of the bar. Say we had a bar 10 inches long of some very elastic material, which stretches 1 inch by 20 lbs. per square inch; then obviously, to stretch it 10 inches, and thereby double the original length, we require 200 lbs. per square inch, which is, therefore, the Modulus of Elasticity for that material. It is here assumed that the elasticity is perfect, or that the extensions would be exactly proportional to the strain throughout, which would not be strictly true with any known material:—moreover, it is assumed that the bar would bear stretching to double its original length without rupture, which is true with very few materials. The expression "Modulus of Elasticity" must be regarded as a conventional one, adopted for convenience of calculation, rather than as a statement of fact; thus, as applied to the bar we have just considered, it means that within certain limits and with moderate strains, the bar would stretch $\frac{1}{200}$th of its length for each pound per square inch, tensile strain.

(722.) The Modulus of Elasticity may also be found from the shortening which a bar experiences by a Compressive strain, and is in that case the weight in pounds per square inch capable, *theoretically*, of reducing the length of the bar to *nothing*. Thus taking the same bar as before, say that a compressive strain of 20 lbs. per square inch on the bar 10 inches long was found by experiment to shorten it one inch, or to reduce its length to 9 inches. Then evidently we require 200 lbs. per square inch to shorten it 10 inches, or to reduce its length to *nothing*, which again is, of course, not a statement of fact.

(723.) The Modulus of Elasticity may also be found from the deflection of a beam strained transversely by a known weight. It is well known that, with a beam supported at the two ends

and loaded in the centre, the weight generates a tensile strain on the lower fibres of the beam, which stretch and become longer than before. Similarly, a compressive strain is generated at the upper part of the section, which thereby becomes shorter than before. The combined effect of both is that the beam originally straight becomes curved or deflects, and we may then find the Modulus of Elasticity by the rule:—

(724.) $$E = \frac{w \times l^3}{4 \times b \times d^3 \times \delta}.$$

In which E = the Modulus of Elasticity in pounds per square inch.
b = breadth of rectangular bar, in inches.
d = depth " " "
l = length between supports, in inches.
δ = deflection, in inches.
w = weight in pounds producing that deflection.

This rule assumes that the material resists extension and compression with equal energy, or that the Modulus of Extension and Compression are equal, and without the knowledge that they are so, the result may be taken as giving a *mean* of the two.

(725.) When we proceed to find the Modulus of Elasticity experimentally, by the extension, compression, and deflection of any material, we find departures from the simple laws which we have so far assumed, which complicate the question very considerably.

1st. The Modulus, calculated from deflection, does not agree exactly with those found by direct extension or compression. 2nd. So far as observations on wrought and cast iron enable us to judge, bodies yield more to compressive than to equivalent tensile strains, and as a result of this, the "Modulus of Extension" is greater than the "Modulus of Compression." 3rd. The elasticity of all bodies is more or less imperfect, as manifested by the extensions, compressions, &c., increasing in a more rapid ratio than the strains, the result of which is that the Modulus is not constant, but is progressively reduced as the

strain is increased. 4th. In the case of cast iron the Modulus is considerably affected by the size or least thickness of the casting, small castings having a higher Modulus than large ones from the same iron. We shall consider the effect of these facts separately.

(726.) "*Modulus of Extension, Compression, and Deflection.*"—Taking first the case of wrought iron, whose elasticity when not overstrained is nearly perfect, and of which we have the most perfect experimental knowledge, we find that the Modulus of Elasticity calculated

	Lbs. per Sq. In.
From the Extensions up to 8 tons per square inch (Table 96)	= 28,000,000
" Compressions " 11 " " " (" 98)	= 22,400,000
" Deflections up to the "limit of Elasticity" (" 106)	= 27,603,000

It will be observed that the Modulus of Deflection, which we supposed (724) to be a mean between those of Extension and Compression, namely 25,200,000 lbs., is in this case considerably greater.

(727.) With cast iron, the comparison is obscured by defect of elasticity (688), but we can eliminate the effect of this source of complication by taking for all the strains the same fraction of the breaking weight, say $\frac{1}{3}$rd, and we then have:—

	Lbs. per Sq. In.
From the Extensions with $2\frac{1}{2}$ tons per square inch (Table 88)	= 12,646,500
" Compressions " 14 " " " (" 89)	= 11,482,900
" Deflection of 1-inch bar, with $\frac{1}{3}$rd breaking weight .. .. (" 64)	= 15,242,320

Here the Modulus of Deflection, so far from being an arithmetical mean between those of Extension and Compression (724), is much greater than either.

(728.) The principal value of the term "Modulus of Elasticity" is its supposed universal application to all the strains to which materials are subjected, so that the extension, compression, transverse flexure of a beam, and angular torsion of a shaft, &c., should all be calculated with the same constant by the use of appropriate formulæ, and we should be able to reason on any particular strain with data obtained from another kind of strain.

(729.) We have seen, however, that this is only approximately true, and thus this distinctive advantage of the term "Modulus of Elasticity" is reduced considerably, in fact, it would be more correct to use the terms "Modulus of Extension," "Modulus of Compression," and "Modulus of Deflection," but the multiplication of terms would be objectionable; we shall therefore retain the old general term "Modulus of Elasticity," distinguishing the method by which it was determined, and to which alone it applies with absolute correctness, by characteristic affixes; thus E_E, E_C, E_D will indicate the Modulus of Elasticity by Extension, Compression, and Deflection respectively.

(730.) The Modulus found by experiment for any particular strain may be applied, as we have seen, with approximate correctness to other strains in the absence of more precise data, and this is very convenient in many cases. For instance, we have absolutely no experimental knowledge of the extension and compression of Timber, but experiments on deflection are very numerous, and E_D derived from these may be used to determine the extension and compression approximately.

(731.) "*Differences in Value of* E_E *and* E_C."—Table 91 gives a collective comparison of the relative elasticity of Cast and Wrought iron under equivalent tensile and compressive strains, and shows that at least with small strains those materials yield more to compression than to extension, and the effect of this on the Moduli E_E and E_C is shown by Tables 88, 89; 96, 98. Thus for Wrought iron with 1 ton per square inch E_E is 28,000,000 lbs., and E_C is 22,400,000 lbs.; similarly for Cast iron E_E = 13,449,400, and E_C = 12,844,000 lbs. With Cast iron the case is affected by defect of elasticity, so that with strains greater than 2½ tons per inch E_C becomes the greater of the two.

(732.) "*Effect of Defect of Elasticity.*"—The elasticity of all bodies is more or less imperfect (688), and the effect of this fact on the Modulus of Elasticity may be illustrated clearly by the case of Cast iron, whose elasticity is very imperfect (688). Thus by col. 10 of Table 88, E_E is progressively reduced from 13,695,200 lbs., with ½ ton tensile strain per square inch, to 9,223,500 lbs. with 7 tons, which is nearly the mean breaking

weight. Similarly, by col. 10 of Table 89, E_C is progressively reduced from 12,844,000 lbs. with 1 ton, to 3,862,400 lbs. with 42 tons per square inch compressive strain. Table 108 gives the result of the experiments of Mr. Hodgkinson on $3 \times 1\frac{1}{2}$ bars of Blaenavon Iron, which were made with special ca:e, having friction rollers to support the ends, &c. The effect of defective elasticity is clearly shown by col. 11, the Modulus E_D being regularly and progressively reduced from 15,216,300 lbs. with $\frac{1}{30}$th of the breaking load to 8,353,770 lbs. with breaking weight.

(733.) This great variation in the Moduli with the ratio of the strain applied in proportion to the breaking weight, not only complicates the question, but also renders it necessary to distinguish the Modulus between two given strains, from that *at* a given strain. For instance, col. 6 of Table 88 gives the Extension by every successive half-ton throughout:—thus between 3 and $3\frac{1}{2}$ tons, the extension by that half-ton is ·00010681 of the length, and the mean Modulus between those weights, or at the mean weight of $3\frac{1}{4}$ tons, will be $1120 \div \cdot 00010681 = 10,485,900$ lbs. Then, between $3\frac{1}{2}$ and 4 tons the extension by that particular half-ton is ·0001141, and the mean Modulus between those weights, or at a mean weight of $3\frac{3}{4}$ tons, is $1120 \div 0001141 = 9,816,000$ lbs.

Having thus found the Modulus at $3\frac{1}{4}$ and $3\frac{3}{4}$ tons, that at the mean weight of $3\frac{1}{2}$ tons may be found, and becomes $(10,485,900 + 9,816,000) \div 2 = 10,150,950$ lbs., as given in col. 11, but between 0 and $3\frac{1}{2}$ tons, the Modulus by col. 10 is 12,040,900 lbs. The meaning of this is, that between 0 and 3 tons per square inch a bar of cast iron extends in a gradually increasing ratio to the strain, but at such a *mean* rate that if thenceforth it were continued uniformly proportional to the strain, 12,040,900 lbs. per square inch would stretch the bar to double its original length, or, in other words, each pound would stretch the bar $\frac{1}{12040900}$th of the length. But when already loaded with 3 tons per square inch, a small further strain would stretch the bar $\frac{1}{10150950}$th of the length per pound. Cols. 10 and 11 of Table 88, and cols. 10, 11 of Table 89, have been calculated in this way.

(734.) The elasticity of wrought iron is practically perfect nearly up to the "limit of Elasticity" (692), and as a result, the Modulus of Elasticity is constant within that limit. This is shown by Table 96, where E_E by direct experiment in col. 5 is nearly constant up to 9 or 10 tons per square inch tensile strain, the small differences being due to errors of observation which are unavoidable. The mean value is given by col. 7 at 28,000,000 lbs., and as constant up to 8 tons. Similarly, Table 98 gives E_C nearly constant by direct experiment in col. 5, its mean value being given at 22,400,000 lbs., and as constant up to 11 tons per square inch by col. 7. The same results are given by Table 106, where E_D is nearly constant, up to the limit of Elasticity, the mean of the whole of the experiments up to that point being 27,645,000 lbs., agreeing nearly with the mean Modulus 27,603,000 lbs. derived from general observations (726). Beyond the strains named, the Modulus falls off rapidly and irregularly, and is in fact a question of *time*, as may be inferred from the Tables, which show that with greater strains, the extensions, &c., go on increasing even with constant weights.

(735.) The elasticity of Steel is nearly perfect up to the "limit of Elasticity":—we have no experiments giving the value of E_E or E_C, but that of E_D is given by Table 107, and is nearly constant with strains less than the "limit of Elasticity" as calculated by the rule $d^2 \times b \times 5600 \div L = w$. The mean value of E_D for cast steel calculated from the mean deflection in col. 4 of Table 105 by Rule (724) is 30,146,600 lbs. per square inch, col. 7:—that for shear steel being 31,169,000 lbs.; and it should be observed that these results were obtained from the deflections of bars of unwrought and untempered steel. With tempered spring steel we may admit an elasticity practically perfect, and that the Modulus is constant.

(736.) The elasticity of Timber is very imperfect, and the Modulus of Elasticity very variable, as shown by Table 112, which gives the value of E_D from two experiments by Mr. E. Clark on American Red Pine: the Modulus of Elasticity E_D is in col. 4 reduced progressively from 1,712,350 lbs. with $\frac{1}{5}$th to 583,350 lbs. with the breaking weight. The value of the

Modulus for the same kind of Timber is given by col. 7 of Table 105 at 1,623,450 lbs., which is correct for strains up to the Safe load, or say $\frac{1}{5}$th of the Breaking weight (888).

(737.) The effect of defective elasticity on the Moduli E_B, E_C, E_D is shown graphically by Diagrams, Figs. 214, 218, in which all the experimental strains have been reduced to fractions of the Ultimate, or breaking weight, so as to render the results directly comparable with one another. If the elasticities were perfect, all the lines indicating the value of the Modulus would have been horizontal.

(738.) "*Effect of Size of Casting.*"—In searching for the Modulus E_D for cast iron, from the experimental deflection of square and rectangular bars of various sizes, another, and perhaps an unexpected complication is discovered: the modulus is found to vary considerably with the size of the bar, or more correctly with the least dimension, in the case of rectangular bars. We found in (932) that the transverse strength of rectangular bars of cast iron is inversely proportional to the size of the casting, bars 1, 2, and 3 inches square having specific strengths in the ratio 1, ·7519, and ·6364 respectively, and experiments have shown that the Modulus E_D is similarly affected.

(739.) In order to show distinctly the effect of size alone, it is necessary to clear the subject from complications arising from the varying elasticities of different kinds of iron by selecting and comparing the experiments on one and the same kind of iron, varying only in size. Then again, to eliminate the effect of defective elasticity (688), it is necessary to bring the strains in all cases to equality, by reducing them to fractions of the ultimate strain or breaking weight. This is done in Diagram, Fig. 214, where the Moduli given by Tables 114, 118, from the experiments of Mr. Hodgkinson, on bars of different sizes, but all of the same kind of iron, namely, Blaenavon No. 2, are plotted to the same scale, and the actual strains being reduced to fractions of the breaking weight, the effects of the size of casting, defect of elasticity on the different sizes, &c., are made manifest. This diagram shows:—

(740.) 1st. That the Modulus of Elasticity E_D decreases as the

strain is increased, or, is inversely proportional to the strain, this effect being due to defect of elasticity, for obviously with perfect elasticity the modulus would be the same for all strains, and the lines in the diagram would be horizontal and perfectly straight.

(741.) 2nd. That this decrease in the Modulus is in inverse *arithmetical* ratio to the strain, as shown by the lines being approximately straight ones. Of course there are considerable irregularities due to errors of observation (not necessarily errors of the *observer*), but the general result is that the lines are straight.

(742.) 3rd. That the Modulus is inversely proportional to the size of bar, or rather to its least dimension, but not in arithmetical ratio; thus the line of the 2-inch bar is not exactly midway between those of the 1-inch, and 3-inch, but much nearer the latter, agreeing to some extent with the transverse strength, as shown by Table 142.

(743.) 4th. That the difference in the Modulus between castings of different sizes, but loaded to the same extent, varies directly as the strain, being a minimum with very small strains, and increasing progressively to a maximum with the breaking weight. This is shown in the diagram by the non-parallelism of the lines, which diverge from one another as the strain is increased:—the line of the small bars being more nearly horizontal than those of the large ones, shows more perfect elasticity in the former, and that defect of elasticity is more influential on the Modulus E_D as the size of the bar is increased.

(744.) 5th. That in rectangular bars of unequal dimensions, the Modulus E_D is governed potentially by the least dimension, rather than the greater, and does not differ materially from that of a square bar of that least dimension. Thus, the bar 1×2 has practically the same modulus as the bar 1 inch square, and the bars $6 \times 1\frac{1}{2}$ and $3 \times 1\frac{1}{2}$ occupy in the diagram the position of a bar $1\frac{1}{2}$ inch square or nearly so, not exactly, however, the $6 \times 1\frac{1}{2}$ bar being somewhat affected by the larger dimension, which reduces its modulus below that of the $3 \times 1\frac{1}{2}$ bar.

(745.) The straight lines F, G, H, J are intended to equalize the anomalies of the experiments and represent approximately

the mean Modulus E_D for the different sizes of No. 2 Blaenavon cast iron, and from them we have obtained Table 114, which gives the combined effect of size of casting and defect of elasticity or ratio of the strain to the breaking weight, on the Modulus of that particular iron.

TABLE 114.—Of the MODULUS of ELASTICITY, by Deflection, E_D of CAST-IRON BARS, showing the effect of Size of Casting, and Ratio of Strain to the Breaking Weight.

Ratio of Strain to Breaking Weight.	Least Dimension of Rectangular Bar.			
	1 Inch.	1½ Inch.	2 Inches.	3 Inches.
	Modulus of Elasticity in Lbs. per Square Inch.			
·0	15,555,000	15,000,000	14,625,000	14,000,000
·1	15,000,000	14,400,000	14,000,000	13,333,000
·2	14,445,000	13,800,000	13,375,000	12,667,000
·3	13,885,000	13,200,000	12,750,000	12,000,000
·4	13,330,000	12,600,000	12,125,000	11,333,000
·5	12,775,000	12,000,000	11,500,000	10,667,000
·6	12,220,000	11,400,000	10,875,000	10,000,000
·7	11,665,000	10,800,000	10,250,000	9,333,000
·8	11,110,000	10,200,000	9,625,000	8,667,000
·9	10,555,000	9,600,000	9,000,000	8,000,000
1·0	10,000,000	9,000,000	8,375,000	7,333,000

This Table applies strictly to Blaenavon No. 2 iron only.

(746.) The effect of size of casting on the Modulus E_D is also clearly shown by Table 118, where 1, 2, and 3-inch bars are all reduced to one standard for the purpose of direct comparison. Taking for illustration from col. 5, the same load on bars of all three sizes as nearly as possible, say 1210 lbs. for 1-inch bars, giving E_D = 11,767,600 lbs. by col. 4: for 1212 lbs. on *reduced* 2-inch bars E_D = 9,304,260 lbs.; and for 1260 lbs. on *reduced* 3-inch bars, E_D = 7,995,000 lbs., &c., thus showing a great reduction as the size of the bar is increased.

The effect of defect of Elasticity on the Modulus E_D is also clearly manifested with bars of all sizes, by the almost perfect regularity with which its value falls off as the strain is pro-

gressively increased up to the breaking load, when the 1-inch bars give 9,738,700 lbs., the reduced 2-inch, 8,412,600 lbs.; and the reduced 3-inch, 7,614,440 lbs.; see (769).

(747.) The Diagram, Fig. 218, gives a general comparison of the three Moduli, namely, of Extension E_E, Compression E_C, and Deflection E_D, all reduced to one uniform standard series of strains from 0 to the breaking loads increasing by $\frac{1}{10}$th.

It will be observed that E_E and E_C are nearly parallel to each other up to $\frac{1}{3}$rd of the breaking weight, although the *actual* compressive strain is then about 6 times the tensile. Beyond the strain of $\frac{1}{3}$rd the modulus of Extension E_E continues to decrease with great regularity up to the breaking weight, but in an increasing ratio, as manifested by the line being curved. The line of E_C is remarkable, falling off at first in nearly arithmetical ratio so far as $\frac{3}{10}$ths the breaking weight as is due with perfect elasticity, after which the Modulus decreases very rapidly up to about ·547 of the Ultimate weight, and then returning nearly to its first rate of decrease. But it should be observed that beyond ·547, we have no experimental evidence, and that the results are based on assumed compressions obtained by plotting all the experimental compressions in a Diagram, Fig. 215, and continuing the curve by judgment up to the Crushing strain. They are therefore more or less problematical and of uncertain accuracy, but it will be obvious from inspection that if the experimental curve were continued at its normal rate up to the breaking weight, the Modulus E_C would be reduced to *nothing*, which is manifestly incorrect, and indeed absurd (722).

(748.) The values of E_E and E_C were determined from bars 1 inch square, they therefore are strictly comparable with the value of E_D from bars of the same size only, and we then find that instead of being intermediate between E_E and E_C as might have been expected (727), it is for all strains greater than either: hence it is expedient, whenever practicable, to use the particular Modulus adapted to the case, having been derived from similar cases. Thus, when the extensions are required, E_E should be used; for compressions E_C, and for deflections E_D, taking care, where accuracy is desired, to use the Modulus

specially applicable to the ratio of the strain to the breaking weight, kind of iron, and size of casting. It is advisable to adopt this course, not only with such materials as cast iron and timber whose elasticity is very imperfect, but also with the most perfectly elastic materials, such as wrought iron and steel, as may be seen by (726).

(749.) We may now give some illustrations of the application of the various Moduli E_E, E_C, E_D to practice.

Say, we have a bar of cast iron 10 feet, or 120 inches long, loaded with a tensile strain of 4 tons per square inch, and we require the extension by that strain:—now by col. 10 of Table 88 the mean value of E_E is then 11,709,200 lbs., and the extension, with 4 tons, or 8960 lbs., will be $8960 \div 11{,}709{,}200 = {\cdot}000765$ parts of the length, or ${\cdot}000765 \times 120 = {\cdot}0908$ inch. Now, say that when the bar is loaded with 4 tons, we require the effect of 4 cwt., or 448 lbs., more; then by col. 11, the Modulus when already loaded with 4 tons, is 9,472,000 lbs., hence the extension with 448 lbs. more, would be $448 \div 9{,}472{,}000 = {\cdot}0000473$ parts of the length, or ${\cdot}0000473 \times 120 = {\cdot}005676$ inch.

The application of E_C to find the compressions by crushing strains is of course precisely similar to that of E_E, the proper value of the Modulus being taken from col. 10 or col. 11 in Table 89.

To find the deflection of a rectangular bar, the value of E_D must be selected, proper for the particular iron (739); ratio of the strain to the breaking weight (740); and size of casting (738); and we can then find the deflection by the rule:—

(750.) $$\delta = \frac{w \times l^3}{E_D \times 4 \times d^3 \times b}.$$

In which δ = the deflection in inches, w = weight in pounds, l = length between supports in inches, d = depth in inches, b = breadth in inches, E_D = Modulus of deflection in pounds per square inch. Thus with a wrought-iron bar in which $d = 1\frac{1}{2}$ inch, $b = 5\frac{1}{2}$ inches, $l = 13\frac{1}{2}$ feet, or 162 inches, and w = 840 lbs.: taking E_D from col. 7 of Table 105 at 27,600,000,

the deflection becomes $\delta = \dfrac{840 \times 162^3}{27600000 \times 4 \times 1\frac{1}{2}^3 \times 5\frac{1}{2}} = 1 \cdot 743$ inches, &c. It is shown in (303) that the strength of Timber pillars may be found direct from the Modulus of Elasticity.

CHAPTER XIX.

ON PERMANENT SET.

(751.) "*Defect of Elasticity.*"—The elasticity of all bodies is more or less imperfect, and manifests itself in two principal ways. 1st, by the extensions, compressions, deflections, &c., increasing more rapidly than the strains, whereas with perfect elasticity they would be simply proportional to those strains; the effect of this fact is shown in (604), (613), (688). The 2nd result of defective elasticity is that when once strained, the body never returns to its primitive form, but takes a "permanent set" varying in amount very greatly with the nature of the material and the extent of the strain; this will form the subject of the present chapter.

(752.) In earlier days it was assumed that within what was termed the *limit of elasticity*, or with strains less than about one-third of the breaking weight, the elasticity of ordinary materials, such as cast and wrought iron, timber, &c., was perfect, that is to say, the extensions, &c., being simply proportional to those strains, and giving no permanent set. But the refined experiments of Mr. Hodgkinson have shown that, although this may be practically true with some materials, such as wrought iron and steel, it is in all probability not strictly true with any; he found that the sets were generally proportional to the *square* of the strains, or nearly so: see (688) and Table 111, from which it would follow that if with a moderate strain the set were considerable, then, 1st, that with very slight strains the sets would be excessively small, and possibly unmeasurable, if not inappreciable; but, 2nd, that law would show that there must be sets with all strains, however small.

PERMANENT SET UNDER TENSILE AND COMPRESSIVE STRAINS.

(753.) "*Cast Iron.*"—Table 115 gives the result of Mr. Hodgkinson's experiments on the set from both strains; those with tensile strain were made on round bars of iron united together at the ends, so that the whole length exclusive of the couplings was 50 feet: the diameter was about $1\frac{1}{8}$ inch, equal to one square inch area. There were nine experiments upon four kinds of cast iron: namely, Lowmoor No. 2, Blaenavon No. 2, Gartsherrie No. 3, and a mixture of Leeswood No. 3 and Glengarnock No. 3 in equal proportions. There were two experiments upon each of the simple irons, and three upon the mixture:—the mean experimental tensile strains are given in col. 1, and the corresponding sets in col. 4.

The sets under compressive strains were obtained by experiments on the same kinds of iron, the bars being 10 feet long, and 1 inch square nearly, and to prevent lateral flexure they were enclosed in a strong iron frame, &c. The mean experimental compressive strains are given in col. 1, and the corresponding sets in col. 5.

(754.) By plotting these experimental results in diagrams we have obtained Fig. 217, and from that we have found the sets for even tons and half-tons given in col. 3 of the Table. In col. 6 we have given the ratio of the sets under the same amount of tensile and compressive strains:—it will be observed that with small strains, the compressive sets are much greater than the tensile ones, but with heavy strains the tensile sets are much the greater. The fact is that the amount of set is governed, not by the absolute strain alone, but by the relative strain with reference to the ultimate or breaking weight. As the ultimate strain is approached, the set is very rapidly increased; the ultimate strength of cast iron being six times greater for compressive than for tensile strains, defect of elasticity, as manifested by the set, tells more rapidly with the latter, so that while with the small strain of half a ton per square inch, the set with a compressive strain is greater than with a tensile in the ratio of 1·713 to 1, they become about equal with $3\frac{1}{2}$ tons, and only about half or ·574 to 1 with 7 tons per square inch,

TABLE 115.—Of the PERMANENT SET of CAST IRON under TENSILE and COMPRESSIVE STRAINS: by Direct Experiments.

Strain per Square Inch.			Permanent Set in Parts of the Length.		Ratio.
Experimental Weight. Lbs.	Interpolated. Lbs.	Interpolated. Tons.	By Tensile Strain.	By Compressive Strain.	
..	1,120	½	·00000167	·000002860	1·713
1,581	..	..	·00000183	..	..
2,065	..	..	..	·000003917	..
2,108	..	..	·00000454	..	..
..	2,240	1	·00000500	·00000793	1·586
3,161	..	..	·00000892	..	..
..	3,360	1½	·00000983	·00001413	1·437
4,130	..	..	..	·00001884	..
4,215	..	..	·0000146	..	..
..	4,480	2	·0000163	·00002147	1·317
5,269	..	..	·0000221	..	..
..	5,600	2½	·0000245	·00002940	1·200
6,194	..	..	·0000333	..	..
6,323	..	..	..	·00003100	..
..	6,720	3	·0000347	·00003720	1·072
7,676	..	..	·0000431	..	..
..	7,840	3½	·0000467	·00004710	1·008
8,259	..	..	..	·00005375	..
8,430	..	..	·0000553	..	..
..	8,960	4	·0000615	·00005700	·927
9,848	..	..	·0000703	..	..
..	10,080	4½	·0000807	·00006810	·844
10,538	..	..	·0000885	..	..
..	11,200	5	·0001033	·00007930	·768
11,591	..	..	·0001088	..	..
..	12,320	5½	·0001313	·00009180	·699
12,388	..	..	..	·00009063	..
12,645	..	..	·0001340	..	..
..	13,440	6	·0001613	·0001045	·648
13,700	..	..	·0001747	..	..
14,453	..	..	..	·0001171	..
..	14,560	6½	·0001950	·0001181	·606
14,793	..	..	·0002008	..	..
..	15,680	7	·0002300	·0001320	·574
16,518	..	..	..	·0001427	..
(1)	(2)	(3)	(4)	(5)	(6)

TABLE 115.—Of the PERMANENT SET of CAST IRON under TENSILE and COMPRESSIVE STRAINS—*continued.*

Strain per Square Inch.			Permanent Set in Parts of the Length.		Ratio.
Experimental Weight. Lbs.	Interpolated. Lbs.	Interpolated. Tons.	By Tensile Strain.	By Compressive Strain.	
..	17,920	8	..	·0001627	..
18,583	..	..	..	·0001709	..
..	20,160	9	..	·0001953	..
20,647	..	..	..	·0002070	..
..	22,400	10	..	·0002307	..
..	24,640	11	..	·0002713	..
24,777	..	..	..	·0002683	..
..	26,880	12	..	·0003160	..
28,906	..	..	..	·0003583	..
..	29,120	13	..	·0003710	..
..	31,360	14	..	·0004430	..
33,030	..	..	..	·0005080	..
..	33,600	15	..	·0005320	..
..	35,840	16	..	·0006430	..
37,159	..	..	..	·0007018	..
..	38,080	17	..	·0007870	..
..	40,320	18	..	·0009520	..
41,347	..	..	..	·001072	..
..	42,560	19	.	·001157	..
(1)	(2)	(3)	(4)	(5)	(6)

that being about the breaking weight by tensile strain, but only one-sixth of the breaking weight by compressive strain.

(755.) The ultimate compressive strength of cast iron is about 42 or 43 tons per square inch (132), but the set was not observed with strains greater than about 19 tons, as in the Table. With $2\frac{1}{2}$ tons per square inch Tensile strain, which is about $\frac{1}{3}$rd of the strength of cast iron, the extension by col. 4 of Table 88 is ·00044625, and the set with that strain by col. 4 of Table 115 being ·0000245, is equal to ·0000245 $\div$ ·00044625 = ·055, or $5\frac{1}{2}$ per cent. With 14 tons per square inch crushing strain, which is about $\frac{1}{3}$rd of the ultimate compressive strength of cast iron, the compression by col. 5 of Table 89 is ·002731,

and the set by col. 5 of Table 115 being $\cdot000443$, is equal to $\cdot000443 \div \cdot002731 = \cdot016$, or $1\cdot6$ per cent. only.

By cols. 4, 5 the permanent set with any given tensile or compressive strain may be easily calculated by interpolation, remembering that it is proportional to the *square* of the strain (752). Thus, the set of a cast-iron bar after a tensile strain of 6000 lbs., or $2\cdot7$ tons, may be found from col. 4, which gives $\cdot0000245$ for $2\frac{1}{2}$ tons, therefore $\cdot0000245 \times 2\cdot7^2 \div 2\cdot5^2 = \cdot0000286$ for $2\cdot7$ tons, which with a length of say $72\frac{1}{2}$ feet, or 870 inches, gives $\cdot0000286 \times 870 = \cdot25$, or $\frac{1}{4}$ inch permanent set, &c.

(756.) "*Wrought Iron.*"—The results of Mr. Hodgkinson's experiments on the set of wrought iron under tensile strains are given in cols. 5, 5 of Tables 94, 95, the "sets" being reduced to parts of the length of the bar. The observed sets with very small strains are very anomalous, due no doubt to the difficulty of measuring with accuracy such minute distances as the sets really are with light strains, even when the bars are of great length:—thus the observed set with 7571 lbs. was only $\cdot0022$, or $\frac{1}{454}$ inch on a length of 50 feet. The sets for even tons were obtained by interpolation between the next greater and lesser experimental sets.

By plotting the experimental observations in a large diagram, we have obtained the sets in col. 2 of Table 116; the mean set per ton given in col. 3 will serve to calculate the set for strains intermediate between those given in the Table, by simple interpolation, which will be accurate enough for ordinary purposes. Thus, with a bar 100 feet, or 1200 inches long, with say $12\cdot4$ tons per square inch Tensile strain, the set after the strain was taken off would be $\cdot00000725 \times 12\cdot4 \times 1200 = \cdot108$ inch; more accurately it would be $\cdot108 \times 12\cdot4^2 \div 12^2 = \cdot115$ inch. For strains under 8 tons, it has been assumed in Table 116 that the sets are proportional to W^2 (752). Tables 94, 95 show that with heavy strains, *time* is very influential on the Permanent Set.

(757.) Cols. 6, 6 in Tables 94 and 95 show that the permanent sets give very clearly and definitely the point at which a wrought-iron bar is overstrained by a tensile strain. Thus, in

TABLE 116.—Of the PERMANENT "SET" of WROUGHT IRON under TENSILE STRAINS.

Tons per Square Inch.	Set in Parts of the Length of the Bar.				
	By Diagram.		By Direct Experiment.		
	Total.	Mean, per Ton.	Bar ½ Inch Diameter.	Bar ¾ Inch Diameter.	Mean.
1	·000000138	·000000138	..	..	..
2	·000000552	·000000276	..	..	..
3	·000001242	·000000414	..	..	..
4	·000002208	·000000552	·00000285	..	..
5	·000003450	·000000690	·00000356	·00000489	·00000422
6	·000004968	·000000828	·00000427	·00000664	·00000545
7	·000006762	·000000966	·00000499	·00000882	·00000690
8	·000008827	·000001103	·00000631	·0000120	·00000915
9	·0000132	·000001467	·0000101	·0000225	·0000163
10	·0000250	·000002500	·0000208	·0000327	·0000268
11	·0000441	·000004000	·0000455	·0000516	·0000485
12	·0000870	·00000725	·0000968	·0000844	·0000908
13	·0002708	·0000208	·000247	·0003760	·0003110
14	·000827	·000059	·000919	..	..
15	·00356	·000237	·00229	..	..
16	·0080	·000500	·00543	..	..
17	·0125	·000735	·00864	..	..
18	·0169	·000939	·0100	..	..
19	·0214	·00113	·0156	..	..
20	·0260	·00130	·0191	..	..
21	·0305	·00145	·0217	..	..
22	·0351	·00160	..	..	..
(1)	(2)	(3)	(4)	(5)	(6)

Table 94, the set with 27,761 lbs. is 9·3 per cent. of the extension, but, with 30,284 lbs. becomes 36·5 per cent.: again, in Table 95, 26,675 lbs. gives a set of 8·2 per cent. of the extension; but 29,343 lbs. gives 20 per cent., &c. With both these bars therefore about 12 tons per square inch will decidedly overstrain the iron: with very heavy strains the permanent set is nearly *equal to the extension*, being as much as 92 per cent. in Table 94, and 94 per cent. in Table 95.

We have unfortunately no experiments on the set of wrought iron under compressive strains; neither have we any experi-

ments showing the effect of variations in the thickness or size of bar in the case of cast iron on the sets under tensile and compressive strains; the effect of size on the set of bars loaded transversely we shall find (769) to be very considerable.

(758.) The longitudinal set may be determined approximately from the observed set of a bar loaded transversely by the rules in (511), (642). Thus, by col. 5 of Table 106 a bar of wrought iron in which $d = 1 \cdot 515$ inch, $b = 5 \cdot 523$ inches, $l = 13\frac{1}{2}$ feet, or 162 inches, loaded with a weight $w = 952$ lbs. in the centre, has a set of $\cdot 02$ inch. This by the rule (511) is equivalent to a strain at the upper and lower edges of the section, or $f = \frac{3 \times 162 \times 952}{2 \times 5 \cdot 523 \times 1 \cdot 515^2} = 17920$ lbs., or 8 tons per square inch; and the longitudinal set with that strain by the rule in (642) becomes $E_x = \frac{3 \times 1 \cdot 515 \times \cdot 02}{2 \times 81^2} = \cdot 000006927$ of the length of a bar strained longitudinally with 8 tons per square inch. By direct experiment, the set by Table 116 $= \cdot 00000827$, the $\frac{1}{2}$-inch bar gave $\cdot 00000678$ with 18,672 lbs. by Table 95; and the $\frac{3}{4}$ bar in Table 94 gave $\cdot 0000109$ with 17,666 lbs. tensile strain. This method is useful in those numerous cases where we have no direct experimental information.

PERMANENT SET UNDER TRANSVERSE STRAINS.

(759.) "*Cast Iron.*"—The elasticity of cast iron varies so much with different kinds of iron, and with the size of the casting, or rather with its thickness (744), that it is difficult to give rules for permanent set which will be even approximately correct for general cases.

For rectangular bars of Cast iron we have the Rules:—

(760.) For W_T in tons, $S = \frac{L^4 \times (W_T \div z)^2 \times \cdot 012}{d^5 \times b^2}$.

(761.) W_C in cwts., $S = \frac{L^4 \times (W_C \div z)^2 \times \cdot 00003}{d^5 \times b^2}$.

(762.) W_P in lbs., $S = \frac{L^4 \times (W_P \div z)^2 \times \cdot 000000002391}{d^5 \times b^2}$.

With square sections these Rules become:—

(763.) For W_T in tons, $S = \frac{L^4 \times (W_T \div z)^2 \times \cdot 012}{d^7}$.

(764.) W_C in cwts., $S = \frac{L_4 \times (W_C \div z)^2 \times \cdot 00003}{d^7}$.

(765.) W_P in lbs., $S = \frac{L^4 \times (W_P \div z)^2 \times \cdot 000000002391}{d^7}$.

In which L = length between supports in feet: d = depth in inches: b = breadth in inches: z = the ratios of strength as governed by the least dimension of the casting, and as given in (934), &c.: S = Permanent set in inches.

Thus, with the bar in Table 108 say we take $W_P = 112$ lbs.; L being 162 inches, $d = 1 \cdot 522$ inch: $b = 3 \cdot 066$ inches, &c. Then taking z from (934) = ·8141 for $1\frac{1}{2}$ inch thick, the Rule (762) gives $S = \frac{13 \cdot 5^4 \times (112 \div \cdot 8141)^2 \times \cdot 000000002391}{1 \cdot 522^5 \times 3 \cdot 066^2} = \cdot 0196$ inch Permanent set; col. 10: experiment gave ·0192 inch, col. 8. The cols. 8 and 10 do not agree very well with heavy strains.

(766.) The mean Transverse breaking weight of a cast-iron bar 1 inch square and 1 foot long is 2063 lbs. by col. 7 of Table 64, hence with Factor 3 we have 2063 ÷ 3 = 688 lbs. safe load, with which Rule (765) gives

$$S = \frac{1^4 \times (688 \div 1)^2 \times \cdot 000000002391}{1^7} = \cdot 001132 \text{ inch set}$$

with safe load, from which we obtain the Rules:—

(767.) Permanent set with Safe Load $S_S = \frac{L^2}{d \times 900}$.

(768.) „ „ with Breaking weight, $S_B = \frac{L^2}{d \times 100}$.

In which S_S = Permanent set with Safe Load, and S_B with Breaking weight; the rest as before. These simple rules may be applied with approximate accuracy to cast-iron beams of all sections, the effect of size being eliminated (773): for example, a

girder of any ordinary section, say 20 feet long, and 8 inches deep, gives $S_B = \frac{20^2}{8 \times 100} = \cdot 5$, or $\frac{1}{2}$ inch set with Breaking weight: and $S_s = \frac{20^2}{8 \times 900} = \cdot 0555$, or $\frac{1}{18}$th inch with safe load, namely $\frac{1}{3}$rd the breaking weight, &c.

Table 117 gives a summary of Mr. Hodgkinson's experiments on the mean set of 1-inch bars of cast iron, $4\frac{1}{2}$ feet long: col. 3 has been calculated by Rule (765): thus with 56 lbs. we obtain $S = \frac{4 \cdot 5^4 \times (56 \div 1)^2 \times \cdot 000000002391}{1^7} = \cdot 003075$ inch, &c.

TABLE **117**.—Of the PERMANENT SET in CAST-IRON BEAMS, 1 inch square and $4\frac{1}{2}$ feet long.

Weight.	By Experiment.	By Calculation.	Ratio of Load to Breaking Weight.
lbs.	inch.	inch.	Per Cent.
56	·0037	·003075	12
112	·0127	·012300	24
168	·0280	·027675	36
224	·049	·049200	49
280	·076	·076875	61
336	·109	·110700	73
392	·149	·150675	85
448	·196	·196800	97
(1)	(2)	(3)	(4)

(769.) "*Effect of Size of Casting.*"—This effect will be most clearly shown by reducing the experimental results from bars of different sizes to the equivalent loads, deflections, and permanent sets with a "Unit" beam, or a bar 1 foot long, 1 inch deep, and 1 inch wide, and in order to clear the investigation from the complications arising from the varying properties of different kinds of iron, it will be necessary to take one and the same iron for all the bars. We shall then obtain the deflection and set for bars of different sizes in a directly comparable form and shall show the effect of size alone cleared from all obscuring circumstances; see (746).

For this purpose we will take Blaenavon No. 2 iron: bars about 1, 2, and 3 inches square, with lengths of $4\frac{1}{2}$, 9, and $13\frac{1}{2}$

feet respectively, were experimented upon by Mr. Hodgkinson: the loads, deflections, and permanent sets are given by cols. 1, 2, 3 of Table 118.

To reduce the loads in col. 1 to equivalent ones of a "Unit" beam we have the Rule:—

(770.) $$W = w \times L \div (d^2 \times b).$$

In which W = reduced weight on Unit beam: w = the experimental weight on a bar whose depth = d, and its breadth = b, both in inches: while its length = L in feet. Thus a weight = 224 lbs. on a bar 13½ feet long, 3·05 inches deep, and 3·095 inches wide is by the Rule equivalent to $W = 224 \times 13{\cdot}5 \div (3{\cdot}05^2 \times 3{\cdot}095) = 105$ lbs. on a bar 1 × 1 inch × 1 foot long, as in col. 5 of Table 118.

Then to reduce the experimental deflection with say the same 3-inch bar to that due to the equivalent load on Unit bar we have the Rule:—

(771.) $$\delta_1 = \delta \times d \div L^2.$$

In which δ = the deflection by experiment with a bar whose depth = d in inches, and its length L in feet: δ_1 = the deflection with *equivalent* weight on Unit beam: thus in our case $\delta_1 = {\cdot}195 \times 3{\cdot}05 \div 13{\cdot}5^2 = {\cdot}00326$, as in col. 6 of Table 118.

The reduced permanent set may be found by a modification of the same Rule:—

(772.) $$S_1 = S \times d \div L^2.$$

In which S = the permanent set by experiment with a bar whose depth = d in inches and its length L in feet, and S_1 = the equivalent set on a bar 1 inch square and 1 foot long:— thus in our case ${\cdot}003 \times 3{\cdot}05 \div 13{\cdot}5^2 = {\cdot}0000502$ inch, as in col. 7, &c.

We thus find, that with a bar 13½ feet long, 3·05 inches deep, 3·095 inches wide, a load of 224 lbs. produces by experiment a deflection of ·195 inch, and a permanent set of ·003 inch, and that this is equivalent to a load of 105 lbs. on a bar 1 foot long and 1 inch square, which load would give a deflection of ·00326 inch, and a permanent set of ·0000502 inch, &c. Table 118 has been calculated in this way throughout.

TABLE 118.—Of the DEFLECTION and PERMANENT SET of BARS of BLAENAVON CAST IRON, 1, 2, and 3 inches square: showing the effect of *Size*.

Bar, L = 4½ Feet: d = 1·075: b = 1·081.				Reduced to 1 In. Square, 1 Ft. Long.		
Central Load.	Deflection.	Permanent Set.	Modulus of Elasticity E_D.	Central Load.	Deflection.	Permanent Set.
lbs.			lbs.	lbs.		
28	·060	·0025	13,679,800	101	·00318	·0001317
56	·116	·006	14,151,500	202	·00616	·0003185
112	·238	·012	13,794,700	403	·01263	·000639
168	·368	·022	13,382,400	606	·01953	·001168
224	·513	·038	13,054,300	807	·02723	·002017
280	·665	·058	12,342,400	1009	·03530	·003078
336	·837	·094	11,767,600	1210	·04443	·004991
392	1·019	·131	11,276,800	1412	·05409	·006954
448	1·225	·181	10,720,500	1614	·06502	·009608
476	1·348	..	10,351,200	1714	·07155	..
504	1·462	·257	10,105,500	1815	·07760	·01364
532	1·567	..	9,952 120	1916	·08318	..
550 Broke	1·656	·325	9,738,700	1980	·08790	·01725
Bar, L = 9 Feet: d = 2·008: b = 1·963.				Reduced to 1 In. Square, 1 Ft. Long.		
112	·160	·006	13,877,000	127	·00397	·0001487
224	·337	·024	13,170,970	255	·00835	·000595
336	·533	·050	12,491,400	383	·01321	·001240
448	·745	·084	11,915,800	510	·01847	·002082
560	·975	·126	11,381,100	638	·02417	·003124
672	1·224	·179	10,879,000	765	·03034	·004438
784	1·497	·245	10,377,500	893	·03710	·006074
896	1·758	·318	10,099,200	1020	·04358	·007883
1008	2·129	·435	9,844,000	1148	·05278	·01078
1064	2·266	..	9,304,260	1212	·05617	..
1120	2·499	·572	8,880,830	1275	·06195	·01418
1207 Broke	2·843	..	8,412,600	1372	·07048	·01780
Bar, L = 13½ Feet: d = 3·05: b = 3·095.				Reduced to 1 In. Square, 1 Ft. Long.		
224	·195	·003	13,903,900	105	·00326	·0000502
448	·407	·014	13,323,200	210	·00681	·0002343
672	·633	·038	12,849,600	315	·01060	·000636
896	·882	·072	12,296,000	420	·01476	·001205
1120	1·149	·121	11,797,300	525	·01923	·002025
(1)	(2)	(3)	(4)	(5)	(6)	(7)

TABLE 118.—Of the DEFLECTION and PERMANENT SET of BARS of BLAENAVON CAST IRON, 1, 2, and 3 inches square—*continued.*

Bar, L = 13½ Feet: d = 3·05: b = 3·095.				Reduced to 1 In. Square, 1 Ft. Long.		
Central Load.	Deflection.	Permanent Set.	Modulus of Elasticity E_D.	Central Load.	Deflection.	Permanent Set.
lbs.			lbs.	lbs.		
1344	1·447	·186	11,244,900	630	·02422	·003113
1568	1·779	·275	10,667,300	735	·02977	·004613
1792	2·183	·388	9,935,910	840	·03653	·006493
2016	2·568	·531	9,501,240	945	·04298	·008887
2240	2·997	·688	9,046,580	1050	·05016	·01151
2464	3·637	·981	8,199,400	1155	·06086	·01642
2688	4·095	1·218	7.945,000	1260	·06853	·02038
2863 Broke	4·451	1·630	7,614,440	1342	·07617	·02728
(1)	(2)	(3)	(4)	(5)	(6)	(7)

(773.) In the Diagram, Fig. 219, these reduced deflections and permanent sets are represented graphically, and show clearly the superior strength and stiffness of small castings.

It is important to observe, however, that the permanent set with the safe load of ⅓rd the breaking weight is about the same with all the bars: thus at A or ⅓rd the breaking weight of the 1-inch bar, the set is about the same as at B, which is about ⅓rd the breaking weight of the reduced 2 and 3-inch bars. This shows that the Rule (767) applies to bars of all sizes.

That defect of elasticity increases with the size of the casting is manifested in the Diagram by the permanent set and the deflections increasing therewith: it is also shown by the high ratio of the two to one another, which also increases with the size of the bar, as shown by Table 118. Thus the ratio of the *ultimate* set to the *ultimate* deflection is with the 1-inch bar ·0879 ÷ ·01725 = 5·096: with the 2-inch bar ·07048 ÷ ·0178 = 3·96: and with the 3-inch bar ·07617 ÷ 02728 = 2·793.

Again: taking from col. 5 the same load as nearly as possible on the reduced 1, 2, and 3-inch bars, we have say 1009, 1020, and 1050 lbs., and might expect the Permanent sets in col. 7 to be all alike, but so influential is *size* on the strength, and thereby on the set, that col. 7 gives ·003078; ·007883 and

·01151, or in the Ratio 1·0; 2·56, and 3·74. The fact is that although the loads were nearly identical in actual amount, they were very different in their Ratio to the respective strengths of the bars: with the 1-inch bars the load was 1009 ÷ 1980 = ·51, or 51 per cent. of the breaking weight: with the reduced 2-inch, 1020 ÷ 1372 = ·74, or 74 per cent.: and with the reduced 3-inch bars 1050 ÷ 1342 = ·78, or 78 per cent. The Permanent Set increasing very rapidly as the Breaking weight is approached, that fact tells on the results, as we have seen.

CHAPTER XX.

ON IMPACT.

(774.) "*General Principles.*"—The power of bodies to resist an impulsive strain or a blow is second only in importance to their power in bearing a statical load or dead weight. Many of the forces which the Engineer has to deal with in practice are dynamic ones, or forces in motion, and as the laws governing the strength of Materials in that case differ entirely from those for a statical force it will be necessary to investigate those laws somewhat exhaustively.

We will take first the case of impact on Beams, not only because it is the most important but also as giving the greatest facility for the illustration of general principles. It will be expedient in explaining the theory and leading facts, to take first a very light beam or a case in which the weight of the beam itself is so small in proportion to the strength that it may be neglected without sensible error. The results we thus obtain may be afterwards modified for practical cases in which the weight of the beam or the load upon it is considerable and its effect influential.

(775.) Let A in Fig. 186 be a beam or elastic spring, say of steel, fixed at one end, and let its elasticity be such that it deflects 1 inch for each pound at the end, then with W = 10 lbs. as in the figure, the deflection will be 10 inches, or from B

to C. Now, a certain amount of *power* has to be expended in thus bending the beam, which may be expressed in inch-lbs., or pounds falling 1 inch. At first sight it might appear that in our case we have 10 lbs. falling 10 inches, or $10 \times 10 = 100$ inch-lbs., but the strain or bending weight is not uniform throughout the fall;—at first it is 0 and increases from B to C in arithmetical ratio from 0 to 10, hence the *mean* weight is evidently $(0 + 10) \div 2 = 5$ lbs. which, falling 10 inches, gives $5 \times 10 = 50$ inch-lbs. as the power required to bend the spring 10 inches. It follows from this, that if the weight instead of being placed steadily on the beam were allowed to *fall* through the 10 inches deflection, we should require only 5 lbs. to deflect the beam from B to C, instead of 10 lbs. dead weight. If, therefore, we place 5 lbs. at B and suddenly release it, the spring would be deflected by that weight to C, the point due to a dead weight of 10 lbs.; or, if on the other hand the weight of 10 lbs. had been suddenly released at B it would have deflected the beam not to C only, but to D, 20 inches below B; or in other words, it would have strained the spring to an extent double that produced by a similar dead weight. That this is a fact is proved and illustrated by the experiments of Captain James, R.E., given in Table 119, the mean ratio of the 14 experiments is 1 to 1·938, or very nearly double as given by theory. The same effect would be produced by any weight less than 5 lbs., if the height of fall were proportionally increased; thus a weight of 1 lb. falling 50 inches from E to C would deflect the beam as before from B to C.

It should be observed that the height of fall must be measured from C, the point to which the beam is deflected, and not from B, the point where the falling weight first strikes the Beam.

RESILIENCE.

(776.) The *Power* of a beam in resisting Impact has been termed its "Resilience," and it will be seen from the foregoing investigation that it may be expressed by taking half the product of the deflection by the weight producing it: hence we have the general Rule:—

(777.) $$R = \delta \times W \times \tfrac{1}{2}.$$

TABLE 119.—Of the DEFLECTIONS by given DEAD LOADS on CAST-IRON BARS, compared with those produced by the same Loads, laid on the Beam and suddenly released.

Length. Feet.	Depth. Inches.	Breadth. Inches.	Kind of Load, &c.	Weight or Load in Lbs.								Breaking Weights. Lbs.
				112	224	448	560	784	896	1080	1120	
				Deflections in Inches.								
9	2	1	By dead loads	·25	·58	1·03	1·39	..	..	..	..	1000
"	"	"	By released loads	·51	·93	2·12	2·93	..	..	..	..	588
			Ratio of Deflections, &c.	2·04	1·61	2·06	2·11	..	..	..	..	1·70
9	3	1	By dead loads	..	·16	..	·41	·61	·72	·84	·96	2008
"	"	"	By released loads	..	·30	..	·72	1·08	1·28	1·55	1·91	1230
			Ratio of Deflections, &c.	..	1·88	..	1·76	1·77	1·78	1·85	1·99	1·633
9	$1\frac{1}{2}$	4	By dead loads	..	..	·65	..	1·24	1·47	1·71	..	1911
"	"	"	By released loads	..	..	1·51	..	2·25	3·14	3·41	..	1110
			Ratio of Deflections, &c.	..	..	2·32	..	1·82	2·14	2·00	..	1·722

The mean Ratio from the 14 sets of experiments is 1·94 to 1: theory giving 2 to 1.

In which δ = the deflection, say in inches: W = the deflecting weight in lbs., tons, &c., and R = the Resilience of the Beam in inch-lbs., or inch-tons depending on W. The most useful values of R are for the two standard loads;—Breaking Weight,—and Working Load; for convenience we may indicate them by R and r respectively.

Table 67 gives in cols. 5 and 6, the values of R, r, in inch-lbs. for a Standard bar 1 inch square and 1 foot long, which may be termed the "Specific Resilience" of those Materials. Thus for Cast iron $R = \delta \times W \times \frac{1}{2}$ becomes $R = \cdot 0785 \times 2063 \times \frac{1}{2} = 81 \cdot 0$, as in col. 5: similarly cols. 3, 4 give $r = \cdot 0197 \times 688 \times \frac{1}{2} = 6 \cdot 78$, as in col. 6.

The same reasoning and Rule will apply to cases other than beams, say to the driving of a nail by a falling weight: thus, Mr. Bevan found that to drive a sixpenny nail $1\frac{1}{2}$ inch into dry Christiania Deal required a steady pressure of 400 lbs. To do the same work by impact required 4 blows of a hammer weighing 6·275 lbs., falling 12 inches at each stroke, the mechanical work done being $6 \cdot 275 \times 12 \times 4 = 301$ inch-lbs.: by the Rule we obtain $R = 1\frac{1}{2} \times 400 \times \frac{1}{2} = 300$ inch-lbs.

(778.) We can now search for the Laws by which the depth, breadth, and length of a beam govern R, or its power in resisting Impact. By (659) it is shown that $\delta = \dfrac{L^3 \times W \times C}{d^3 \times b}$, hence when all are constant except d and W, we have $\delta = \dfrac{W}{d^3}$. Now by (324), the strength of beams or W varies as d^2, hence with depths in the ratio 1, 2, 3, 4, the transverse strength for a dead load will vary in the *ratio* $1^2, 2^2, 3^2, 4^2$, or 1, 4, 9, 16, and with those loads, the deflections by rule $\delta = \dfrac{W}{d^3}$ become $\dfrac{1}{1^3}, \dfrac{4}{2^3}, \dfrac{9}{3^3}, \dfrac{16}{4^3}$, or $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, hence the *ratio* of R or $\delta \times W$ comes out $1 \times 1 = 1$: $4 \times \frac{1}{2} = 2$: $9 \times \frac{1}{3} = 3$, and $16 \times \frac{1}{4} = 4$, or in the simple direct ratio of the depth d, namely, 1, 2, 3, 4.

Then, for the breadths: the same Rule (659) shows that δ varies as $\dfrac{W}{b}$, therefore with breadths in the ratio 1, 2, 3, 4,

the weights W, are 1, 2, 3, 4 also, and $\frac{W}{b}$ becomes $\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4} =$ 1·0 in all cases, showing that when the loads are proportional to the breadths the deflections are constant. But R, or $\delta \times W$, will not be constant, but become $1 \times 1 = 1 : 1 \times 2 = 2 : 1 \times 3 = 3 :$ and $1 \times 4 = 4$, &c., showing that R is simply and directly proportional to b.

Then, for the influence of the length: the same Rule (659) shows that $\delta = L^3 \times W$; then with lengths 1, 2, 3, 4, W, or the dead loads, will obviously be in the simple *inverse* ratio, $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, &c.: hence the deflections, or $\delta = L^3 \times W$, become $1^3 \times 1 = 1 : 2^3 \times \frac{1}{2} = 4 : 3^3 \times \frac{1}{3} = 9 :$ and $4^3 \times \frac{1}{4} = 16$, and R, or the power to resist Impact, or $\delta \times W$, becomes $1 \times 1 = 1 : 4 \times \frac{1}{2} = 2 : 9 \times \frac{1}{3} = 3 :$ and $16 \times \frac{1}{4} = 4$, which are in the simple *direct* ratio of the lengths. This is remarkable, being precisely the reverse of the effect of length on the dead load, where of course the strength is *inversely* as the length.

We thus find that the power of Rectangular Beams in resisting Impact, or R, is simply and directly proportional to the length multiplied by the depth, and by the breadth. Putting these results into the form of Rules, we have for Rectangular beams :—

(779.) $$h = \frac{d \times b \times L \times R}{w}.$$

(780.) $$w \times h = d \times b \times L \times R.$$

In which d = the depth of the Beam in inches : b = breadth in inches : L = length between supports in feet : w = falling weight, say in lbs. : h = height fallen by w in inches, and R = the Specific Resilience of the Material, as given by cols. 5 or 6 in Table 67 ; cols. 9, 12 in Table 64, &c.

These Rules do not allow for the resistance from the Inertia of the Beam or of any dead load it may bear before receiving the force of the blow, and by which the results may be modified very considerably.

(781.) "*Effect of the Inertia of the Beam.*"—Mr. Hodgkinson has shown by his experiments that in resisting impact, the

power of a heavy beam is to that of a light one as the inertia of the beam plus the falling weight is to the falling weight alone, or as $\frac{I + w}{w}$, hence the Rule (780) becomes $h = \frac{d \times b \times L \times R}{w} \times \frac{I + w}{w}$, from which we obtain the general Rules:—

(782.) $$h = \frac{d \times b \times L \times R \times (I + w)}{w^2}.$$

(783.) $$d = \frac{w^2 \times h}{b \times L \times R \times (I + w)}.$$

(784.) $$b = \frac{w^2 \times h}{d \times L \times R \times (I + w)}.$$

(785.) $$d \times b = \frac{w^2 \times h}{L \times R \times (I + w)}.$$

In which, the letters have the same signification as in (780), except I, which is the Inertia of the beam and the load upon it. The inertia of a beam uniform in section from end to end, supported at the ends, and struck in the centre, may be taken at half the weight between supports. To this has to be added the whole central load (if any), or if otherwise distributed, it must be reduced to an equivalent central load.

(786.) The application of these rules may be illustrated by an example:—say we have a beam of English Oak 12 inches deep, 6 inches wide, 20 feet long between supports, and we require the height from which a weight of 5 cwt., or 560 lbs., must fall to strain the beam to $\frac{1}{5}$th of the breaking strain.

First we have to find the inertia of the beam from its weight:—we have $1 \times \frac{1}{2} \times 20 = 10$ cubic feet, and by col. 2 of Table 150, $48 \cdot 4 \times 10 = 484$ lbs. for the weight, the inertia is therefore 242 lbs., and taking the value of r at $2 \cdot 04$ from col. 6 of Table 67, we get by Rule (782):—

$$h = \frac{12 \times 6 \times 20 \times 2 \cdot 04 \times (242 + 560)}{560^2} = 7 \cdot 51 \text{ inches fall.}$$

To find the height of fall to *break* the beam with the same

falling weight, we obtain the value of R from col. 5 of the same Table $= 78 \cdot 4$, and h becomes

$$\frac{12 \times 6 \times 20 \times 78 \cdot 4 \times (242 + 560)}{560^2} = 288 \cdot 7 \text{ inches,}$$

or 24 feet:—hence the ratio of the breaking and safe heights is $288 \cdot 7 \div 7 \cdot 51 = 38 \cdot 4$ to 1, as in col. 9: see (825).

(787.) To vary the illustration, say that the beam was loaded with a central dead weight of 10 cwt., or 1120 lbs., adding which to the inertia of the beam itself we obtain $\mathrm{I} = 1120 + 242 = 1362$ lbs., and for $\frac{1}{5}$th of the breaking strain, h becomes

$$\frac{12 \times 6 \times 20 \times 2 \cdot 04 \times (1362 + 560)}{560^2} = 18 \text{ inches fall,}$$

whereas when unloaded the fall was $7 \cdot 51$ inches only (800).

If the extra load of 10 cwt. had been equally distributed all over the length of the beam, it would have been equivalent to a central load of 5 cwt., or 560 lbs., hence I would be $560 + 242 = 802$ lbs., and h becomes

$$\frac{12 \times 6 \times 20 \times 2 \cdot 04 \times (802 + 560)}{560^2} = 12 \cdot 75 \text{ inches fall.}$$

(788.) The three rules in (783), &c., by which d, b, and $d \times b$ are respectively determined, are difficult in application, because the inertia of the beam depends on the dimensions which are unknown, but we can assume dimensions and solve the question by repeated approximations to any desirable degree of accuracy. Say we require the dimensions of a beam of Elm 15 feet long to bear safely a weight of 8 cwt., or 896 lbs., falling 12 inches. We will assume the dimensions at 12 inches square, hence the beam contains 15 cubic feet, weighs by Table 150, $36 \cdot 65 \times 15 = 550$ lbs., its inertia $550 \div 2 = 275$ lbs.; the value of r from col. 6 of Table 67 is $2 \cdot 13$, and the rule (785) will now give $d \times b = \dfrac{896^2 \times 12}{15 \times 2 \cdot 13 \times (275 + 896)} = 257$,

and $\sqrt[2]{257} = 16$ inches square, instead of 12 inches as we assumed. But 16 inches would be too much, because the weight, and thereby the inertia, would be greater than we

assumed. Assuming 15 inches square, or 1·25 feet, as a second approximation, the weight comes out $1 \cdot 25 \times 1 \cdot 25 \times 15 \times 36 \cdot 15 = 847$ lbs., hence the inertia $= 424$ lbs., and $d \times b$ becomes $\frac{896^2 \times 12}{15 \times 2 \cdot 13 \times (424 + 896)} = 228$, and $\sqrt[2]{228} = 15 \cdot 1$ inches square, agreeing sufficiently closely with the sizes we assumed. By (821) it is shown that the beam might be of any dimensions at pleasure, so long as $d \times b = 228$; thus it might be $14 \times 16 \cdot 3 = 228$, or $12 \times 19 = 228$, &c., &c., and it would be unimportant whether it was struck by the falling weight in the direction of its greatest or least dimension, as shown in (824).

It will be observed that these Rules apply only to beams of rectangular sections; the following apply to beams of all sections, but are not so facile in application:—

(789.) $$h = \frac{f_0^2 \times W \times (I + w)}{F_0 \times w^2 \times 2}.$$

(790.) $$f_0 = \sqrt{\left(\frac{h \times F_0 \times w^2 \times 2}{W \times (I + w)}\right)}.$$

In which W = a given Statical weight on the beam, in pounds.
F_0 = Flexure or deflection produced by W, in inches.
w = falling weight, in pounds.
h = height fallen by w, in inches.
f_0 = flexure in inches produced by the impulse of w.
I = Inertia of the beam and its load, as in (785), &c.

(791.) These rules connect together the laws of the Statical and Dynamic forces, enabling us to reason from one to the other:—thus from the known strength and stiffness of a beam of any form, material, mode of fixing, &c., under a statical load or dead weight, we may calculate the effect of an impulsive strain upon it.

As an illustration of the application of these rules, we may take Mr. Bevan's experiments in Table 120:—they were made on a beam whose inertia or half-weight between supports was 63·5 lbs., a deflection of 1 inch was produced by a dead weight of 148 lbs., and the falling weight was 28 lbs. The deflections

produced by various heights of fall are given in col. 2 and as calculated by the rule in col. 3. Thus, with 12 inches fall, $f_0 = \left(\frac{12 \times 1 \times 28^2 \times 2}{148 \times (63 \cdot 5 + 28)}\right)\sqrt{\ } = 1 \cdot 18$ inch deflection: by experiment it was $1 \cdot 25$ inch.

TABLE 120.—Of IMPACT on BEAM of WOOD, 18 feet long, &c.

Height of Fall: Inches.	Deflection: Inches.	
	By Experiment.	By Calculation.
12	1·25	1·180
24	1·66	1·667
36	2·35	2·042
48	2·62	2·358
(1)	(2)	(3)

(792.) Again:—say that we have a beam in which a dead weight of 400 lbs. produces a deflection of 1·2 inch, the inertia or half-weight between supports being 600 lbs., and the falling weight 200 lbs.; we require the fall to produce a deflection of say $2\frac{1}{2}$ inches. Then by Rule (789):—

$$h = \frac{2 \cdot 5^2 \times 400 \times (600 + 200)}{1 \cdot 2 \times 200^2 \times 2} = 20 \cdot 8 \text{ inches fall.}$$

It will be observed that in both these examples, the *dimensions* of the beam are not given, nor are they required by the rules in (789), &c.

(793.) But these rules are based on the supposition that the elasticity of the deflecting beam is perfect, or that the deflections are strictly and simply proportional to the loads even up to the breaking point; this is far from the truth with cast iron (688), and is not strictly true perhaps with any material, although sufficiently so for practical purposes, with wrought iron and steel up to the "limit of Elasticity" (692). This will be seen by comparing Tables 121 and 122, for the rule in (789) shows that, other things being the same, the fall h should vary as f_0^2, and a comparison of cols. 2 and 3 of Table 121 shows that

this is practically true for wrought iron, but a similar comparison of cols. 2 and 3 of Table 122 shows that the defect of elasticity in cast iron causes a considerable departure from the rule (816).

When f_0 and F_0 have the same value, this source of error is eliminated, and even when they have nearly the same value, the rule may be used without serious error.

(794.) When f_0 is equal to F_0, that is to say, when the falling weight has to produce the same deflection as the statical or

TABLE 121.—OF EXPERIMENTS on RESISTANCE of WROUGHT-IRON BARS to IMPACT, showing that the Power is as the *Square* of the Deflection.

Deflection.	Fall in Feet.		Work done by Ball in Foot-lbs.		Weight of Ball in Lbs.	Length.	Depth.	Breadth.	Weight between Supports.
	By Experiment.	Calculated as δ^2.	By Experiment.	Calculated as δ^2.					
inches.						feet.	inches.	inches.	lbs.
½	·0477	·0456	3·601	3·440	75½	13½	1·027	5·51	252
1	·1823	·1823	13·764	13·764	"	"	"	"	"
1½	·3863	·4101	29·166	30·96	"	"	"	"	"
2	·7083	·7292	53·477	55·056	"	"	"	"	"
2½	1·0827	1·1393	81·744	86·000	"	"	"	"	"
3	1·5234	1·6407	115·017	123·84	"	"	"	"	"
3½	2 0541	2·2331	155·084	168·56	"	"	"	"	"
4	2·7044	2·9216	204·182	220·224	"	"	"	"	"
1	·0808	·0808	12·221	12·2	151¼	"	"	"	"
2	·3155	·3232	47·719	48·8	"	"	"	"	"
3	·6877	·7272	104·015	109·8	"	"	"	"	"
4	1·2578	1·2928	190·242	195·2	"	"	"	"	"
5	1·9645	2·020	297·131	305·0	"	"	"	"	"
6	2·7862	2·9088	421·413	439·2	"	"	"	"	"
½	·0123	·0123	7·417	7·4	603	13½	1·515	5·523	372
1	·0492	·0492	29·67	29·6	"	"	"	"	"
1½	·1061	·1107	63·98	66·6	"	"	"	"	"
2	·1876	·1968	113·1	118·4	"	"	"	"	"
2½	·2940	·3075	177·3	185·0	"	"	"	"	"
3	·4222	·4428	254·6	266·4	"	"	"	"	"
3½	·5745	·6027	346·4	362·6	"	"	"	"	"
4	·7428	·7872	447·9	473·6	"	"	"	"	"
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

TABLE 122.—Of EXPERIMENTS on IMPACT with CAST-IRON BARS, 1·53 inch deep, 6·122 inch wide, 13½ feet long, Falling Weight 603 lbs.

Deflection.	Fall in Feet.			Work done by the Ball in Foot-lbs.		
	By Experiment.	Calculated as δ^2.	Calculated as $\delta^{1\cdot 64}$.	By Experiment.	Calculated as δ^2.	Calculated as $\delta^{1\cdot 64}$.
inches.						
1½	·0638	·0638	·0638	38·47	38·47	38·47
3	·2095	·2552	·1988	126·33	153·88	119·9
4½	·4063	·5742	·3866	245·00	346·23	233·1
6	·6390	1·0208	·6197	385·32	615·52	373·7
7½	·9040	1·5950	·8915	545·11	961·75	537·6
9	1·1994	2·2968	1·2050	723 24	1384·92	726·7
(1)	(2)	(3)	(4)	(5)	(6)	(7)

dead weight W, then $\frac{f_0^2 \times W}{F_0}$ is equivalent to $\frac{f_0^2 \times W}{f_0}$ or to $\frac{F_0^2 \times W}{F_0}$, therefore to $F_0 \times W$ simply, and the rule in (789) becomes:—

(795.) $$h = \frac{F_0 \times W \times (I + w)}{w^2 \times 2}.$$

Say that we have a girder 30 feet long, 20 inches deep, capable of carrying safely 10 tons in the centre, the weight of the beam between supports being 2 tons, hence the inertia = 1 ton, and let the falling weight w be ½ a ton. We have first to find F, or the deflection produced by the statical weight, which by the rule in (702) will be $\delta = \frac{30^2 \times \cdot 02}{20} = \cdot 9$ inch:— then the rule (795) becomes $h = \frac{\cdot 9 \times 10 \times (1 + \frac{1}{2})}{\frac{1}{2}^2 \times 2}$ or $\frac{13 \cdot 5}{\cdot 5} = 27$ inches fall of ½ a ton to produce the same deflection and strain as the dead load of 10 tons. By Rule (789) we obtain $h = \frac{\cdot 9^2 \times 10 \times (1 + \frac{1}{2})}{\cdot 9 \times \frac{1}{2}^2 \times 2}$ or $\frac{12 \cdot 15}{\cdot 45} = 27$ inches, as before.

(796.) Again: a beam weighing 160 lbs. between supports, deflects 4 inches with a dead load of 600 lbs., and we require the height from which 50 lbs. must fall to produce the same

deflection. The inertia being $160 \div 2 = 80$ lbs., h becomes $\frac{4 \times 600 \times (80 + 50)}{50^2 \times 2} = 62 \cdot 4$ inches fall measured from the point to which the beam is deflected (775), therefore $62 \cdot 4 - 4 = 58 \cdot 4$ inches from the level of the unloaded beam.

We should obtain the same result in this case (defect of elasticity not interfering) by the rule in (789), which becomes $h = \frac{4^2 \times 600 \times (80 + 50)}{4 \times 50^2 \times 2} = 62 \cdot 4$ inches fall, as before.

(797.) "*Impact on Cast-iron Beams.*"—We may combine the laws of deflection for cast-iron girders with those of impact, so as to obtain simple general rules specially applicable to the two most important loads, namely, the safe and breaking weights.

Let W = the statical or dead load on the centre of a girder, in Tons or lbs.
w = the dynamic or falling weight, in the same terms as W.
h = the height fallen by w, in inches.
d = depth of the girder, in inches.
L = Length of the girder, in feet.
M = Multiplier, or ·02 for safe load, and ·0785 for breaking weight (Table 64).
I = Inertia of the beam, or half the weight between supports, in the same terms as W and w.

We then have the Rules:—

(798.) $$h = \frac{L^2 \times M \times W \times (I + w)}{d \times w^2 \times 2}.$$

(799.) $$d = \frac{L^2 \times M \times W \times (I + w)}{h \times w^2 \times 2}.$$

Taking the same example as in (795) we have

$$h = \frac{30^2 \times \cdot 02 \times 10 \times (1 + \frac{1}{2})}{20 \times \frac{1}{2}^2 \times 2} \text{ or } \frac{270}{10} = 27 \text{ inches fall}$$

for safe strain, as before.

If we required the fall to *break* the beam, we may find it by the same rule, observing that the breaking weight will be

30 tons, and M = ·0785, then $h = \frac{30^2 \times \cdot 0785 \times 30 \times (1 + \frac{1}{2})}{20 \times \frac{1}{2}^2 \times 2}$ = 318 inches, or $26\frac{1}{2}$ feet fall, to break the beam; being 11·8 times the fall which would strain the beam to one-third of the breaking weight (825).

(800.) "*Effect of a Dead Load on a Beam subjected to Impact.*"—In beams subjected to impact, there are two contrary results produced by a dead load. The first effect is the obvious one, that being deflected by the dead weight before receiving the blow, the falling weight has less work to do in breaking the beam, or in producing any given strain, therefore the height of fall with a given weight or the falling weight with a given height will be *less* as the dead load is increased. But the other result is, that the inertia of the dead load being added to that of the beam itself, the resistance to impact is increased, and the height fallen or the falling weight *increases* as the dead load is increased. The question becomes thus rather complicated, and requires investigation.

(801.) Taking the same spring as before (775), Fig. 186, which deflected 1 inch per pound of statical load, we will now observe the effect of loading it before impact with varying amounts of dead load, the weight of the beam itself being still for the sake of illustration taken as nothing. Say that the maximum strain, or that which breaks the beam, is 9 lbs. dead weight, deflecting the beam 9 inches:—then if unloaded, a weight of 1 lb. must fall by the rule (789) or $h = \frac{f_0^2 \times W \times (I + w)}{F_0 \times w^2 \times 2}$, in our case $\frac{9^2 \times 9 \times (0 + 1)}{9 \times 1^2 \times 2}$ = 40·5 inches. Then, having found that a weight of 1 lb. falling 40·5 inches deflects the unloaded beam 9 inches, we will observe the flexure which that weight falling always the same height will produce in the loaded beam. Thus, say we have a dead load of 4 lb. producing 4 inches deflection before impact, we then by Rule (790) obtain

$$f_0 = \sqrt{\left(\frac{40 \cdot 5 \times 9 \times 1^2 \times 2}{9 \times (4 + 1)}\right)} = 4 \cdot 02 \text{ inches flexure}$$

by the impulse of w falling 40·5 inches; but the beam being

previously deflected 4 inches by the dead load, the total flexure is $4 + 4 \cdot 02 = 8 \cdot 02$ inches, whereas when unloaded the same weight falling the same height deflected the beam 9 inches. Calculating in this way we obtain col. 5 of Table 123, and it will be observed that while the flexure produced by the impulse of w is continuously reduced by increasing the load on the beam col. 4, the flexure due to the dead weight, col. 3, is continuously increased, but in a different ratio, the result being that the final combined flexure or strain on the beam is a minimum when the dead load plus the falling weight, or $I + w$, is 3 or $\frac{1}{3}$rd of the breaking dead load; the deflection being then only 7·19 inches, instead of 9 inches as when unloaded.

TABLE 123.—Of the FLEXURE produced by IMPACT.

Dead Load.	I + w.	Flexure produced by		
		Dead Load.	Impact of w.	Total.
0	1	0	9·00	9·00
1	2	1	6·36	7·36
2	3	2	5·19	7·19*
3	4	3	4·50	7·50
4	5	4	4·02	8·02
5	6	5	3·67	8·67
6	7	6	3·40	9·40
7	8	7	3·18	10·18
8	9	8	3·00	11·00
9	10	9	2·85	11·85
(1)	(2)	(3)	(4)	(5)

(802.) This may be further illustrated if we take the same spring, and find the height of fall with a constant falling weight of 1 lb. to produce always the same deflection of 9 inches with varying dead loads. Thus with a dead load of $2\frac{1}{2}$ lbs. we have $2\frac{1}{2}$ inches deflection, leaving $9 - 2\frac{1}{2} = 6\frac{1}{2}$ inches to be produced by impact, the height of fall for which will be $h = \frac{6\frac{1}{2}^2 \times 9 \times (2\frac{1}{2} + 1)}{9 \times 1^2 \times 2} \doteq 73 \cdot 94$ inches, whereas when unloaded the fall with the same weight and deflection was 40·5 inches only, so that the resistance to impact has been

increased by loading the beam. The cols. 6, 8, 10, and 12 of Table 124 have been thus calculated, and they show that resistance to impact, or height of fall, is a maximum when the load plus the falling weight, or $I + w$, is $3\frac{1}{2}$, the breaking weight being 9.

(803.) Engineers have been guided by experience to fix on $\frac{1}{3}$rd of the statical breaking weight, as the safe statical load for ordinary beams, neglecting usually the weight of the beam itself, and it is remarkable that the power of a beam to resist impact is a maximum with that load. But this requires explanation, and must not be understood to mean that a beam loaded with $\frac{1}{3}$rd of the breaking load, is at the maximum of its power to resist a blow from an extra falling weight because the dead load already strains it to the safe limit assigned. Take the case of a beam whose breaking weight is 9, as in Table 124, which would have 3 assigned as the safe load, but say that when loaded with 2, the rest, or 1 *falls* on the beam, which often happens by the accidental failure of a rope, &c. Then, if we admit the weight of the beam between supports to be 1, its inertia would be $\frac{1}{2}$, hence $I + w$ becomes $(2 + \frac{1}{2}) + 1 = 3\frac{1}{2}$ when by col. 6 of Table 124 the resistance is a maximum, h being $73 \cdot 94$ inches.

Again: say that when the dead load is 1, the rest, or 2 falls, then $I + w$ becomes $(1 + \frac{1}{2}) + 2 = 3\frac{1}{2}$, when by col. 10 the height $h = 24 \cdot 61$ inches and is still a maximum.

(804.) It will also be found that a factor for safety less or greater than $\frac{1}{3}$rd (or $\frac{3}{9}$) will give a less resistance. Say we adopt $\frac{5}{9}$ instead of $\frac{3}{9}$; then if when loaded with $\frac{4}{9}$ or 4 in the Table, the rest, or 1 falls; $I + w$ becomes $(4 + \frac{1}{2}) + 1 = 5\frac{1}{2}$, and by col. 6 the height of fall is $55 \cdot 7$ inches, whereas the same weight of 1 fell $73 \cdot 94$ inches to break the beam when the factor was $\frac{1}{3}$ or $\frac{3}{9}$. Again; when the dead load is 3 and the rest, or 2 falls, we get $I + w = (3 + \frac{1}{2}) + 2 = 5\frac{1}{2}$, and by col. 10 the height of fall is $20 \cdot 8$ inches instead of $24 \cdot 61$ inches with the same falling weight of 2 when the factor was $\frac{1}{3}$rd.

If we adopt a lower factor, say $\frac{2}{9}$ instead of $\frac{3}{9}$, then, if when loaded with 1 the rest, or 1 falls, we get $(1 + \frac{1}{2}) + 1 = 2\frac{1}{2}$ as the value of $I + w$, and by col. 6, the height of fall is $70 \cdot 31$ inches instead of $73 \cdot 94$ inches as with a factor of $\frac{1}{3}$rd.

TABLE **124.**—Of the RESISTANCE of an ELASTIC BEAM to IMPACT: showing that it varies with the Dead Load on the Beam.

Central Load on the Beam, or Inertia, = I in Lbs.	Deflection produced in Inches. By the Statical Load before Impact.	By the Impact of w, f.	Total.	Falling Weight, $w = 1 \cdot 0$. $I + w$.	Calculated Fall in Inches, h.	Falling Weight, $w = 1\frac{1}{4}$. $I + w$.	Calculated Fall in Inches, h.	Falling Weight, $w = 2 \cdot 0$. $I + w$.	Calculated Fall in Inches, h.	Falling Weight, $w = 3$. $I + w$.	Calculated Fall in Inches, h.
0	0	9	9	1	40·50	1½	27·00	2	20·25	3	13·50
½	½	8½	„	1½	54·18	2	32·11	2½	22·58	3½	14·05
1	1	8	„	2	64·00	2½	35·56	3	24·00	4	*14·22
1½	1½	7½	„	2½	70·31	3	37·50	3½	*24·61	4½	14·06
2	2	7	„	3	73·50	3½	*38·11	4	24·50	5	13·61
2½	2½	6½	„	3½	*73·94	4	37·55	4½	23·76	5½	12·91
3	3	6	„	4	72·0	4½	36·00	5	22·50	6	12·0
3½	3½	5½	„	4½	68·0	5	33·61	5½	20·8	6½	10·92
4	4	5	„	5	62·5	5½	30·55	6	18·7	7	9·72
4½	4½	4½	„	5½	55·7	6	27·00	6½	16·4	7½	8·44
5	5	4	„	6	48·0	6½	23·11	7	14·0	8	7·11
6	6	3	„	7	31·5	7½	15·00	8	9·0	9	4·50
7	7	2	„	8	16·0	8½	7·55	9	4·5	10	2·22
8	8	1	„	9	4·5	9½	2·11	10	1·2	11	0·61
9	9	0	„	10	0·0	10½	0·00	11	0·0	12	0·00
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)

(805.) In Table 125 are given the results of Mr. Hodgkinson's experiments on the resistance to impact of bars of Cast iron 3 inches square, $13\frac{1}{2}$ feet long between supports. To obtain exact data for calculation, precisely similar bars of the same iron were subjected to experiments with a dead load, the bars being strained horizontally so as to eliminate the complications arising from the weight of the bar itself. The mean breaking weight was 2865 lbs., and the ultimate deflection $4 \cdot 939$ inches: to obtain the deflection with smaller weights before defect of elasticity (688) became considerable, it was observed that with 672 lbs. the deflection was $\cdot 633$ inch or $\cdot 633 \div 672 = \cdot 000942$ inch per lb. in the centre, which is equivalent to $\cdot 000942 \times \frac{5}{8} = \cdot 00059$ inch per lb. distributed all over the beam.

(806.) Taking the 3rd experiment in the Table, the deflection due to the weight of the beam is $376 \times \cdot 00059 = \cdot 222$ inch: that due to the central weight of 28 lbs. is $\cdot 000942 \times 28 = \cdot 026$ inch, the sum is $\cdot 222 + \cdot 026 = \cdot 248$ inch, col. 6, so that the flexure required from impact is $4 \cdot 939 - \cdot 248 = 4 \cdot 691$ inch, col. 7. The inertia of the bar is $376 \div 2 = 188$ lbs., which added to 28 lbs., the central load, gives $I = 188 + 28 = 216$ lbs., col. 4, and the falling weight being 303 lbs., the Rule (789) becomes $h = \dfrac{4 \cdot 691^2 \times 2865 \times (216 + 303)}{4 \cdot 939 \times 303^2 \times 2} = 36 \cdot 08$ inches, col. 11: experiment gave 42 inches, col. 9, but the last observed fall which did *not* break the bar was 39 inches. Obviously the fall which really broke the beam was somewhere between 39 and 42 inches, taking it at a mean of the two we obtain $40\frac{1}{2}$ inches, col. 10: hence we have $36 \cdot 08 \div 40 \cdot 5 = \cdot 891$, showing a difference of $1 \cdot 0 - \cdot 891 = \cdot 109$, or $- 10 \cdot 9$ per cent., col. 12. The mean difference or error of all the experiments was $- 4\frac{1}{2}$ per cent., ranging from $+ 24 \cdot 4$ to $- 30 \cdot 3$ per cent., showing a nearly equal divergence in both directions.

(807.) "*Impact out of the Centre of Beams.*"—We have so far considered only the case of a beam struck in the centre, but the same reasoning will apply to other cases; we have only to find the load out of the centre, and the deflection at the point of application, and the resistance to Impact R will be $\frac{1}{2}$ the product, or $R = W \times \delta \times \frac{1}{2}$, as before (777).

TABLE 125.—Of EXPERIMENTS on CAST-IRON BARS, 3 inches square, 13½ feet long, with a Falling Weight of 303 lbs.: showing that Resistance to Impact is increased by loading the Bar with a Dead Weight.

Dead Load on the Bar in Lbs.		Weight of Bar between Supports. Lbs.	Inertia.		Flexure.		Height of Fall in Inches.			Height of Fall: Calculated.	Error Per Cent.
Equally Dis-tributed.	In the Centre.		Of the Bar and Load. Lbs. I.	Of Bar and Load, plus the Falling Weight (I + w).	Due to Dead Load. Inches.	Required from the Impact. Inches.	Fall. The Bar not Broken.	Fall, which Broke the Bar.	Mean Height of Fall.		
0	0	376	188	491	·222	4·717	27	28½	27¾	34·52	+ 24·4
0	4	382	195	498	·229	4·710	31½	33	32¼	34·90	+ 8·2
0	28	376	216	519	·248	4·691	39	42	40½	36·08	− 10·9
166	4	387	280	583	·330	4·609	45	48	46½	39·12	− 15·8
389	4	386	391	694	·461	4·478	42	48	45	43·96	− 2·3
389	0	387	388	691	·458	4·481	42	48	45	43·83	− 2·6
391	4	379	389	692	·459	4·480	60	66	63	43·89	− 30·3
956	4	383	674	977	·795	4·145	54	60	57	53·03	− 7·0
										Mean =	− 4·5
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)

Say we take the case of a bar of wrought iron 1 inch deep, 4 inches wide, and 16 feet long between end-bearings: the load in the centre, straining the bar to the "limit of Elasticity," with $M_T = 2000$ lbs. (374) by Rule (324) becomes $W = 1^2 \times 4 \times 2000 \div 16 = 500$ lbs., with which the deflection by Rule (659) becomes $\delta = \dfrac{16^3 \times 500 \times \cdot 00001565}{1^3 \times 4} = 8$ inches, A, Fig. 190.

By the Rule (650) the *same* weight of 500 lbs. at $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$ span, or 4 feet, 2 feet, and 1 foot from one of the props, would give with $\frac{1}{2}$ L = 8 feet, and δ = 8 inches

Inches deflection.

at 4 feet from prop, $d = \dfrac{8}{8^4} \times \Big\{16 \times 4) - 4^2\Big\}^2 = 4 \cdot 5$

at 2 „ „ $d = \dfrac{8}{8^4} \times \Big\{16 \times 2) - 2^2\Big\}^2 = 1 \cdot 531$

at 1 „ „ $d = \dfrac{8}{8^4} \times \Big\{16 \times 1) - 1^2\Big\}^2 = 0 \cdot 4395$

(808.) But if the loads were all proportional to the strength at each point, we should have had at B in Fig. 190, or at 4 feet from the prop, by the Rule in (420), $500 \times (8 \times 8) \div (12 \times 4) = 666 \cdot 7$ lbs., and as 500 lbs. gave 4·5 inch, so by proportion 666·7 lbs. would give $4 \cdot 5 \times 666 \cdot 7 \div 500 = 6$ inches deflection at B. Similarly, at C, or 2 feet from a prop, we should have $500 \times (8 \times 8) \div (14 \times 2) = 1143$ lbs., and a deflection of $1 \cdot 531 \times 1143 \div 500 = 3 \cdot 5$ inches: and finally at D, or 1 foot from a prop, we obtain $500 \times (8 \times 8) \div (15 \times 1) = 2133$ lbs., giving a deflection of $\cdot 4395 \times 2133 \div 500 = 1 \cdot 875$ inch.

We can now find the resistance to Impact by a blow at A, B, C, or D in the figures, by simply multiplying the load at each point by the corresponding deflection, and taking $\frac{1}{2}$ the product, as explained in (776). At the centre A, we obtain $500 \times 8 \times \frac{1}{2} = 2000$ inch-lbs.: at B, or 4 feet from one prop, we have $666 \cdot 7 \times 6 \times \frac{1}{2} = 2000$ inch-lbs: at C, or 2 feet from a prop, $1143 \times 3 \cdot 5 \times \frac{1}{2} = 2000$ inch-lbs.: and at D, or 1 foot from a

prop, $2133 \times 1 \cdot 875 \times \frac{1}{2} = 2000$ inch-lbs., being thus precisely *the same in all the four cases.* We have here neglected the Inertia due to the weight of the beam, the effect of which is to disturb perfect equality, as shown in (811).

(809.) From this we find that while the safe load increases with the distance from the centre, the deflection with that increased load is reduced in exactly the same ratio, so that their product, or $W \times \delta$, is everywhere constant. The same fact is shown by col. 6 of Table 104.

The result is, that the power of a beam to resist Impact is the same at whatever part of the length it is struck: that is to say, a given weight which falling on the centre from a certain height, will break the beam, or strain it to a given fraction of the breaking weight, will have the same effect at any other point in the length.

This remarkable result has been confirmed by experiment, as shown by Table 126. Mr. Hodgkinson found that a bar of cast iron 1 inch square and 4 feet long, struck horizontally at the centre by a ball of $20\frac{3}{4}$ lbs., acting as a pendulum with a radius of 12 feet, required a fall through a chord of 6 feet to break it: and two similar bars struck at $\frac{1}{4}$ span, or midway between the centre and one support, broke with the same fall precisely.

(810.) Let Fig. 205 represent the case: it is an axiom that the velocity at V acquired by a body falling by gravity say from Q, is the same whether the body falls through the arc Q, V, or vertically through the height X, V, &c. If we take chords 1, 2, 3, as in the figure, the vertical heights are in the ratio 1, 4, 9 as shown, and as the velocities are proportional to the square-roots of the heights we have velocities of $\sqrt{1} = 1$, $\sqrt{4} = 2$, and $\sqrt{9} = 3$; that is to say, the velocities are simply as the chords, and the vertical heights as the square-roots of the chords: hence we have the Rule

(811.) $$h_P = C_P{}^2 \div (R_P \times 2).$$

In which R_P = the radius of the pendulum: C_P = the chord fallen through, and h_P = the corresponding vertical fall, all in the same terms (feet or inches): thus in our case $h_P = 6^2 \div (12 \times 2) = 1 \cdot 5$ foot, as in col. 6 of Table 126.

In calculating the effect of blows at the centre and elsewhere, perfect equality will be disturbed by the Inertia of the bar (781): with the 1-inch bar, the weight between supports being 11·2 lbs., the Inertia I = 11·2 ÷ 2 = 5·6 lbs., and by Rule (782) we obtain $h = \frac{1 \times 1 \times 4 \times 81 \times (5\cdot6 + 20\cdot75)}{20\cdot75^2} =$ 19·8 inches, or 1·65 feet, as in col. 8. When struck at $\frac{1}{4}$ span, I becomes 5·6 × 2 = 11·2 lbs., and

$$h = \frac{1 \times 1 \times 4 \times 81 \times (11\cdot2 + 20\cdot75)}{20\cdot75^2} = 24 \text{ inches,}$$

or 2 feet, &c.

With the 3-inch bars, the weight of the bar between supports = 186 lbs., hence when struck at the centre I = 93 lbs., and Rule (782) gives $h = \frac{3 \times 3 \times 6\cdot75 \times 81 \times (93 + 603)}{603^2} =$ 9·42 inches, or ·785 feet. When struck at $\frac{1}{4}$ span, I = 93 × 2 = 186 lbs., and $h = \frac{3 \times 3 \times 6\cdot75 \times 81 \times (186 + 603)}{603^2} =$ 10·67 inches, or ·8898 feet, &c.

TABLE 126.—Of EXPERIMENTS on the ULTIMATE RESISTANCE of CAST IRON to IMPACT, when struck at the Centre and at $\frac{1}{4}$ Span.

Length.	Depth and Breadth.	Falling Weight. Lbs. W.	Radius of Pendulum.	Chord Fallen through.	Corresponding Vertical Fall: Ft. h.	Work Done by the Ball. W × h.	Calculated Fall in Feet.	Where Struck.
feet.	inches.		feet.	feet.				
4	1	$20\frac{3}{4}$	12	6	1·5	31·13	1·65	Centre
"	"	"	"	"	"	"	2·00	$\frac{1}{4}$ Span
"	"	"	"	"	"	"	2·00	"
$6\frac{3}{4}$	3	603	$17\frac{1}{2}$	4·84	·669	402	·7849	Centre
"	"	"	"	4·96	·703	424	"	"
"	"	"	"	4·46	·568	333	"	"
"	"	"	"	4·59	·602	363	"	"
			Mean=	4·71	·634	382	"	"
"	"	"	"	5·62	·902	544	·8898	$\frac{1}{4}$ Span
"	"	"	"	5·34	·815	491	"	"
"	"	"	"	5·75	·944	570	"	"
"	"	"	Mean=	5·57	·887	535	"	"
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

(812.) "*Resistance to Impact as Deflection Squared.*"—With beams whose elasticity is perfect the statical or dead loads and the corresponding deflections are simply proportional to each other, thus, deflections 1, 2, 3, &c., require loads 1, 2, 3, &c., but the dynamic or falling weights are proportional to the deflection *squared*. Thus deflections 1, 2, 3 require with the same fall, weights in the ratio 1^2, 2^2, 3^2, or 1, 4, 9, &c., for obviously, as the *power* required to bend a beam, or R, is proportional to the deflection multiplied by the weight producing it, or to $\delta \times W$, it follows that with statical weights, 1, 2, 3 and corresponding deflections in the ratio 1, 2, 3, $\delta \times W$ becomes $1 \times 1 = 1$: $2 \times 2 = 4$: and $3 \times 3 = 9$, &c., or as the deflection squared. Conversely when the falling weight is constant, the height of fall is in the ratio of deflection squared; this is proved to be true experimentally by Table 120, deflections of 1·25 and 2·62 inches, which are in the ratio 1 to 2 nearly, required falls of 12 and 48 inches, which are in the ratio 1^2 to 2^2, or 1 to 4.

(813.) In the case (775) and Fig. 186, in which the deflection is throughout 1 inch per pound for the statical weights

1	2	3	4	5	6	7	8	9	10 lbs.

the power is respectively

$\frac{1}{2}$	2	$4\frac{1}{2}$	8	$12\frac{1}{2}$	18	$24\frac{1}{2}$	32	$40\frac{1}{2}$	50

inch-lbs., and for each successive inch

$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$	$5\frac{1}{2}$	$6\frac{1}{2}$	$7\frac{1}{2}$	$8\frac{1}{2}$	$9\frac{1}{2}$

inch-lbs. While therefore 1 lb. statically produces 1 inch deflection throughout, the first inch takes $\frac{1}{2}$ inch-lb. only dynamically, and the last, $9\frac{1}{2}$ inch-lbs. It is apparently an anomaly that the last inch deflection requires $9\frac{1}{2}$ inch-lbs., and yet that it is produced by the addition of 1 lb. falling 1 inch, and it is still more remarkable because the dynamic effect of the latter is only $\frac{1}{2}$ an inch-lb., for, as in the first inch so with the last, the *mean* weight during the 1-inch fall is $\frac{1+0}{2} = \frac{1}{2}$ inch-lb. But it should be observed that while the addition of the first pound had no other effect than to yield the $\frac{1}{2}$ inch-lb. due to it, the addition of the last pound had also the effect of causing the 9 lbs. with which

the beam was already loaded to descend 1 inch with the full uniform weight, thus yielding 9 inch-lbs., which added to the $\frac{1}{2}$ inch-lb. due to the weight itself, makes the total dynamic weight = $9\frac{1}{2}$ inch-lbs. for the last inch, as before stated.

(814.) The *power* to deflect the beam from 0 to 9 inches, as shown by (801), being $40\frac{1}{2}$ inch-lbs., and from 0 to 5 inches $12\frac{1}{2}$ inch-lbs., it follows that to increase the deflection from 5 to 9 inches will be $40\frac{1}{2} - 12\frac{1}{2} = 28$ inch-lbs., or the same result as in (813).

Table 121 gives the results of Mr. Hodgkinson's experiments on the resistance of wrought-iron bars to impact: comparing cols. 4 and 5 it will be seen that the height of fall is practically as the square of the deflection, as due by theory.

(815.) The imperfect elasticity of cast iron causes a considerable divergence from the rule, as is shown by Table 122, which gives the result of similar experiments on cast-iron bars: comparing cols. 2 and 3, or 5 and 6, it will be seen that the experimental fall in col. 2 or the power in col. 5, is in a much lower ratio than δ^2, the difference being due to defect of elasticity (688). The ordinary rule supposes that the deflections are simply proportional to the statical loads, but col. 4 of Table 108 shows that this is not even nearly true of cast iron, the deflections increasing with successive 56 lbs. from ·3754 inch with the first, to 1·157 inch with the last.

Say for illustration, that we had a material in which defect of Elasticity was such, that to produce deflections 1, 2, 3, required loads 1, $1\frac{1}{2}$, and 2, respectively, instead of 1, 2, 3, as due with perfect elasticity. Then $\delta \times W$ becomes $1 \times 1 = 1$: $2 \times 1\frac{1}{2} = 3$: and $3 \times 2 = 6$: we thus obtain the ratio 1, 3, 6, instead of 1, 4, 9, which is that of the square of the deflection.

(816.) Comparing the first experiment in Table 122 with the last, the deflections being in the ratio $1\frac{1}{2}$ to 9, or 1 to 6, the vertical fall, or the work done by the falling ball, should be as 6^2 or 36 to 1, but experiment gave $1·1944 \div ·0638 = 18·8$ to 1, which is in the ratio of the 1·64 power of the deflection, or $\delta^{1·64}$ instead of δ^2. Thus in our case the ratio of the deflections was 1 to 6, and the log. of 6 = ·788: the ratio of the work done in producing the deflections = 1 to 18·8, the log. of

which $= 1 \cdot 274$: then we obtain $1 \cdot 274 \div \cdot 778 = 1 \cdot 638$, say $1 \cdot 64 =$ the power of the deflection to which the work done is proportional. Cols. 4 and 7 were calculated by that ratio, taking the experimental numbers for $1\frac{1}{2}$ inch deflection as a basis. It should be observed that the difference between $\delta^{1 \cdot 64}$ and δ^2 is so great as to double the work with deflections in the ratio 1 to 7, for $7^{1 \cdot 64} = 24 \cdot 3$ and $7^2 = 49$ or about double.

(817.) "*Effect of Inertia not Constant.*"—The inertia of the beam and its load is most influential when the falling weight is light in proportion thereto. If the beam were unloaded, and itself without weight, it would be quite immaterial what the falling weight might be so long as the fall multiplied by that weight was constant: thus 100 lbs. falling 10 feet would produce the same effect in straining the beam as 10 lb. falling 100 feet, &c. But this is not true when the inertia of the beam and its load is considered: say that we have a beam whose inertia, or half-weight between supports = 1000 lbs., and we require the heights from which weights of 1, 10, and 100 lbs. must fall to produce one and the same deflection. Of course if the inertia was nothing, those heights would be inversely proportional to the weights simply, or 100, 10, and 1.

We require only proportional numbers, and may take from (782) $h = (\mathrm{I} + w) \div w^2$, which becomes in our cases $(1000 + 1) \div 1^2 = 1001$; $(1000 + 10) \div 10^2 = 10 \cdot 1$; and $(1000 + 100) \div 100^2 = \cdot 11$ respectively, or nearly in the ratio 100, 1, and $\frac{1}{10}$th, instead of 100, 10, and 1, as with a beam whose inertia was nothing.

The *Mechanical power* R, required to bend a beam being by (780) equal to $w \times h$, and h being proportional to $(\mathrm{I} + w) \div w^2$, we have the ratio of the power $\mathrm{R} = \left\{\mathrm{I} + w) \div w^2\right\} \times w$, or

(818.) $$\mathrm{R} = (\mathrm{I} + w) \div w,$$

which in our three cases becomes $(1000 + 1) \div 1 = 1001$; $(1000 + 10) \div 10 = 101$; and $(1000 + 100) \div 100 = 11$ respectively, which agrees with the preceding calculations of the heights of fall with weights of 1, 10, and 100 lbs. falling 1001, $10 \cdot 1$, and $0 \cdot 11$, the products of the weight by the height

becoming $1001 \times 1 = 1001$; $10 \cdot 1 \times 10 = 101$; and $0 \cdot 11 \times 100 = 11$ respectively.

(819.) The experiments of Mr. Hodgkinson in Table 127, show very clearly that the resistance of inertia is most influential with light falling weights:—for instance, for $1\frac{1}{2}$ inch deflection, with the first bar, the work done by the three falling balls of the respective weights $75\frac{1}{2}$, $151\frac{1}{4}$, and 603 lbs. is, $167 \cdot 127$; $104 \cdot 015$; and $63 \cdot 978$ foot-pounds respectively, whereas, but for the inertia, all three would have been alike. The same bar was previously deflected $1 \cdot 547$ inch by a central Statical weight of 784 lbs., hence $1\frac{1}{2}$ inch deflection would require $784 \times 1 \cdot 5 \div 1 \cdot 547 = 762$ lbs., the power R being thus $762 \times 1\frac{1}{2} \times \frac{1}{2} = 571 \cdot 5$ inch-lbs., or $571 \cdot 5 \div 12 = 47 \cdot 62$ foot-pounds. The inertia of the bar, or half-weight between supports, being 186 lbs., the rule (818) or $R = (I + w) \div w$, becomes with $w = 75\frac{1}{2}$ lbs., $R = (186 + 75\frac{1}{2}) \div 75\frac{1}{2} = 3 \cdot 464$ times the power with no inertia, therefore in our case $47 \cdot 62 \times 3 \cdot 464 = 165$ foot-lbs.: experiment gave $167 \cdot 127$. With $w = 151\frac{1}{4}$ lbs., we have $R = (186 + 151 \cdot 25) \times 47 \cdot 62 \div 151 \cdot 25 = 106 \cdot 2$ foot-lbs.: by experiment $104 \cdot 15$. With $w = 603$ lbs., $R = (186 + 603) \times 47 \cdot 62 \div 603 = 62 \cdot 31$ foot-lbs., and by experiment $63 \cdot 978$, &c.

These calculations prove that the great differences shown by the experiments are confirmed by theory; and also that the resistance of beams to impact may be calculated accurately from the deflections with statical weights.

(820.) If we had not known by direct experiment the stiffness of the particular bar, we might have obtained nearly the same results from general rules. Thus, the statical weight to produce $1\frac{1}{2}$ inch deflection in our wrought-iron bar with $L = 13 \cdot 5$, $d = 1 \cdot 515$, $b = 5 \cdot 523$ by the rule in (662), namely $W = (d^3 \times b \times \delta) \div (L^3 \times C)$, in our case becomes $(1 \cdot 515^3 \times 5 \cdot 523 \times 1 \cdot 5) \div (13 \cdot 5^3 \times \cdot 00001583) = 739 \cdot 7$ lbs.: hence R or $W \times \delta \times \frac{1}{2}$ becomes $739 \cdot 7 \times 1\frac{1}{2} \times \frac{1}{2} \div 12 = 46 \cdot 23$ foot-lbs. Then with $w = 75\frac{1}{2}$ lbs., we get $R = (186 + 75 \cdot 5) \times 46 \cdot 23 \div 75 \cdot 5 = 160 \cdot 1$ foot-lbs.: with $w = 151\frac{1}{4}$, $R = (186 + 151 \cdot 25) \times 46 \cdot 23 \div 151 \cdot 25 = 103 \cdot 1$ foot-lbs.:—and with 603 lbs. $R = (186 + 603) \times 46 \cdot 23 \div 603 = 60 \cdot 49$ foot-lbs., &c., being nearly the same result as we found before with data derived by

direct experiment with a statical weight on the particular bar. The value of C was taken from a special experiment.

TABLE 127.—Of EXPERIMENTS on RESISTANCE of WROUGHT-IRON BARS to IMPACT, showing that the Inertia of the Bar is most influential with Light Falling Weights.

Deflection.	Weight of Falling Ball: Lbs.			Length.	Depth.	Breadth.	Weight between Supports.
	75½	151¼	603				
	Work done by Ball in Deflecting the Bar, or $w \times h$.						
inches.				feet.	inches.	inches.	lbs.
½	18·807	12·992	7·417	13½	1·515	5·523	372
1	74·201	49·519	29·688	„	„	„	„
1½	167·127	104·015	63·978	„	„	„	„
2	..	187·28	113·123	„	„	„	„
2½	..	294·75	177·28	„	„	„	„
3	..	430·32	254·59	„	„	„	„
1	13·764	12·221	..	13½	1·027	5·51	262
2	53·477	47·719	..	„	„	„	„
3	115·017	104·015	..	„	„	„	„
4	204·181	190·242	..	„	„	„	„
½	26·274	20·267	15·376	6¾	„	„	„
¾	59·683	45·677	35·095	„	„	„	„
1	106·485	80·314	61·747	„	„	„	„
1¼	166·817	126·747	94·792	„	„	„	„
1½	238·051	180·895	135·253	„	„	„	„
1¾	327·572	242·454	..	„	„	„	„
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

(821.) "*Resistance to Impact as the Weight of the Beam simply.*"—The Rule (780) shows that $w \times h$, or the power of a beam in resisting Impact is proportional to $d \times b \times L$: now obviously those three dimensions multiplied into each other give the cubic capacity of the beam, and thereby the *weight* between supports, so that substituting W_B, the weight of the beam in lbs., for $d \times b \times L$, we have the rule $w \times h = W_B \times M_B$. In which M_B is a Multiplier adapted to the case. But $R = w \times h$, from which we find that with any given material, the resistance of rectangular beams to Impact, R, will be simply proportional

to the weight of the beam between supports, and may be found without knowing any of the dimensions. Thus in Table 128, we have in col. 4 the weight of bars of very different sizes, varying from 384 to 15 lbs. Now, taking the value of M_B approximately = 2, the rule $w \times h = W_B \times M_B$ gives the results in col. 8, agreeing remarkably with experiment, col. 7, although two important matters were neglected, namely, the resistance of Inertia, and the variation in strength with different sizes of cast iron (931).

TABLE 128.—Of the ULTIMATE RESISTANCE of CAST-IRON BARS to IMPACT, showing that it is as the Weight of the Bar simply.

No. of Experiment.	Length	Depth.	Breadth.	Weight of Bar between Supports.	Weight of Ball in Lbs., w.	Vertical Descent of Ball in Feet, h.	Work done by the Falling Ball to Break the Bars $w \times h$. By Experiment.	Col. 4 $\times$ 2.
	feet.	inches.	inches.	lbs.				
1	$13\frac{1}{2}$	3	3	378	603	1·238	746	756
2	"	$1\frac{1}{2}$	6	381	"	1·207	728	762
3	"	6	$1\frac{1}{2}$	384	"	1·270	766	768
4	$6\frac{3}{4}$	3	3	192	"	0·639	385	384
5	"	$1\frac{1}{2}$	6	186	"	0·5521	333	372
6	9	2	2	111	$151\frac{1}{4}$	1·964	297	222
7	"	"	"	108	603	·3159	190	216
8	"	"	"	108	$151\frac{1}{4}$	1·2856	194	216
9	"	"	"	106	$75\frac{1}{2}$	3 0506	230	212
10	"	"	"	106	$151\frac{1}{4}$	1·4130	214	212
11	"	1	2	57	$75\frac{1}{2}$	1·4334	108	114
12	$4\frac{1}{2}$	2	2	54	$75\frac{1}{2}$	1·925	145	108
13	"	"	"	54	603	0·1704	103	108
14	"	1	1	15	$75\frac{1}{2}$	0 5469	41·3	30
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

(822.) We have now to find the correct value of M_B. With a falling weight the power exerted $= w \times h$, or the falling weight w multiplied by the height of fall. The resistance of the beam to this force, as we have seen (777), is $\delta \times W \times \frac{1}{2}$, which is also the value of R:—therefore $w \times h : \delta \times W \times \frac{1}{2}$, and R, have all one and the same value.

By col. 5 of Table 67, the value of R for a Standard bar of cast iron 1 inch square and 1 foot long = 81 inch-lbs.: now,

the weight of this bar between supports by Table 151 is $\cdot 2556 \times 12 = 3$ lbs.: hence the value of R in inch-lbs. will be given by the Rule $R = W_B \times 27$, or in our case $R = 3 \times 27 = 81$ inch-lbs.: for foot-lbs. the Rule becomes $R = 3 \times 2\cdot 25 = 6\cdot 75$ foot-lbs.

Applying the two corrections for Inertia and thickness of casting, we obtain the Rule:—

$$(823.) \qquad R = \frac{W_B \times z \times M_B \times (I + w)}{w}.$$

In which W_B = the weight of the beam between supports, in lbs.

w = the falling weight, in lbs.

z = the Multiplier for thickness of casting, as in (931).

M_B = 27 for R in inch-lbs. with cast-iron bars; and 2·25 for foot-lbs.

I = the Inertia of the bar, or $\frac{1}{2}$ weight between supports, in lbs.

R = Ultimate resilience, in inch-lbs., or foot-lbs., depending on M_B.

Taking from (934) the experimental value of M_0 for 3-inch, 2-inch, and 1-inch bars at ·6195, ·7184, and 1·0 respectively: for experiment No. 1 in Table 128, we get for 3-inch bars $R = 378 \times 2\cdot 25 \times \cdot 6195 \times (189 + 603) \div 603 = 692$ foot-lbs.: experiment gave 746. The 2-inch bars, No. 7, give $R = 108 \times 2\cdot 25 \times \cdot 7184 \times (54 + 603) \div 603 = 190\cdot 2$ lbs.: experiment gave 190 lbs. The 1-inch bars, No. 4, give $R = 15 \times 2\cdot 25 \times 1 \times (7\cdot 5 + 75\cdot 5) \div 75\cdot 5 = 37\cdot 1$ foot-lbs.: experiment gave 41·3 foot-lbs., &c.

The effect of size on the Resilience may be shown clearly by the *reduced* experiments on 1, 2, and 3-inch bars in Table 118: thus by cols. 5 and 6, the ultimate value of R for 1-inch bars by Rule (777) is $R = \cdot 0879 \times 1980 \times \frac{1}{2} = 97$: for the reduced 2-inch, $R = \cdot 07048 \times 1372 \times \frac{1}{2} = 48\cdot 35$: for the reduced 3-inch, $R = \cdot 07617 \times 1342 \times \frac{1}{2} = 51$, which is anomalous, being greater than the 2-inch (932).

Rule (823) must not be applied to beams of all sections: for instance, the ⊥ girders in (342) and Figs. 71, 72, are similar in section and therefore in weight. But when broken with the flange uppermost $W = 1120$ lbs., and $\delta = 5$ inches, hence $R = 5 \times 1120 \times \frac{1}{2} = 2800$ inch-lbs.; with flange downwards $W = 364$, $\delta = 1 \cdot 138$, and $R = 1 \cdot 138 \times 364 \times \frac{1}{2} = 207$ inch-lbs. Hence, so far from being alike, or simply as the weight, we have the ratio $13\frac{1}{2}$ to $1 \cdot 0$.

(824.) "*Effect of Depth and Breadth.*"—With a rectangular beam of unequal dimensions, say 6 inch by 1 inch, the resistance to Impact is the same, whether the bar be struck on its broad side or on its edge. This fact is implied and indeed involved in the Rules (780), &c., for as the height h which a given weight must fall is simply proportional to $d \times b$, it is obviously immaterial which dimension is made the depth and which the breadth. To show that this is theoretically correct more clearly, we may take a bar of cast iron 1×6 inches and say 10 feet long. Taking M_T for Breaking weight at $\cdot 921$ from col. 6 of Table 66, when laid flat $d = 1$, and $b = 6$, then Rule (324) gives $W = 1^2 \times 6 \times \cdot 921 \div 10 = \cdot 5526$ ton Breaking weight, with which the deflection by Rule (695) becomes $\delta = 10^2 \times \cdot 0785 \div 1 = 7 \cdot 85$ inches: hence R, or the mechanical power producing that deflection and breaking the beam, is $\cdot 5526 \times 7 \cdot 85 \times \frac{1}{2} = 2 \cdot 169$ inch-tons (776).

When fixed on edge, $d = 6$, and $b = 1$: then $W = 6^2 \times 1 \times \cdot 921 \div 10 = 3 \cdot 316$ tons, Breaking weight, with which the deflection becomes $\delta = 10^2 \times \cdot 0785 \div 6 = 1 \cdot 308$ inches; hence the mechanical power $R = 3 \cdot 316 \times 1 \cdot 308 \times \frac{1}{2} = 2 \cdot 169$ inch-tons, being precisely the same as in the other position.

Mr. Hodgkinson's experiments in Table 129 prove the correctness of this reasoning: thus, in Nos. 1, 2, 3, $d \times b = 9$ in all cases, and col. 7 shows that the power required to break the bars, or $w \times h$, was practically the same in all: the other experiments in the Table show similar results.

(825.) "*High Ratio of Safe and Breaking Dynamic Loads.*"—The resistance to impact being proportional to the statical load on a beam multiplied by the deflection produced by it, the

TABLE 129.—PROVING that RECTANGULAR BEAMS RESIST IMPACT with EQUAL ENERGY when STRUCK on their NARROW or BROAD DIMENSIONS.

Length.	Depth, d.	Breadth, b.	Area, or $d \times b$.	Weight of Ball, w.	Vertical Descent of Ball, h.	Work done by the Ball to Break the Beam, $w \times h$.
feet.				lbs.	feet.	
$13\frac{1}{2}$	3	3	9	603	1·238	746·51
$13\frac{1}{2}$	$1\frac{1}{2}$	6	9	603	1·2071	727·88
$13\frac{1}{2}$	6	$1\frac{1}{2}$	9	603	1·270	765·81
$6\frac{3}{4}$	3	3	9	603	·639	385·32
$6\frac{3}{4}$	$1\frac{1}{2}$	6	9	603	·5521	332·916
$4\frac{1}{2}$	1	2	2	$75\frac{1}{2}$	·971	73·31
$4\frac{1}{2}$	2	1	2	$75\frac{1}{2}$	1·02	77·01
(1)	(2)	(3)	(4)	(5)	(6)	(7)

result is that necessarily the ratio between the Dynamic Safe and Breaking weights is very much higher than between the like Statical weights. For instance, if the elasticity of a beam is perfect (688), the deflection being in that case exactly proportional to the weights, and say that the ratio of the Statical safe and breaking loads is 1 to 3, then as we have with the latter three times the weight and three times the deflection, we get $3 \times 3 = 9$ times the resistance to impact; in fact, the Dynamic ratio is the *square* of the Statical ratio, and the latter being say 3, 4, 5, &c., the former will be 9, 16, 25, &c.

(826.) When the elasticity is imperfect, and the deflections increase more rapidly than the weights, the ratio is higher still: for instance, for cast iron Table 67 shows that with a statical ratio of 3 to 1, the deflections by cols. 8 and 11 are $\cdot 0785 \div \cdot 01971 = 3 \cdot 983$, or nearly 4 to 1, and the resistance to impact $3 \cdot 983 \times 3 = 12$ to 1 nearly. Table 67 also shows by col. 9 that Timber, with a Statical Ratio of 5 to 1, has a Dynamic ratio varying from 53·0 to 1 in Ash, to 27·1 to 1 in Elm.

(827.) This high ratio, which we have seen to be a result of the nature of the case, is a great practical advantage, because in most cases the dynamic strain is uncertain as to its amount, being frequently the result of accident, such as the failure of a

rope, &c. Thus, if we arrange the proportions of a cast-iron beam to bear safely a falling load that will strain it to the limit of safety, say $\frac{1}{3}$rd of the Statical breaking weight; it will be strained to only $\frac{1}{12}$th of the Dynamic breaking weight; leaving thus a very large margin for safety: see (786).

(828.) "*Stiffness a Source of Weakness.*"—In many cases stiffness or rigidity is necessary; for instance, in girders carrying a water-tank where undue flexure would throw a strain on the joints, &c., and would be likely to cause leakage if not rupture: but when an impulsive strain has to be borne, the most flexible beam is the strongest, other things being equal. For instance, if we had two beams whose breaking weights were the same, but one deflecting twice as much as the other, then the latter would bear twice the strain dynamically, or a given weight falling double the height, &c.

(829.) It is shown in (778) that R, or the resistance to Impact, is *directly* as the length of the Beam, other things being equal, which of course is directly contrary to the case of a dead load, where the strength is *inversely* as the length. This remarkable result is shown to be experimentally true by Table 130, where bars similar in depth and breadth, but differing in length, are arranged in groups. The Ratios of the lengths are 2 to 1 in all cases, and col. 7 shows that R, or the mean resistance to Impact, is 1·99 to 1.

(830.) "*Summary of Remarkable Laws.*"—There are several remarkable laws of Impact which it may be interesting and instructive to collect from the foregoing investigation: excluding the effect of Inertia for the moment we find:—

1st. The resistance to Impact, or the Resilience of a Beam, R, is the same whether the blow is struck at the centre or elsewhere in the length: see (809).

2nd. In rectangular beams of unequal dimensions the resistance R is the same whether the bar is struck on its narrow, or broad dimension: see (824).

3rd. With rectangular beams of the same material, the resistance to Impact R is simply proportional to the weight of the beam between supports irrespective of the particular dimensions: (821).

TABLE 130.—PROVING that RESISTANCE to IMPACT is *directly* as the LENGTH of the BEAM.

Length.	Depth.	Breadth.	Weight of Ball.	Vertical Descent of Ball to Break the Beam.	Work done by the Ball. Foot-lbs. $w \times h$.	Ratio.
ft. in.	inches.	inches.	lbs.	feet.		
6 9	3	3	603	0·639	385	1·938
13 6	3	3	"	1·238	746	
6 9	$1\frac{1}{2}$	6	"	0·5521	333	2·186
13 6	$1\frac{1}{2}$	6	"	1·2071	728	
4 6	2	2	"	0·1704	103	1·845
9 0	2	2	"	0·3159	190	
					Mean =	1·99
(1)	(2)	(3)	(4)	(5)	(6)	(7)

Combining 1, 2, 3, we are conducted to this remarkable fact, that the weight of the rectangular beam being the same, it is immaterial whether it be long or short, broad or narrow, deep or shallow. Moreover, whether the beam is struck on edge or on the flat;—in the centre or anywhere out of the centre, the result is the same in all cases.

4th. The power R is *directly* as the length, instead of *inversely* as for a dead load: see (829).

5th. The power R of a beam in resisting Impact is *increased* by loading it with a dead load up to a certain point: (801). Thus by col. 9 of Table 125 it is more than doubled.

6th. With $\frac{1}{3}$rd of the Breaking weight, which is the ratio adopted by most Engineers for the Working dead load, the resistance to Impact is a maximum: (803).

7th. The power required to produce given deflections in any beam by an impulsive strain is proportional to the deflection *squared*, not as the deflection simply, as with dead loads: (812).

8th. The stiffest beams are the weakest, and *vice versâ*, other things being equal (828).

9th. The Ratio between the Breaking and Safe strains by Impact, or between R and *r*, is exceedingly high, being as the *square* of the Ratios with dead loads, as shown by (825).

IMPACT FROM ROLLING LOAD.

(831.) "*Rolling Load at High Velocity.*"—When the load on a horizontal beam rolls over it at a high velocity, the strain becomes more or less a dynamic one, but under certain limitations as governed by the speed of the transit. Let A in Fig. 193 be an unloaded beam, W, a weight, which as a dead or statical load deflects the beam to B. But by (775), and Table 119, it is shown that if that same weight were laid quietly on the centre of the beam A and suddenly released, it would deflect it to C, producing double the deflection and thereby double the strain, the weight really *falling* as the beam deflects, and acting therefore as an impulsive load.

Now let the weight W, roll horizontally upon the beam with a high velocity, such, that in travelling half the length of the beam, or from w to W, it would, if free, follow the line d, e, f, and fall by gravity from W to W_1, and deflect the beam as before, or as when it fell vertically the same height. The horizontal velocity necessary to effect this, is easily calculated:—for instance, let the beam be 10 feet long between bearings, and the dynamic deflection A C, = 4 inches. Then by the laws of falling bodies:—

(832.) $$t = \sqrt{(h \div 193)},$$

in which $h =$ the height fallen in inches, and $t =$ time in seconds, we obtain in our case $(4 \div 193)\sqrt{} = \cdot 144$ second, in which time the weight must travel half the length of the beam, or 5 feet, hence its horizontal velocity must be $5 \div \cdot 144 = 34$ feet per second, or $34 \times 3600 \div 5280 = 23 \cdot 18$ miles per hour. In this case then, a load passing over this beam at a velocity of 23·18 miles per hour, will deflect that beam, and thereby strain it to the same extent as a double load acting as a dead weight; or, in other words, the strain with any load is doubled on this particular beam by a horizontal velocity of 23·18 miles per hour. The deflection is a maximum with this velocity, that is to say, with a higher or a lower velocity the deflection would be less. With a higher velocity the weight W would not have time to fall the height A, C, or 4 inches:—for instance, with

double velocity, or $23 \cdot 18 \times 2 = 46 \cdot 36$ miles per hour, it would reach the centre of the beam in $\cdot 144 \div 2 = \cdot 072$ second, the fall due to which time by the laws of gravity or:—

(833.) $$h = {}^2 \times 193,$$

becomes in our case $\cdot 072^2 \times 193 = 1$ inch only, instead of 4 inches, so that with a velocity of 46·36 miles per hour the deflection and therefore the strain would have been $\frac{1}{4}$th only of that due to 23·18 miles per hour, and half only of that with the same weight acting statically or as a dead load in the centre. So that in this case, while a velocity of 23·18 miles per hour *doubles* the effect of a given statical load, a double velocity or 45·36 miles reduces the effect of a given dead load to *half*. Again, with velocities lower than 23·18 miles per hour, the deflections would be less than with that velocity, rising with the velocity from A, B, or that with a dead load, to A, C with 23·18 miles per hour.

(834.) These theoretical results are confirmed by the experiments of Captain James, R.E., who found with bars of steel $2\frac{1}{4}$ feet long, 2 inches broad, and $\frac{1}{4}$ inch deep, that with velocities of

0	15	24	29	34	44

feet per second, the central deflections were

0·70	1·02	1·32	1·45	1·30	1·03

inches, attaining a maximum with the velocity of 29 feet per second, which was reduced by an increase in velocity.

Similar results were obtained by wrought-iron bars 9 feet long, 3 inches deep, and 1 inch wide:—with a load of 1778 lbs. travelling at velocities of—

0	15	29	36	43

feet per second, the central deflections were

·29	·38	·50	·62	·46

inches respectively, attaining a maximum with a velocity of 36 feet per second.

(835.) Table 131 gives the results of experiments by Captain James, on Cast-iron bars of various sizes, all 9 feet long, the effect of velocity being here shown not as affecting the deflection, but as governing the breaking weight. In these experiments, the breaking statical weight was first found for each size of bar and the corresponding ultimate deflection; then lighter loads were caused to pass over similar bars at a certain fixed velocity, the load being increased continuously by increments of 56 lbs. until the bar broke, &c.

Theoretically, as we have seen (775), the rolling load with which the beam breaks should be *half* the equivalent dead weight, the maximum effect being attained by a certain velocity such that the rolling body can fall by gravity the height due to the ultimate deflection in the same time as it takes to traverse the half-length of the beam.

TABLE 131.—Of the STRENGTH of BEAMS of CAST IRON, 9 feet long, to bear Loads rolling over them at different Velocities.

Velocity in Feet per Second.	$d = 1\frac{1}{4}$ inch, $b = 4$ inches.		$d = 2$ inches, $b = 1$ inch.		$d = 3$ inches, $b = 1$ inch.	
	Breaking Weight.	Ratios.	Breaking Weight.	Ratios.	Breaking Weight.	Ratios.
	lbs.		lbs.		lbs.	
0	2075	1·000	1140	1·0000	2167	1·000
15	1649	·795	921	·8075	1700	·803
24	..	..	761	·6677	..	..
29	1335	·643	608	·5331	1522	·718
33	..	..	606	·5318	..	..
36	1059	·510	588	·5155	1203	·568
43	854	·431	..	..	1091	·515
(1)	(2)	(3)	(4)	(5)	(6)	(7)

(836.) We can calculate with approximate accuracy the velocities with which the different bars should break:—thus, the *observed* ultimate deflection of the 3 × 1-inch bars was 2·25 inches; the load would fall that height by gravity by the Rule (832) in $(2 \cdot 25 \div 193)\sqrt{\ } = 0 \cdot 107$ second, and as the rolling load has to travel half the length of a 9-foot bar, or $4\frac{1}{2}$ feet in that time, its horizontal velocity must be $4 \cdot 5 \div$

$0 \cdot 107 = 42$ feet per second. By experiment, the bar broke with a velocity of 43 feet per second with $\cdot 515$, or nearly half the dead breaking load, as in col. 7 of Table 131.

Again, with the bar 2×1 inch, the observed ultimate deflection was $3 \cdot 2$ inches: the load would fall that height in $(3 \cdot 2 \div 193) \surd = 0 \cdot 1288$ second, hence the horizontal velocity would be $4 \cdot 5 \div 0 \cdot 1288 = 35$ feet per second: the experimental velocity was 36 feet, with $\cdot 5155$, or nearly half the dead breaking load, as in col. 5.

Again: with the bars $4 \times 1\frac{1}{2}$ inch, the observed ultimate deflection was $4 \cdot 45$ inches; the load would fall that height in $(4 \cdot 45 \div 193) \surd = 0 \cdot 1516$ second, hence the horizontal velocity will be $4 \cdot 5 \div 0 \cdot 1516 = 30$ feet per second: the experimental velocity, however, was 43 feet per second, with $\cdot 431$ the dead breaking load, as in col. 3, &c.

(837.) "*Effect of the Inertia of the Beam.*"—We have so far regarded the beam as having no weight, therefore yielding no resistance to impact by its vis-inertia, and for small bars, such as we have considered, the effect of inertia on the result would be so small that it might be neglected without serious error. For instance, the bar 2 inches deep, 1 inch wide, and 9 feet long, should break by the ordinary rules (324) with $2^2 \times 1 \times 2063 \div 9 = 917$ lbs. in the centre, with an ultimate deflection of $\cdot 0785 \times 9^2 \div 2 = 3 \cdot 18$ inches (695). Now, that deflection being produced by a statical or dead load of 917 lbs., would equally be produced by a dynamic or falling load of half that amount, or 458 lbs., as shown in (775), neglecting for the moment the inertia of the beam. By (781) it is shown that in resisting impact, the power of a heavy beam is to that of a light one, as the inertia of the beam plus the falling weight, is to the falling weight alone, or $\frac{I + w}{w}$. The inertia of a beam being taken as equal to half its weight between bearings, and the total weight in our case being 56 lbs., the inertia would be 28 lbs.; hence the effect of the inertia would be to increase the falling weight from 458 lbs., to $458 \times \frac{458 + 28}{458} = 486$ lbs., an

increase of 28 lbs. only, or 5 per cent., which is so small that it may be neglected with impunity.

(838.) But where the weight of the beam is great in proportion to the falling load, the case is very different. Say we have a beam weighing 2 tons between bearings, its inertia being therefore 1 ton, and that a rolling load was calculated to produce a deflection of 2 inches when the inertia was neglected, then the effect of the inertia would be to reduce the deflection to $2 \times \frac{1}{1+1} = 1$ inch, or to half. In that case the rolling load might be increased to 2 tons, and the dynamic deflection would then be 2 inches, or the same as that due to a statical or dead load of 2 tons, the inertia doubling the power of this particular beam in resisting a falling or rolling load.

(839.) In most practical cases of Railway bridges the highest attainable velocity is very much below that which we have shown (832) to be necessary, in order to obtain the maximum dynamic deflection. This fact, together with the resistance from the inertia of the bridge itself, causes the deflection from a rolling load to be in most cases very slightly in excess of the statical deflection from a dead load. Thus, in the case of the Ewell bridge experimented upon by H.M. Commissioners, the length between bearings was 48 feet, the weight of the bridge 30 tons, hence its inertia = 15 tons, and the central *statical* deflection produced by an Engine and Tender weighing 39 tons, was ·215 inch. With velocities of

0	25	30·9	32·3	53·7	75

feet per second, or,

0	17	21	22	36·6	51

miles per hour, the central deflections were

·215	·215	·230	·225	·245	·235

inches respectively, which are irregular, and increase very slightly with increase of velocity; this is what might have been expected, as we shall see.

(840.) We have shown (831) that at a certain horizontal

velocity the dynamic deflection would be double the statical or $\cdot 215 \times 2 = \cdot 43$ inch in our case: but to produce that deflection, the load must traverse the half-length of the beam in the time necessary for a body to fall that height by gravity, which in our case by Rule (832) would be $(\cdot 43 \div 193)\sqrt{\ } = \cdot 047$ second. The horizontal velocity must therefore be $24 \div \cdot 047 = 511$ feet per second, or 348 miles per hour! whereas the highest velocity attained was 51 miles per hour only, or about $\frac{1}{7}$th of that necessary to produce the maximum deflection:—it would therefore (833) have little more effect than a dead load, which was the fact, as shown by the experiments. Moreover, even with 348 miles per hour, if that had been attainable, we should not have had the full dynamic deflection, because the inertia of the bridge would reduce it from $\cdot 43$ to $\cdot 43 \times \frac{39}{15 + 39} = \cdot 311$ inch.

CHAPTER XXI.

COLLAPSE OF TUBES.

(841.) "*Experimental Results.*"—The laws governing the strength of cylindrical tubes in resisting collapse under external pressure are so obscure that it seems hopeless to attempt to discover them by any theoretical investigation, and we are compelled to obtain Rules from experiment. Nothing was experimentally known until Mr. Fairbairn investigated the matter, and Engineers had to work in the dark. The laws obtained by Mr. Fairbairn are very remarkable, and differ entirely from the theoretical ones; for example, with a perfectly cylindrical tube the strain generated by external pressure would be simply a crushing one, and the strength in that case should be directly proportional to the thickness, and inversely as the diameter,—the length having no effect on the result. But Mr. Fairbairn found with tubes made of thin wrought-iron plates, that the strength was directly proportional to the $2 \cdot 19$ power of the thickness, and inversely as the *length* as well as the diameter.

(842.) Table 132 gives the general results of Mr. Fairbairn's experiments; they were for the most part on tubes without cross-joints, and with one longitudinal joint only, a fact which it is necessary to observe, as the strength is affected by it considerably (848). Direct evidence of the effect of *length* is given by many of these experiments; for example, Nos. 4, 5, and 9 were all of the same dimensions except the lengths, which were in the ratio 1, 2, 3; col. 4 shows that the strengths were almost precisely in inverse ratio, or 3, 2, 1, being in fact 140, 93, and 47 lbs. respectively.

(843.) In five cases the ends of the tubes were free, in all the rest they were fixed as with an ordinary boiler flue, but either way the result was about the same. We should expect the pressure to be less with free ends than with fixed ones, but this result was realised in two cases only, Nos. 1 and 16, the difference by col. 6, being 11·6 and 14 per cent. only. In the other three cases the free-ended tubes were *stronger* than the average, Nos. 5, 9, and 19 giving 11·8, 12·7, and 4·5 per cent. respectively: it would appear from this, that fixing the ends of a tube has no effect on the strength; which again is an unexpected result. No. 28 was not collapsed by 450 lbs. per square inch, nor was it likely to fail with that pressure, the calculated collapsing strain being 1396 lbs. by col. 5.

(844.) The tube No. 33 was of Steel, with which we should have expected greater strength than with iron, but a comparison of cols. 4 and 5 shows that the calculated pressure by the Rules for *iron* tubes = 298 lbs., but experiment gave with steel 220 lbs. only, showing that for some unknown reason, the strength of the steel tube was 26 per cent. less than that of an iron one, which is an unsatisfactory result requiring further experimental investigation: a similar anomaly, however, was observed with steel chain (102).

From these experiments Mr. Fairbairn obtains for cylindrical wrought-iron tubes the Rules:—

(845.) $$P = 33 \cdot 6 \times (100\,t)^{2 \cdot 19} \div (L \times d).$$

(846.) $$p = 5 \cdot 6 \times (100\,t)^{2 \cdot 19} \div (L \times d).$$

TABLE 132.—Of EXPERIMENTS on the COLLAPSING STRENGTH of WROUGHT-IRON TUBES.

No.	Thickness.	Diameter.	Length.	Collapsing Pressure. Lbs. per Square Inch.			Remarks.
				By Experiment.	By Calculation.	Error per Cent.	
	inches.	inches.	inches.				
1	·043	4	15	147	164	+ 11·6	Ends free.
2	"	"	19	170	130	− 23·6	
3	"	"	19	137	130	− 5·0	
4	"	"	20	140	124	− 11·4	{60 inches long; 2 rings.
5	"	"	30	93	82	− 11·8	Ends free.
6	"	"	38	65	65	0·0	
7	"	"	40	65	62	− 4·6	
8	"	"	60	43	41	− 4·6	
9	"	"	60	47	41	− 12·7	Ends free.
10	"	6	29	47	56	+ 19·0	
11	"	"	30	48	54	+ 12·0	
12	"	"	30	52	54	+ 4·0	
13	"	"	30	65	54	− 17·0	
14	"	"	59	32	28	− 12·5	
15	"	8	30	39	41	+ 5·0	
16	"	"	30	36	41	+ 14·0	Ends free.
17	"	"	39	32	32	0·0	
18	"	"	40	31	31	0·0	
19	"	"	60	22	21	− 4·5	Ends free.
20	"	10	30	33	33	0·0	
21	"	"	50	19	19	0·0	
22	"	12	30	22	27	+ 23·0	
23	"	"	58½	11	14	+ 27·0	
24	"	"	60	12·5	13·6	+ 9·0	
25	$\frac{1}{8}$	15	21½	150	323	+115·0	{59¾ inches long; 2 rings.
26	·14	9	37	262	392	+ 50·0	Lap-joint.
27	·14	"	37	378	392	+ 3·7	Butt-joints.
28	¼	"	37	450	1396	..	Not collapsed.
29	¼	18¾	61	420	406	− 3·3	
30	⅜	42	300	127	131	+ 3·1	Ordinary joints.
31	⅜	"	420	97	94	− 3·1	" "
32	$\frac{1}{8}$	14$\frac{11}{16}$ × 14$\frac{1}{8}$	60	125	114	− 8·8	
33	$\frac{1}{8}$	15⅝ × 15$\frac{5}{16}$	21¼	220	298	+ 35·6	{Steel, 59½ inches long; 2 rings.
34	·043	14 × 10¾	60	6·5	5·84	− 10·14	
35	¼	20¾ × 15½	61	127·5	127·7	0·0	
	(1)	(2)	(3)	(4)	(5)	(6)	

In which

P = the collapsing pressure, in pounds per square inch.
p = the safe working " " "
d = diameter of tube, in inches.
t = thickness of plate, in inches.
L = length of tube, in feet.

The col. 5 in Table 132, from No. 1 to No. 29 inclusive, has been calculated by this rule.

(847.) To find $(100\,t)^{2\cdot19}$ we must use logarithms: thus, for $\frac{1}{8}$ inch we have $100 \times \cdot125 = 12\cdot5$, the logarithm of which, or $1\cdot09691 \times 2\cdot19 = 2\cdot4022$, the natural number due to which, or $252\cdot5$, is the $2\cdot19$ power required: col. 5 of Table 133 has been calculated in this way. Thus to calculate No. 29 in Table 132, we take $(100 \times \cdot25)^{2\cdot19} = 1152$ from col. 5 of Table 133, then the rule (845) becomes $P = 33\cdot6 \times 1152 \div (5\cdot083 \times 18\frac{3}{4}) =$ 406 lbs.; experiment gave 420 lbs.: hence $406 \div 420 = \cdot967$, and $1\cdot0 - \cdot967 = \cdot033$, or an error of $-\,3\cdot3$ per cent., as in col. 6.

(848.) "*Tubes with Ordinary Riveted Joints.*"—It should be observed that the small experimental tubes from which the rule (845) was derived were virtually *jointless*, for although they had for the most part one longitudinal seam or joint, the collapsing strength would not be affected thereby. When a cylindrical tube, Fig. 191, is compressed into an ellipsis the parts from m to n and from o to p are flattened, while those from m to o and n to p are more curved, but at some point between E, H, &c., the curvature and the position or distance from the centre are unchanged. If, therefore, the longitudinal joint took up its position at one of those points, which it would be sure to do in consequence of its superior strength over other parts of the tube, it would be subjected to no special strain, and would add nothing to the normal strength of a jointless tube.

(849.) With ordinary boiler flues there are usually numerous lap-joints, both cross-ways and longitudinally: at every cross-joint there is of course a double thickness of metal, which will act partially as a ring (862), not so perfect in its effect as a strong ring of ⊥ or L iron, but still very influential on the

TABLE 133.—Of the STRENGTH of BOILER TUBES to RESIST EXTERNAL OR COLLAPSING PRESSURE.

Thickness of Plate.	With Ordinary Joints.		Without Joints.		$(100\,t)^{2\cdot19}$
	$P \times d \times L$, Collapsing Strain.	$p \times d \times L$, Working Strain.	$P \times d \times L$, Collapsing Strain.	$p \times d \times L$, Working Strain.	
$\frac{1}{16}$	2,729	455	1,860	310	55·3
$\frac{1}{8}$	12,450	2,075	8,484	1,414	252·5
$\frac{3}{16}$	30,250	5,042	20,620	3,440	613·6
$\frac{1}{4}$	56,810	9,468	38,720	6,450	1152
$\frac{5}{16}$	92,630	15,440	63,130	10,520	1878
$\frac{3}{8}$	138,100	23,017	94,110	15,680	2800
$\frac{7}{16}$	193,500	32,250	131,900	22,000	3924
$\frac{1}{2}$	259,300	43,218	176,700	29,450	5257
$\frac{9}{16}$	335,500	55,917	228,700	38,120	6835
$\frac{5}{8}$	422,600	70,433	288,000	48,000	8609
$\frac{11}{16}$	520,800	86,800	354,900	59,150	10560
$\frac{3}{4}$	630,200	105,000	429,500	71,600	12785
$\frac{13}{16}$	750,800	125,100	511,700	85,300	15225
$\frac{7}{8}$	883,000	147,200	601,800	100,300	17906
$\frac{15}{16}$	1,027,000	171,200	700,100	116,700	20826
1	1,183,000	197,200	806,300	134,400	23990
	(1)	(2)	(3)	(4)	(5)

strength. The increase due to such joints can be determined only by experiments on the large scale from flues in actual practice, and from these we find that the rules in (845) require to be modified into—

(850.) $$P = 49\cdot3 \times (100\,t)^{2\cdot19} \div (L \times d).$$

(851.) $$p = 8\cdot2 \times (100\,t)^{2\cdot19} \div (L \times d).$$

Mr. Fairbairn made two experiments on 42-inch flues, the results of which are given by Nos. 30 and 31 in Table 132; col. 5 has been calculated by the rule, which shows an error of $+\,3\cdot1$ and $-\,3\cdot1$ per cent. respectively, as in col. 6.

(852.) From this it appears that a tube with ordinary lap-joints is $49\cdot3 \div 33\cdot6 = 1\cdot47$, or 47 per cent. stronger than similar tubes without any joints, or (what amounts nearly to the same thing) with one longitudinal joint only (848).

Although this increase is considerable, it is small compared

to the effect of strong rings at each joint: for example, a boiler tube say 24 feet long, would have cross-joints about every 3 feet, and with strong rings, the collapsing pressure would be $24 \div 3 = 8$ times that due with a 24-foot tube, but the lap-joints, as we have seen, add only 47 per cent. to the strength.

The rules in (845), &c., show that $P \times L \times d$ is constant for the same thickness of plate, hence we have for tubes without joints the rules:—

(853.) $$P \times L \times d = 33 \cdot 6 \times (100\,t)^{2 \cdot 19}.$$

(854.) $$p \times L \times d = 5 \cdot 6 \times (100\,t)^{2 \cdot 19}.$$

And for ordinary boiler tubes with lap-joints longitudinally and cross-ways the rules:—

(855.) $$P \times L \times d = 49 \cdot 3 \times (100\,t)^{2 \cdot 19}.$$

(856.) $$p \times L \times d = 8 \cdot 2 \times (100\,t)^{2 \cdot 19}.$$

Tables 133 have been calculated by these rules, and from them the strength or thickness of any tube may be easily found by the rules:—

(857.) $$P = T_C \div (L \times d).$$

(858.) $$p = T_S \div (L \times d).$$

(859.) $$T_C = P \times L \times d.$$

(860.) $$T_S = p \times L \times d.$$

In which T_C = the Tabular number for collapsing strain given by col. 1 or col. 3 in Table 133, and T_S = the Tabular number for the working strain by col. 2 or 4. For example, to find the collapsing pressure of a boiler flue with ordinary joints, 42 inches diameter, 35 feet long, and $\frac{3}{8}$ thick, we take T_S from Table 133 at 138,100, and we obtain $138100 \div (35 \times 42) =$ 94 lbs. per square inch: see No. 31 in Table 132.

Again, say that we required the thickness of plate for a tube 36 inches diameter, 20 feet long, to bear safely a working pressure of 45 lbs. per square inch; then the rule $T_S = p \times L \times d$, becomes $45 \times 20 \times 36 = 32400$, the nearest number to which in col. 2 of Table 133, is 32,250, opposite $\frac{7}{16}$ inch, the required thickness, &c.

(861.) Tables 134, 135, have been calculated by the rules in (854), (856), and will further simplify calculation, for the strength being inversely proportional to the length, we have only to divide the numbers given in those Tables by the length in feet to find the working pressure in any given case. Thus a flue 2 feet 3 inches diameter, and 10 feet long, with thicknesses $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$ inch, would have working pressures of 35, 85, and 160 lbs. per square inch respectively, &c.

TABLE 134.—Of the STRENGTH of BOILER TUBES with ordinary LAP-JOINTS, in RESISTING EXTERNAL PRESSURE.

Internal Diameter.	Thickness of Plate.								
	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$
	Working Pressure in Lbs. per Square Inch, for 1 Foot Long.								
ft. in.									
1 0	173	420	789	1287	..	..	..	..	..
1 3	138	336	631	1030	1535	..	..	..	..
1 6	115	280	526	858	1279	1792	..	..	..
1 9	..	240	451	735	1096	1536	2058	..	..
2 0	..	210	394	643	959	1344	1800	2330	..
2 3	..	187	350	572	853	1195	1600	2149	2608
2 6	..	168	315	515	767	1075	1441	1864	2348
2 9	..	..	287	468	698	977	1310	1694	2134
3 0	..	..	263	429	640	895	1200	1553	1956
3 3	..	..	..	396	590	827	1110	1433	1806
3 6	..	..	..	367	548	767	1006	1331	1677
3 9	..	..	..	343	511	717	960	1242	1570
4 0	..	..	..	..	480	672	900	1165	1467
4 3	..	..	..	..	451	632	848	1096	1381
4 6	..	..	..	..	426	597	800	1035	1305

(862.) "*Strengthening Rings.*"—Two of the tubes in Table 132 were of considerable length, but were in effect reduced to short tubes by two rings riveted on them. Thus No. 4 was 60 inches long, but became virtually 20 inches only, and, as shown by col. 4, the strength was proportional to the latter. No. 25 was divided into unequal lengths by two rings, the central length being 17 inches and the ends $21\frac{1}{4}$ and $21 \cdot 5$ inches respectively; of course in such a case the strength would be governed by the

TABLE 135.—Of the STRENGTH of BOILER TUBES without JOINTS, in RESISTING EXTERNAL PRESSURE.

Internal Diameter.		Thickness of Plate.								
		$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$
		Working Pressure in Lbs. per Square Inch, for 1 Foot Long.								
ft.	in.									
1	0	118	286	538	877	..	..	..	..	..
1	3	94	229	430	701	1046	..	..	..	..
1	6	79	191	358	585	871	1221	..	..	..
1	9	..	164	307	501	747	1047	1400	..	..
2	0	..	156	269	438	653	916	1227	1588	..
2	3	..	128	239	390	581	814	1091	1411	1778
2	6	..	114	215	351	523	733	982	1270	1600
2	9	..	..	195	319	475	666	892	1155	1455
3	0	..	..	179	292	436	610	818	1059	1334
3	3	..	..	..	270	402	563	755	979	1231
3	6	..	..	..	250	373	523	701	909	1143
3	9	..	..	..	234	348	488	655	848	1067
4	0	..	..	..	..	327	458	614	795	1000
4	3	..	..	..	..	308	431	578	748	941
4	6	..	..	..	..	296	407	545	707	889

greatest length, or 21·5 inches. In many cases it would be commercially economical to use a thin plate with one or more rings, rather than a thick plate without such support. For example, a tube 36 inches diameter, 30 feet long, $\frac{1}{2}$ inch thick, would give $1200 \div 30 = 40$ lbs. per square inch working pressure:—with a ring in the centre the effective length becomes 15 feet, and with say $\frac{3}{8}$ plate, the working pressure $= 640 \div 15 = 42 \cdot 7$ lbs., and this no doubt would be the most economical.

(863.) There are, however, some practical objections to rings of L or T iron riveted or screwed on the tube in the usual manner, one being the risk of leakage at the rivet-holes, and another, that the thickness of metal at the flange of the angle-iron will cause the boiler-plate at that point to be unduly heated, and a more or less destructive action to be set up.

An ingenious method of overcoming both of these objections is shown by Fig. 191, in which a cylindrical tube, A, B, C, D, is collapsed into the oval E, F, G, H. This change of form

may be resisted in two ways, namely, by preventing E and F from giving *in*, or G, H from giving *out*; if G, H be prevented from bulging out, E and F will be effectually prevented from collapsing. To apply this principle to practice, thin, deep rings are slipped on the tube, as at J in Fig. 192; the sizes will vary with the diameter of the tube and the pressure, and must be determined by judgment; for a 3-foot tube $\frac{3}{8}$ thick, they might be say 3 inches deep and $\frac{5}{8}$ inch thick. It would be necessary in most cases to make the inside diameter of the ring somewhat larger than the outside diameter of the tube, in order to enable it to pass over the heads of the rivets, &c.; in that case *narrow* wedges would have to be inserted at frequent intervals, so that the tube may be effectively supported by the ring.

OVAL TUBES.

(864.) It has been generally admitted that the collapsing strength of an oval or elliptical tube may be calculated by the rules for cylindrical ones by taking as the effective diameter that of the *osculating* circle, or a circle whose radius is the same as that of the flattest part of the ellipsis which is given by the rule

(865.) $$d_0 = A^2 \div a.$$

In which A = the major, and a the minor axis of the oval, d_0 being the effective diameter governing the strength.

Thus, with No. 35 in Table 132, $d_0 = 20\frac{3}{4}^2 \div 15\frac{1}{2} = 27 \cdot 78$ inches, and by col. 5 of Table 133 $(100 \times \cdot 25)^{2 \cdot 19} = 1152$; then the rule (845) becomes $P = 33 \cdot 6 \times 1152 \div (5 \cdot 083 \times 27 \cdot 78)$ = 274 lbs.; but experiment gave 127·5 lbs. only, or less than half. Again, with No. 34, $d_0 = 14^2 \div 10\frac{1}{4} = 19 \cdot 14$ inches, then $100 \times \cdot 043 = 4 \cdot 3$, the logarithm of which, or $0 \cdot 6335 \times 2 \cdot 19 = 1 \cdot 3874$, the natural number due to which, or $24 \cdot 4 \times 33 \cdot 6 = 819 \cdot 84$. Then the rule (845) becomes $P = 819 \cdot 84 \div (5 \times 19 \cdot 14) = 8 \cdot 56$ lbs.; but experiment gave 6·5 lbs. only; hence $8 \cdot 56 \div 6 \cdot 5 = 1 \cdot 32$, or + 32 per cent. error.

(866.) These two experiments are all we have on tubes decidedly oval, for Nos. 32, 33 were too nearly cylindrical to be of much service. We have seen that the rule as applied to the

cases we have, does not give satisfactory results, and if we apply it to extreme cases of *very* flat oval tubes we should obtain results that are manifestly incorrect. For example, in Fig. 195, A is a tube 12×2, and B another, 12×1: now by the rule (865), B should bear half only of the pressure due with A, but obviously there would be no such difference in the strength as 2 to 1; on the contrary, we feel instinctively that there would be so little difference that practically they would collapse with one and the same pressure. Now, a rule that fails with extreme cases, will be very likely to be incorrect in all other cases, although in a less degree; in fact, the laws governing oval tubes differ entirely form those dominating cylindrical ones, and the two forms cannot be assimilated so as to enable both to be calculated by the same rule.

(867.) We have seen in (841) that, theoretically at least, the strain due to external pressure on a perfectly cylindrical tube is simply a *crushing* one. But when an oval, A, B, C, D, Fig. 191, is compressed into another, E, F, G, H, the curvature from m to n and from o to p is flattened, while from m to o and from n to p the curvature is increased. In either case, this increase or decrease of curvature is resisted by the natural stiffness of the material, the strain thus generated being a *transverse* one.

Let Fig. 194 be a tube 1×2 inches, and, for the purpose of illustration, 1 inch deep, subjected to an external fluid pressure in all directions. We may assume that the pressure in the direction of the arrows a, a, tends to flatten the ellipsis and collapse the tube, but the pressure in the direction b, b, tends, on the contrary, to restore the tube to a circular form, and to partially counteract the collapsing tendency of the pressure a, a. But inasmuch as the area on which a acts is in our case double that on which b acts, the final tendency will be to collapse. In all cases the collapsing pressure is a *differential* one, being, in fact, the difference between the strain due to the two pressures from a and b.

(868.) Say that the pressures a and b were 300 lbs. per square inch, then the strain from a, a, acting on two square inches, would be $300 \times 2 = 600$ lbs., and the strain from b, b = $300 \times$

1 = 300 lbs; hence the effect of both is to leave 600 − 300 = 300 lbs. in the direction a, a, or 300 ÷ 2 = 150 lbs. per square inch, which has to be resisted by the material of which the tube is composed.

Let Fig. 196 be a series of oval tubes all of the same *minor* axes and thickness of plate, A being the same as Fig. 194: B is an oval tube 1 × 3 and 1 inch deep, the thickness being the same as in Fig. 194, which we found to collapse with a *differential* or unbalanced strain of 300 lbs. spread over 2 square inches: then, the *transverse* strength being inversely as the length of the *beam*, with B we should have 300 × 2 ÷ 3 = 200 lbs. unbalanced, to obtain which the total pressure in the direction a, a must be 300, and b, b = 100, leaving 300 − 100 = 200 lbs., as required, and as this is now spread over 3 square inches, we have a *gross* pressure of 300 ÷ 3 = 100 lbs. per square inch, whereas with A we had 150 lbs.

Let C be an oval tube 1 × 4 inches, and 1 inch deep, &c. As A collapsed with a *differential* pressure of 300 lbs., C would collapse with 300 × 2 ÷ 4 = 150 lbs., to obtain which a, a = 200 lbs.; then b, b, being one fourth of that, or 50 lbs., the differential pressure = 200 − 50 = 150 lbs., as required. The gross pressure 200 lbs. being now spread over 4 square inches, we have 200 ÷ 4 = 50 lbs. per square inch.

Again, D is an oval tube 1 × 5 inches, and 1 inch deep, &c. A collapsed with a differential pressure of 300 lbs., D would therefore collapse with 300 × 2 ÷ 5 = 120 lbs., to obtain which a, a must be 150 lbs.; then b, b, being one-fifth of that, or 30 lbs., the *differential* pressure = 150 − 30 = 120 lbs., as required. Here the gross pressure, 150 lbs., being spread over 5 square inches, gives 150 ÷ 5 = 30 lbs. per square inch.

Again, E is an oval tube 1 × 6 inches, and 1 inch deep, &c. A collapsed with a *differential* pressure of 300 lbs., E would collapse with 300 × 2 ÷ 6 = 100 lbs., to obtain which the gross pressure a, a must be 120 lbs.; then b, b, being one-sixth of that, or 20 lbs., the differential pressure = 120 − 20 = 100 lbs., as required. Hence the gross pressure, 120 lbs., being spread over 6 square inches, we have 120 ÷ 6 = 20 lbs. per square inch.

Thus, with the five oval tubes we have considered, the minor axes being 1 inch in all cases, and

The major axes being	=	2	3	4	5	6 inches.,
The gross total pressures	=	600	300	200	150	120 lbs.,
The pressures per square inch	=	300	100	50	30	20 lbs.,

the results of these analytical investigations may be represented by the rule:—

(869.) $$P = \frac{600}{(A - a) \times A}.$$

Thus, with an oval tube 2×1, we have $P = \frac{600}{(2 - 1) \times 2} =$ 300 lbs. per square inch:—with another, 6×1, we obtain $P = \frac{600}{(6 - 1) \times 6} = 20$ lbs., &c., &c., as before.

These numbers give *Ratios* only for the strength of oval tubes of varying proportions but all of the same thickness and length, the special object being to investigate the laws by which the two diameters govern the strength. Theoretically the strength should vary as t^2 directly, and as d inversely, and be independent of the length of the boiler, but we have seen that in the case of cylindrical tubes the theory was incorrect (841), and we must assume, in the absence of experiments on oval tubes of various lengths and thicknesses, that the strength of elliptical tubes, like that of cylindrical ones, is directly as $t^{2\cdot19}$ and inversely as the length; we then have the rule:—

(870.) $$P = \frac{(100\,t)^{2\cdot19} \times 61\cdot4}{(A - a) \times A \times L}.$$

In which A = the major and a = the minor axis in inches, L = the length of the tube in feet, P = pressure in lbs. per square inch, and t = the thickness in inches.

With experiment No. 35 in Table 132 this rule becomes $P = \frac{(100 \times \cdot25)^{2\cdot19} \times 61\cdot4}{(20\frac{3}{4} - 15\frac{1}{2}) \times 20\frac{3}{4} \times 5\cdot083} = 127\cdot7$ lbs.: experiment gave 127·5 lbs. Again, with No. 34, we obtain

$$P = \frac{(100 \times \cdot043)^{2\cdot19} \times 61\cdot4}{(14 - 10\frac{1}{4}) \times 14 \times 5} = 5\cdot84 \text{ lbs.}:$$

experiment gave 6·5 lbs.; hence $5 \cdot 84 \div 6 \cdot 5 = \cdot 8986$, showing an error of $1 \cdot 0 - \cdot 8986 = \cdot 1014$ or $- 10 \cdot 14$ per cent.

(871.) "*Oval Tubes with Cross-joints, &c.*"—It should be observed that the multiplier 61·4 has been derived from experimental tubes without cross-joints, and with one longitudinal joint only. By analogy with cylindrical tubes (852) we may infer that for oval tubes on the large scale, having ordinary lap-joints, crossways and longitudinally, the multiplier would become $61 \cdot 4 \times 49 \cdot 3 \div 33 \cdot 6 = 90$, and hence we have the rule

(872.) $$P = \frac{(100\,t)^{2 \cdot 19} \times 90}{(A - a) \times A \times L}.$$

(873.) "*Tubes Slightly Oval.*"—We have seen in (841) that theoretically the strain due to external pressure on a perfectly cylindrical tube is simply a crushing one, but that practically the strength is given by the rule in (845). With a flat oval tube the strain generated by external pressure is a transverse one, and the strength is given by the rule in (870). When the form differs very little from a perfect cylinder we may suppose that the strength will be given by the former rule rather than the latter. For such cases and for tubes without cross-joints we have the rule:—

(874.) $$P = 33 \cdot 6 \times (100\,t)^{2 \cdot 19} \div (L \times d_0),$$

d_0 being the diameter of the osculating circle, or $A^2 \div a$, as explained in (864), and the rest as in (845). For large tubes with ordinary lap-joints the rule becomes:—

(875.) $$P = 49 \cdot 3 \times (100\,t)^{2 \cdot 19} \div (L \times d_0).$$

With a certain ratio between the two diameters these rules will agree in their results with those for decidedly oval tubes in (870), as shown by Table 136; thus, with a tube $12 \times 9\frac{3}{4} \times \frac{1}{8}$, also with $24 \times 22 \times \frac{1}{4}$, and with $36 \times 34 \cdot 07 \times \frac{3}{8}$, the two sets of rules agree. With ovals more nearly cylindrical than those sizes the rules in (845), (850) will govern the strength; but with flatter ovals the rules in (870), (872) will prevail. The best course in any doubtful case is to calculate by both rules and accept the lowest result as correct.

TABLE 136.—Of the CALCULATED STRENGTH of OVAL TUBES 10 feet long to RESIST EXTERNAL PRESSURE.

Sizes in Inches.	Collapsing Pressure in Lbs. per Square Inch.			Combined Results of the two Rules.	
	Ordinary Rule (874).	New Rule (870).	Ratios.	Lbs.	Ratios.
12 × 12 × ⅛	70·70	Infinite	0·0000	70·70	1·0000
12 × 11 × ⅛	64·77	129·0	0·502	64·77	·9161
12 × 10 × ⅛	58·92	64·5	0·913	58·92	·8333
12 × 9¾ × ⅛	57·4	57·4	1·000*	57·4	·8120
12 × 9 × ⅛	53·0	43·0	1·230	43·0	·6082
12 × 8 × ⅛	47·1	32·5	1·449	32·5	·4600
12 × 7 × ⅛	41·18	25·8	1·596	25·8	·3650
12 × 6 × ⅛	35·35	21·5	(1·644)	21·5	·3041
12 × 5 × ⅛	29·46	18·46	1·600	18·46	·2611
12 × 4 × ⅛	23·6	16·15	1·461	16·15	·2284
12 × 3 × ⅛	17·7	14·35	1·233	14·35	·2030
12 × 2 × ⅛	11·8	12·92	0·913	12·92	·1827
12 × 1 × ⅛	5·9	11·75	0·502	11·75	·1662
24 × 24 × ¼	161·3	Infinite	0·000	161·3	1·0000
24 × 22 × ¼	147·7	147·40	1·000*	147·4	·9137
24 × 18 × ¼	121·0	49·12	2·464	49·12	·3044
24 × 12 × ¼	80·6	24·56	(3·281)	24·56	·1522
24 × 6 × ¼	40·3	16·37	2·464	16·37	·1014
24 × 2 × ¼	13·5	13·39	1·007	13·39	·0830
36 × 36 × ⅜	261·3	Infinite	0·000	261·3	1·0000
36 × 34·07 × ⅜	247·4	247·4	1·000*	247·4	·9467
36 × 27 × ⅜	196·0	53·06	3·69	53·06	·2030
36 × 18 × ⅜	130·6	26·53	(4·923)	26·53	·1015
36 × 9 × ⅜	65 3	17·69	3·69	17·69	·0677
36 × 3 × ⅜	21·8	14·47	1·506	14·47	·0553
(1)	(2)	(3)	(4)	(5)	(6)

(876.) Thus with No. 35 in Table 132 we obtained 127·7 lbs. by one rule (870), and 274 lbs. by the other (845); and we may admit the lower result to be correct; a conclusion supported by experiment, which gave 127·5 lbs.

But in experiment No. 32 we had a tube very nearly cylindrical, there being a difference in the two diameters of $\frac{3}{16}$ inch only. We know beforehand that the rule in (870) will not apply to such a case. Nevertheless, for the sake of illustration,

we may try it, and we obtain $P = \dfrac{(100 \times \cdot 125)^{2\cdot 19} \times 61\cdot 4}{(14\frac{11}{16} - 14\frac{1}{2}) \times 14\frac{11}{16} \times 5}$ $= 1125$ lbs.! or 9 times the experimental pressure, which was 125 lbs. only. By the other rule (845) the osculating circle $d_o = 14\frac{11}{16}^2 \div 14\frac{1}{2} = 14\cdot 88$ inches; then $P = 33\cdot 6 \times 252\cdot 5 \div (5 \times 14\cdot 88) = 114$ lbs.; hence $114 \div 125 = \cdot 912$, giving an error of $1\cdot 0 - \cdot 912 = \cdot 088$, or $- 8\cdot 8$ per cent. only; here evidently the rule in (848) is the more correct.

Again, in experiment No. 33 the two diameters differed only $\frac{7}{16}$ inch; here again the rule in (870) will give a pressure greatly in excess; $P = \dfrac{252\cdot 5 \times 61\cdot 4}{(15\frac{5}{8} - 15\frac{3}{16}) \times 15\frac{5}{8} \times 1\cdot 77} = 1281$ lbs.! or nearly 6 times the experimental pressure, which was 220 lbs. per square inch. By the other rule we obtain $d_o = 15\frac{5}{8}^2 \div 15\frac{3}{16} = 16\cdot 07$ inches, then $P = 33\cdot 6 \times 252\cdot 5 \div (1\cdot 77 \times 16\cdot 07) = 298$ lbs.; hence $298 \div 220 = 1\cdot 35$, or an error of $+ 35$ per cent.: of course the lower result is obviously the more correct.

(877.) Table 136 gives the strength of oval tubes of boiler-plate iron with different thicknesses, and 10 feet long, calculated by the two rules for the sake of comparing results. It is remarkable that not only does the ordinary rule give lower pressures for the tubes which are nearly cylindrical, a result that was to be expected, but does the same with the extremely flat tubes 12×1 and 12×2, shown by Fig. 195. In the absence of experiment we should instinctively suppose that both being such extremely flat tubes, and both having the same major diameter, there would be very little difference in the collapsing pressure, and that the new rule in (870) which gives only $12\cdot 92 \div 11\cdot 75 = 1\cdot 10$ or 10 per cent. difference in strength by col. 3, is more correct than the ordinary rule in (874), which gives double strength, or $11\cdot 8$ to $5\cdot 9$ by col. 2. Combining the two rules we obtain col. 5, in which the tubes from 12×12 to $12 \times 9\frac{3}{4}$ have been calculated by the rule for cylindrical tubes, taking for the acting diameter the osculating circle d_o: all the rest being taken from col. 3.

It will be observed that small departures from the perfectly cylindrical form are not very influential on the strength: thus with a tube 12×11, we have $64\cdot 77 \div 70\cdot 7 = \cdot 92$, or 92 per cent. of the strength of a perfectly cylindrical one 12 inches

diameter: another, 12 × 10, gives $58 \cdot 92 \div 70 \cdot 7 = \cdot 83$, or 83 per cent.

The strength with $\frac{1}{8}$ inch thickness and a cylindrical 12-inch tube being $1 \cdot 0$, would be reduced to half with 12 × 8·3 inches; to one-third with 12 × $6\frac{1}{2}$ inches; to one-fourth with 12 × $4\frac{3}{4}$ inches; to one-fifth with 12 × 3 inches; and to one-sixth with 12 × 1 inch. These relative proportions, however, must not be taken as establishing general ratios for other sizes: for example, a tube 12 × 9 × $\frac{1}{8}$ collapses by col. 6 with ·6082 of the pressure due to a 12-inch cylinder, and we might expect that the same *ratio* would prevail for other sizes where the proportions were the same; but the Table shows, col. 6, that with double sizes, or 24 × 18 × $\frac{1}{4}$, the ratio is ·3044, or half only; and with triple sizes, or 36 × 27 × $\frac{3}{8}$, the ratio is ·2030, or one-third only of that with a tube of one-third the size and thickness, &c.

(878.) Table 132 gives a general comparison of experimental with calculated strengths: Nos. 1 to 29 were calculated by the rule for jointless tubes (845); Nos. 30, 31 by the rule for lap-jointed tubes (850); Nos. 32, 33 by the rule for slightly oval tubes (874); and Nos. 34, 35 by the rule for decidedly oval tubes (870). Omitting Nos. 25, 26, which were anomalous, and No. 28, which was not strained to the collapsing point, we have from 1 to 31 inclusive, five whose error (col. 6) = 0; 11 gave + errors, the sum of which = 131·4; 12 gave − errors, the sum being 114·1. Hence $131 \cdot 4 - 114 \cdot 1 = +17 \cdot 3$, which with 28 comparable experiments gives an average error of $17 \cdot 3 \div 28 = +0 \cdot 618$, or less than $\frac{5}{8}$ per cent.

(879.) "*Factor of Safety.*"—For general purposes the Factor 6, as given by Mr. Fairbairn, may be usually admitted for Boilers with both internal and external pressures, but in practice a much lower Factor is very often permitted with both strains.

The two boilers Nos. 30 and 31 in Table 132 were intended for 40 lbs. per square inch, and were no doubt worked at that pressure, but, as shown by col. 4, the collapsing pressures were 127 and 97 lbs. respectively, giving as the value of the Factor, $127 \div 40 = 3 \cdot 2$ and $97 \div 40 = 2 \cdot 42$ only. It is shown in (78) that with internal pressures, the Factor commonly used in

the best practice varies from 3·45 to 2·76. Nevertheless, it is highly expedient wherever practicable to use Factor 6, and thus allow a wide margin for fluctuations in pressure, deterioration from rust, and other contingencies, which are unavoidable, and should thus be provided for.

CHAPTER XXII.

FACTOR OF SAFETY.

(880.) "*Ratios of Breaking Weight, Proof Strain, and Working Load.*"—The Strength of Materials is usually determined by the ultimate or breaking weight of a specimen, and among the most important questions with which the Engineer has to deal is 1st, to determine the Ratio which the working load may safely bear to the ultimate strain,—or the "Factor of Safety"; and 2nd, the "Proof Strain," or the extent to which work should be tested in order to prove the soundness of the materials, also the correctness of the design and perfection of workmanship in the case of complex structures.

In determining the proper value of the Factor of Safety so as to avoid unnecessary strength on the one hand, and risk of failure from inadequate strength on the other, it is necessary to consider, 1st, the varying conditions under which materials are strained; and 2nd, the variableness in the quality of the materials themselves. These will be very influential, so that the value of the Factor will not be constant for all cases, and the whole matter is thereby complicated considerably.

(881.) "*Variable Conditions of Strain.*"—We may have, 1st, a perfectly dead load, or statical strain; 2nd, a rolling load in rapid motion, as in the case of a Railway Bridge, where the strain becomes more or less a dynamic one, but under certain limitations dependent on the horizontal velocity; 3rd, cases where the load is intermittent, being alternately laid on and taken off repeatedly, as in the case of cranes, single-acting pump-rods, &c.; 4th, alternating strains in opposite directions,

as in the case of the beam, piston-rod, &c., of ordinary steam-engines, double-acting pump-rods, &c.

(882.) "*Variableness in Materials.*"—Besides the variations due to methods of loading, there are at least three others due to the materials themselves, and we have, 5th, the natural variableness in the strength of all materials, even those of apparently the same kind and quality (957); 6th, deterioration from age and decay in such materials as timber, ropes, &c., and from rust in the case of wrought iron, especially when exposed to the weather; and 7th, the effect of thickness or size of casting with cast iron, and probably other cast metals (931).

It will not be necessary, however, to consider each case in detail under these several heads; for practical purposes it will be more convenient to take a Factor so high as to cover many of these contingencies, and we may then reduce the cases to 1st, a dead load; 2nd, a rapidly rolling load; 3rd, an intermittent load; and 4th, an alternating strain in opposite directions. See (960) for Real and apparent "Factors of Safety."

We shall in this Chapter confine ourselves to the simple case of a dead, or statical load; having found the proper value of the Factor of Safety for that case, the modifications necessary for other conditions will be considered in the Chapters on Fatigue (903), Impact (774), &c.

(883.) The earlier writers, Tredgold and others, finding that with $\frac{1}{3}$rd of the breaking weight, beams of cast iron, &c., began to show signs of distress by taking a permanent set (752), assumed that strain to be the *limit of elasticity*, and therefore that 3 should be the Factor of Safety for cast iron. It was considered that with loads not exceeding that limit materials would be quite uninjured, but that with greater loads a beam would go on increasing in deflection with time, until at a period more or less remote it would finally break. The Factor 3, based on these conclusions, has been almost universally accepted for dead loads by practical men, although Mr. Hodgkinson's experiments have shown long ago that, 1st, with cast iron particularly there is no such point as the *limit of elasticity*, or any strain, however small, with which there would be no

permanent set (752), and 2nd, that beams and pillars of cast iron will bear not $\frac{1}{3}$rd only, but $\frac{97}{100}$ths of the breaking weight safely and without increase of deflection for years (905).

(884.) From this last statement it would appear that the Factor of Safety might be very much less than 3, say 2, the beam, &c., being then loaded to half the breaking weight, and perhaps this might be permitted if we knew perfectly the actual ultimate strength of the particular specimen to be dealt with, but this must always be an unknown quantity, being, in fact, incapable of proof except by loading that specimen up to the breaking point, which of course would not answer the purpose.

From the variableness in the qualities of most materials, the assumed Factor 3 becomes often 2, or even $1\frac{1}{2}$ in practice:—for example, say we would calculate the transverse strength of a bar of cast iron 3 inches square. It is shown in (933) that the mean specific strength of a 3-inch bar is only 62 per cent. of the strength of 1-inch bars from which the ordinary Multiplier is usually derived: moreover, by (957) and Table 149 the minimum transverse strength is shown to be only 79 per cent. of the *mean* strength, of which that multiplier is the exponent. Now if it should happen that our 3-inch bar is of weak iron, the Factor 3 would really become in effect $3 \times \cdot 62 \times \cdot 79 = 1 \cdot 47$, or *less than half* its assumed value. Considering possible but unknown contingencies from mode of loading and otherwise, it is evident that even the Factor 3 would in that case be too low for safety.

(885.) "*Cast Iron.*"—The effect of size or thickness of casting is thus shown to be so influential that it becomes expedient to consider it separately in each particular case, because if we adopted a Factor sufficiently high to cover this and all other contingencies in the case of large castings, that Factor would be higher than necessary for ordinary sizes, and would lead to a costly excess of strength. Adopting that course but allowing for variableness as shown by Table 149, then the Factor 3 for transverse strength becomes in effect, with weak bars $3 \times \cdot 79 = 2 \cdot 37$; for tensile strength, $3 \times \cdot 79 = 2 \cdot 37$ also; and for crushing strength, $3 \times \cdot 67 = 2 \cdot 0$. These reduced values being admitted as sufficient for safety, we may adopt 3 as the Factor

for all strains with ordinary thicknesses of cast iron, applying subsequently the correction for size of casting where necessary (932). For example, with a cast-iron girder whose breaking weight calculated by the ordinary Multiplier (335) is 30 tons, we have $30 \div 3 = 10$ tons safe load if the thickness of metal differs little from 1 inch; but if the thickness (of the bottom flange more particularly) is about 2 inches, we have $10 \times \cdot 72 = 7 \cdot 2$ tons; and if 3 inches thick, then $10 \times \cdot 62 = 6 \cdot 2$ tons safe dead load.

(886.) "*Wrought Iron.*"—The Diagram, Fig. 215, shows that under tensile and compressive strains wrought iron practically fails with 12 or 13 tons per square inch, the extensions and compressions becoming excessive and increasing with *time*. The mean tensile breaking weight is 25·7 tons, as shown by Table 1; evidently, therefore, the iron begins to be *crippled* with half the breaking weight, and 2 would be too low for the Factor of Safety even if we were sure that the iron was of average quality. Besides, Table 149 shows that if the bar happens to be of weak iron, Factor 2 would really become $2 \times \cdot 77 = 1 \cdot 54$, and the bar would be very much overstrained. Factor 3 becomes $3 \times \cdot 77 = 2 \cdot 3$, which as the diagrams show by * * may be safely permitted.

We may therefore admit 3 as the Factor of Safety with all strains on wrought iron:—thus the safe tensile strain becomes say $25 \cdot 7 \div 3 = 8 \cdot 6$ tons per square inch with bar iron; and $21 \cdot 6 \div 3 = 7 \cdot 2$ tons, with plate iron, &c.

(887.) "*Steel.*"—The elasticity of steel under transverse strains is wonderfully perfect as shown by the Diagram, Fig. 211, where a bar of untempered steel shows no appreciable signs of distress with even $\frac{3}{4}$ths of the ultimate strain, or that with which the bar sinks down completely. In such a case we might admit that the bar might be loaded safely to $\frac{1}{2}$ the ultimate strength, or that the Factor might be = 2. But Table 149 shows that the variableness in the tensile strength of steel is very great, namely ·68, the mean strength being 1·0, hence if a *weak* bar is loaded with $\frac{1}{2}$ the ultimate load due to an *average* bar it would evidently be strained to $\frac{1}{2} \div \cdot 68 = \cdot 73$, or but little less than $\frac{3}{4}$ of its own ultimate strength,

and this would be too near the crippling strain to be admitted safely.

We may therefore allow 3 as the Factor of Safety with all strains on Steel: with a weak bar this would in effect be reduced to $3 \times \cdot 68 = 2$, giving thus the safe load at $\frac{1}{2}$ the ultimate strength, and this, as the diagrams show, may be permitted safely.

(888.) "*Timber.*"—With Timber it is necessary in fixing the value of the Factor of Safety, to provide not only for variableness in the strength common to all materials, but also for deterioration from age and for decay from exposure to the elements, which is quite another matter. To cover all these contingencies it becomes expedient to adopt a higher Factor than would otherwise be necessary: we will take it at 5 as a mean for dead loads.

(889.) "*Effect of Age.*"—The effect of age may be ascertained approximately from the experiments in Table 1:—Mr. Bevan found as a mean of two experiments the tensile strength of ordinary Oak to be 17,400 lbs. per square inch, while old Oak gave 14,000 lbs., or 80 per cent. By Table 149 the minimum tensile strength is $\cdot 72$, the Factor 5 would therefore in the case of *old* and *weak* oak be, in effect, reduced to $5 \times \cdot 8 \times \cdot 72 = 2 \cdot 88$, which however is not too low for safety. The *transverse* strength of young oak or value of M_T is 964 lbs., and of old oak 660 lbs., according to the experiments of Tredgold, hence old oak is only 68 per cent. of the strength of young oak, and the minimum transverse strength being $\cdot 72$ by Table 149, Factor 5 is in effect reduced to $5 \times \cdot 68 \times \cdot 72 = 2 \cdot 45$ in the case of old and weak oak, not very different from the result we obtained from the tensile strength, which came out $2 \cdot 88$.

The mean tensile strength of Teak is 15,090 lbs. per square inch, but of old Teak 8200 lbs. only, or 55 per cent.: hence Factor 5 becomes in effect $5 \times \cdot 55 \times \cdot 97 = 2 \cdot 67$.

We have here taken extreme cases; with ordinary care in selection Timber need never be weaker than we have assumed, and Factor 5 may be taken as sufficient for safety for all strains on timber.

(890.) "*Effect of Decay.*"—The effect of decay on the

strength of timber exposed to the destructive action of the elements is very difficult to estimate, there being, in fact, no limit to the extent to which the material may be weakened from that cause. Mr. Bevan found that Oak from an old pile taken out of the bed of a river, had a tensile strength of 4500 lbs. per square inch only, which is 23 per cent. only of the mean strength of sound oak. If it is deemed necessary in any structure exposed to air and water, to provide for the eventuality of decay, the Factor of Safety should not be less than 10 for a dead load, when the mean strength of sound timber is taken as the basis of calculation:—evidently in the case of Mr. Bevan's oak pile that factor would in effect be reduced to $10 \times \cdot 23 = 2 \cdot 3$.

(891.) "*Stone, Slate, Brickwork, &c.*"—Except for the crushing strain, our experimental knowledge of these materials is limited, and we have little else but judgment to guide us in fixing the value of the Factor of Safety:—they are all weak in resisting Impact, and as in many cases an unexpected blow may have to be borne, it will be well to make the Factor higher than would otherwise be necessary, say 4. Table 149 shows that for brick exposed to transverse strains 4 may become in effect $4 \times \cdot 75 = 3$ with a weak specimen; and that for crushing strains 4 may be reduced to $4 \times \cdot 5 = 2$ in the case of Red Sandstone; and to $4 \times \cdot 58 = 2 \cdot 32$ in the case of Granite. These reduced numbers show that it is not prudent to adopt a Factor lower than 4 for these materials.

(892.) Collecting these results, we obtain for ordinary cases the series of Ratios and Factors of Safety in Table 137. Special cases, which are very numerous, require special Factors obtained direct from experience, and this is very often the only safe course; the modifying circumstances are in practice so numerous and so complex that satisfactory results are not to be obtained in any other way. Table 138 gives the Ratios of the Breaking, Proof, and Working Loads, with special reference to Railway Bridges, &c., according to the judgment of our most eminent Engineers, as given in Evidence before H.M. Commissioners. Of these, R. Stephenson, W. Fairbairn, J. Hawkshaw, J. Cubitt, and P. W. Barlow, have adopted the Factor 6, as the best for general Railway purposes. It is shown in (491) that

TABLE 137.—Of the RATIOS of the BREAKING WEIGHT, PROOF STRAIN, and WORKING LOAD for different Materials; also the FACTOR of SAFETY: all for DEAD LOADS.

Kind of Material.	Ratios.			Factor of Safety.
	Breaking Weight.	Proof Strain.	Working Load.	
Tempered Steel Spring	1	$\frac{3}{4}$	$\frac{1}{2}$	2
Ordinary Wrought Iron, Steel, Cast Iron, Cast and Wrought Lead, Copper, Brass, &c.	1	$\frac{1}{2}$	$\frac{1}{3}$	3
Slate, York Paving, Stone, Brickwork	1	$\frac{3}{8}$	$\frac{1}{4}$	4
Timber	1	$\frac{3}{10}$	$\frac{1}{5}$	5
SPECIAL CASES.				
Railway Bridges, Wrought Iron ..	1	$\frac{1}{3}$	$\frac{1}{6}$	6
,, ,, Cast Iron	1	$\frac{2}{9}$	$\frac{1}{9}$	9
Boilers	1	$\frac{1}{3}$	$\frac{1}{6}$	6
Ropes—Hempen	1	$\frac{1}{3}$	$\frac{1}{4}$	4
Chain—Short-linked—Crane	1	$\frac{4}{10}$	$\frac{3}{10}$	$3\frac{1}{3}$
,, Stud-linked—Cable	1	$\frac{2}{3}$	$\frac{1}{3}$	3

TABLE 138.—Of the RATIOS of the BREAKING PROOF and WORKING LOADS on BEAMS, according to different Authorities.

Authority.	Breaking Weight.	Proof Strain.	Working Load.	
			Dead Load.	Railway Bridges.
W. Fairbairn	1	$\frac{1}{3}$ or $\frac{1}{2}$	$\frac{1}{4}$ or $\frac{1}{3}$	$\frac{1}{4}$ or $\frac{1}{6}$
J. Cubitt	1	$\frac{1}{2}$	..	$\frac{1}{6}$
R. Stephenson	1	$\frac{1}{3}$	..	$\frac{1}{5}$
W. H. Barlow	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
J. Hawkshaw	1	$\frac{2}{7}$	..	$\frac{1}{7}$ or $\frac{1}{5}$
J. Locke	1	$\frac{2}{3}$ or $\frac{3}{4}$	$\frac{1}{5}$ or $\frac{1}{4}$	..
I. K. Brunel	1	$\frac{1}{3}$,, $\frac{2}{5}$	..	$\frac{1}{5}$ to $\frac{1}{4}$
P. W. Barlow	1	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$
C. Fox	1	$\frac{2}{5}$ or $\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$
C. May	1	$\frac{2}{3}$	..	$\frac{1}{5}$
W. Cubitt	1	$\frac{1}{2}$ to $\frac{2}{3}$	$\frac{1}{5}$	..
H. Grissell	1	..	$\frac{1}{3}$	$\frac{1}{6}$
J. U. Rastrick	1	$\frac{1}{2}$	$\frac{1}{6}$	..
J. Glynn	1	..	$\frac{1}{5}$	$\frac{1}{13}$

with *very* large structures, the highest *possible* Factor = 4, being limited by the great weight of the Beam itself.

PROOF STRAIN.

(893.) The great object of testing or proving Materials is to obtain thereby a guarantee that they will safely bear the permanent load assigned to them. To secure that purpose satisfactorily it will not suffice to test up to the working load only: it is necessary to allow an excess of strength to cover the possible contingencies of irregularities in loading, or imperfection in quality. In considering the proper value of the "Proof Strain" there are two different bases to calculate from, 1st, by making the Proof Strain a given fraction of the Breaking Weight, and 2nd, by making it a given multiple of the working load. If the Factor were constant for all materials, the two methods would be identical in their results; but as we have seen, that Factor has a variable value, which alters the case. Thus, say we take the Factor at 3, and allow the Proof Strain to be half the breaking weight; then the Breaking, Proof, and Working Strains would be in the ratio 1, $\frac{1}{2}$, $\frac{1}{3}$, the Proof Strain being $\frac{1}{2} \div \frac{1}{3} = 1 \cdot 50$, or 50 per cent. in excess of the working load. But with Factor 5, if we made the proof strain half the breaking weight as before, we should evidently have 1, $\frac{1}{2}$, $\frac{1}{5}$ as the ratios of the three strains, and in that case the proof strain would have been $\frac{1}{2} \div \frac{1}{5} = 2 \cdot 50$, or 150 per cent. in excess of the working load.

(894.) Considering that the special object of testing has direct reference to the safe endurance of the working load, it seems expedient to take that load as the basis, rather than the breaking weight.

The earlier authorities considered that $\frac{1}{3}$rd of the breaking weight was the "limit of elasticity," and that materials would be permanently injured by heavier strains. Although that conclusion has been proved to be incorrect (883), the notion still lingers in the minds of practical men, some of whom, such as Brunel, object to the testing of materials beyond the permanent working load which they are intended to carry. This would, however, be an obviously unsafe practice, for there might be

some latent defect in design or quality, such that the structure would break with a strain very little in excess of the working load, and if in practice that load should from some unexpected cause be a little greater than was assigned to it, utter failure might result. It is therefore highly necessary that the Proof Strain should be considerably in excess of the working load, so as to leave a margin for contingencies. On the other hand, it is not prudent to overstrain the material:—Mr. Fairbairn says, "I am not an advocate for testing girders much beyond their permanent load"; however, he takes the working load at $\frac{1}{4}$ or $\frac{1}{3}$ of the breaking weight, and the proof strain at $\frac{1}{3}$ or $\frac{1}{2}$ of the breaking weight. Hence the proof strain would be in one case $\frac{1}{3} \div \frac{1}{4} = 1 \cdot 33$, or 33 per cent., and in the other case $\frac{1}{2} \div \frac{1}{3} = 1 \cdot 50$, or 50 per cent. in excess of the working load.

(895.) Most of the Engineers whose opinions are given in Table 138, consider half the breaking weight to be the extreme limit to which materials should be tested. If we admit that the Proof Strain should be 50 per cent. in excess of the working load, then with Factor 3 we should have $1 \div 3 \times 1 \cdot 50 = \cdot 5$, or half the breaking weight, which agrees with the opinion of the eminent Engineers in that Table. Moreover, the various Diagrams and Tables show that with a strain of half the breaking weight there is no excessive deflection or permanent set, which proves that the materials are not overstrained. With Factor 4 we should have for the proof strain $1 \div 4 \times 1 \cdot 50 = \cdot 375$, or $\frac{3}{8}$ths; and with Factor 5, $1 \div 5 \times 1 \cdot 50 = \cdot 3$, or $\frac{3}{10}$ths of the breaking weight respectively; being in both cases less than half.

We may therefore admit that the Proof Strain should be 50 per cent. in excess of the permanent or working load, giving thus a good margin for contingencies, without unduly straining the material.

ON TEST-BARS, ETC.

(896.) "*Factor determined by Test-bars.*"—In large contracts for cast-iron girders, sleepers, &c., for Railway and other purposes, it is usual, in order to secure a high standard of strength, and uniformity therein, to have test-bars cast from the same

iron as that used for the Girders, usually two or three times a day at given intervals. These sample test-bars are then subjected to transverse strain, and the breaking weights are required to come up to a certain standard load fixed by the Engineers, the Girders being rejected if the test-bars fail to come up to that standard. Thus the "Factor of Safety" is determined by Test-bars.

Another test-standard is to give a certain minimum *tensile* strength for the iron, in which case the iron used for the girders is cast at intervals as before in forms suitable for being torn asunder, and is required to bear a given strain per square inch.

(897.) It has been doubted, however, whether the strength of girders of ordinary sections will be simply proportional to the transverse strength of such "Test-Bars," or to the tensile strength of the iron as thus taken. Mr. Berkley made some valuable experiments for the purpose of settling this question, the reduced results of which are given by Table 139. The girders were all of the ordinary double-flanged type recommended by Mr. Hodgkinson and used in his experiments, the depth being $5\frac{1}{8}$ inches and the length $4\frac{1}{2}$ feet between bearings, the other dimensions are given by Figs. 197, 198, 199. Thus No. 4 broke with 16,730 lbs., or 3346 lbs. per square inch of sectional area, when the test-bar (cast from the same iron), 2 inches deep, 1 inch wide, broke with 25 cwt. in the centre, and the tensile strength = $7 \cdot 142$ tons per square inch.

(898.) By judicious mixture a stronger iron was obtained which gave 36 cwt. for the transverse strength of test-bar, and a tensile strength of $13 \cdot 3$ tons per square inch, as in No. 6; and the question was whether the girders would be stronger in the ratio of the transverse strengths 36 to 25, or in that of the tensile strengths $13 \cdot 3$ to $7 \cdot 142$. By the test-bar ratio the girder in the strong iron should break with $3346 \times 36 \div 25 = 4818$ lbs. per square inch of section, but the actual breaking weight was 5309 lbs., hence $4818 \div 5309 = \cdot 908$, showing a *deficit* by the test-bar Ratio of $1 \cdot 0 - \cdot 908 = \cdot 092$, or $9 \cdot 2$ per cent.

Again: by the tensile ratio we obtain $3346 \times 13 \cdot 3 \div 7 \cdot 142$

TABLE **139.**—Of the RELATIONS between the STRENGTH of same IRON in

Reference.	Area of Section in Square Inches.	Breaking Weight in Lbs.		Test-bar. Cwt.	Strain per Square Inch.		Calculated from Test-bars.	
		Total.	Per Sq. In. of Section.		Tensile.	Calculated Crushing.	Lbs. per Sq. In. of Section.	Difference per Cent.
Fig. 197	4·5	14,462	3214	25	7·142	43·0	..	..
"	"	22,400	4977	30	10·25	58·9	3857	−22·5
"	"	22,400	4977	33½	11·75	57·6	4307	−13·5
Fig. 198	5·0	16,730	3346	25	7·142	43·0	..	..
"	"	26,320	5264	30	10·86	50·2	4015	−23·8
"	"	26,544	5309	36	13·30	58·0	4818	− 9·2
Fig. 199	6·4	26,096	4075	25	7·142	43·0	..	..
"	"	31,920	4988	30	10·50	54·3	4890	− 2·0
"	"	40,320	6300	38	13·94	61·4	6194	− 1·7
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

= 6231 lbs. per square inch of section, but the actual breaking weight was 5309 lbs. only, hence 6231 ÷ 5309 = 1·174 or 17·4 per cent. in *excess* by the tensile Ratio.

(899.) The cols. 8, 9, 10, 11 in Table 139 have been calculated in this way, and show that the strength of the girders do not follow precisely or even very nearly the ratio of the transverse strength of the iron as indicated by test-bars, neither does it follow the ratio of the tensile strength of the iron. The test-bars or beams give the best results, and the safest, being always *less* than the experimental strength of the girders, varying from − 1·7 to − 23·8 per cent., as in col. 9. The calculations from the tensile strength give in four cases out of six, errors in excess, in one case to the extent of 26 per cent., as in col. 11. We should have expected that with girders of such a section failure would ensue from the rupture of the bottom flange under tensile strain, and that the strength would be dominated by the tensile rather than by the transverse strength of the iron, but experiment shows that this is not the case.

(900.) With Mr. Stirling's toughened cast iron (938) the mean transverse strength of test-bars showed an increase of

GIRDERS, and the TRANSVERSE and TENSILE STRENGTH of the TEST-BARS.

Calculated from Tensile Strength.		Increase in Strength per Cent.				Authority.
Lbs. per Square Inch of Section.	Difference per Cent.	Tensile.	Crushing.	Test-bar.	Girder.	
..	..	00·0	00	00	00·0	Fairbairn.
4613	− 7·2	43·5	37	20	51·7	Berkley.
5288	+ 6·2	64·5	34	34	51·7	Berkley.
..	..	00·0	00	00	00·0	Fairbairn.
5088	− 3·4	52·0	17	20	57·3	Berkley.
6231	+17·4	86·2	35	44	58·7	Berkley.
..	..	00·0	00	00	00·0	Fairbairn.
5990	+20·1	47·0	26	20	22·4	Berkley.
7931	+26·0	95·1	43	52	54·6	Berkley.
(10)	(11)	(12)	(13)	(14)	(15)	

60 per cent. over ordinary cast iron, while the tensile test gave 74 per cent., as in col. 5 of Table 143; but when cast in large girders of Mr. Hodgkinson's form, the increase was only 36·6 per cent., as shown by Mr. Owen's experiments in Table 68. In that case, therefore, it is evidently unsafe to calculate the strength of the girders from either the transverse or tensile test-bars. This is the more remarkable because it is the reverse of Mr. Berkley's results with ordinary iron, where the transverse test-bars gave too *low* a result, the mean of col. 9 in Table 139 being − 12·1 per cent. But with Stirling's iron the transverse test-bars gave 60 − 36·6 = − 23·4, and the tensile 74 − 36·6 = − 37·4 per cent. too *high* (940).

(901.) It is evident from all this, that the ordinary test-bar, and tensile tests are unsatisfactory: a more reliable test would be given by the use of "Unit" girders, as in (485), that is to say, by making a model girder to a small scale with precisely the same cross-sectional proportions as the full-sized girders, and calculating the latter from that of the model in the manner explained and illustrated in (483).

(902.) In cols. 12, 13, 14, 15 of Table 139, the effect on the

Girder of a given increase in the Tensile, Crushing, and Transverse strengths of the iron is shown more clearly. Thus with Nos. 8, 9, we find that while the tensile strength is increased 47 per cent., the test-bars are 20, and the girders 22·4 per cent. A further increase in the tensile strength of 95·1 per cent., produces 52 per cent. in the test-bars, and 54·6 per cent. in the girders, &c. The crushing strain in col. 7 was *calculated* from the experimental tensile and transverse strengths by the Rule (498), in the absence of direct experiment.

CHAPTER XXIII.

FATIGUE OF MATERIALS.

(903.) "*General Principles.*"—Experiments have shown that when materials are subjected to heavy strains of any kind, they manifest distress in several ways. 1st, where the load is constant by the extensions, deflections, &c., increasing with time. 2nd, where the load is progressively increased, by the deflections &c., increasing in a higher ratio than the strains. 3rd, by taking a "Permanent set" (751). 4th, by eventually breaking or giving way with a strain much below the normal strength of the Material, as the result of long-continued and oft-repeated intermittent strains. These results of over-strain may be aptly expressed by the general term "Fatigue."

This subject may be considered under two heads, 1st, Statical fatigue from a long-continued dead load; 2nd, Dynamic fatigue from a rolling or moving load, which acts more or less with Impact (831).

Statical Fatigue.

(904.) This case may be divided into two branches. 1st, where a heavy and invariable dead load is borne for a long time continuously. 2nd, where the load is intermittent and variable, being alternately and frequently laid on and relieved wholly or partially, but without impact or shock.

"*Constant Load.*"—The effect of constant tensile strains on wrought iron is shown by cols. 4, 4, in Tables, 94, 95; thus, by the latter, with 17·86 tons per square inch, fatigue was manifested by the extensions continuing to increase with *time*, but at a diminishing rate: the 1st hour gave an increase of 14·5 per cent.; the next 15·5 – 14·5 = 1 per cent.; the next 16·3 – 15·5 = 0·8 per cent., &c., up to 7 hours, when it continued constant up to 10 hours. Even with the heavy strain of 22·63 tons per square inch, or about 22·63 ÷ 25·7 = ·88, or 88 per cent. of the breaking weight, the extensions increased only ·08 per cent. in 7 hours, and then remained stationary up to 12 hours. It would seem from this that fatigue manifests itself with moderate strains even more than with heavy ones, which is very remarkable.

(905.) Mr. Fairbairn's experiments on cast-iron bars strained transversely lead to the same conclusion; it was found that in a series of bars subjected to constant dead loads for long periods, those with the *lightest* loads manifested the most distress from fatigue by *increase* in deflections; thus when strained to

62	75	88	100 nearly

per cent. of the ultimate or breaking weights for periods of

$4\frac{3}{4}$	5	5	5

years, the *increase* in deflection from fatigue in those times was

14·1	11·4	8·6	2·8

per cent. respectively; these were the maximum results in each case. It was found that the deflections increased the most considerably during the first weeks and months up to 12 or 15 months, and then became constant or nearly so. One bar loaded up to the very breaking weight for 5 years, had not any greater deflection than it had taken 3 or 4 years before, and Mr. Hodgkinson concludes that it is probably a law with cast iron that the deflection, &c., will go on increasing with time at first until it becomes a certain quantity, beyond which it will no longer increase, but becomes stationary:—We have seen (904) that wrought iron under tensile strains seems to follow a similar law.

(906.) Mr. Fairbairn made similar experiments on the power of cast-iron pillars to sustain long-continued strains :—they were 1 inch diameter, 6 feet long, with rounded ends, were loaded with

4	7	10	13

cwts., which is equivalent to

30	53	75	97

per cent. of the breaking weight, as found by direct experiment on similar pillars. The pillar loaded with 13 cwt. bore the strain for 5 or 6 months and then broke :—the others bore their respective loads for 3 years, and their deflections were then

0·01	0·025	0·409	—

inch. The deflection of the pillar with 10 cwt. was ·230 inch when first taken, and after each successive year it became ·380, ·380, and ·409 inch respectively.

(907.) The general results seem to be :—

1st. That the elasticity is affected by fatigue from a dead load, but up to a certain extent only, and within a limited time; that is to say, the extensions, &c., do not go on increasing with time indefinitely, nor to an unlimited extent, terminated only by fracture; but that both are limited.

2nd. That the *ultimate* strength and deflection are not affected by fatigue, both being the same whether the material is broken suddenly or after a long-sustained and heavy dead load.

3rd. That on emergency materials may be safely strained to a much greater extent than was admitted by Tredgold and other earlier authorities :—they, finding that with loads greater than $\frac{1}{3}$rd of the breaking weight, the extensions, &c., continually increased with a constant load, supposed that this increase would go on indefinitely until rupture ensued, whereas, as we have seen, although it may go on increasing for a long time, even years, it does so in a continually diminishing ratio until in a certain limited time it ceases, or becomes constant.

(908.) "*Variable Load.*"—We have seen that when the load is constant, materials seem to be capable of bearing strains

approximating to the breaking weight for indefinite periods without apparent injury. But where the load is variable, being wholly or partially relieved and laid on again continuously, the case is entirely altered.

This case divides itself into two very different conditions. 1st, where the load is variable, but acts in one direction only as, for instance, with the rods of single-acting pumps; and, 2nd, strains acting alternately in opposite directions, as is the case with double-acting pump-rods, and many of the parts of ordinary steam-engines:—for example, in a piston-rod the strains are alternately tensile on the down-stroke and compressive on the up-stroke, &c.

(909.) "*Load in One Direction only.*"—This case must be subdivided into two different conditions. 1st, "*Intermittent Strains,*" where the load is entirely relieved and laid on again continuously without shock; and, 2nd, "*Differential Strains,*" where the load is intermittent, but is only partially relieved at each stroke.

1st. Wöhler's experiments have shown that where the load is totally relieved each time, the best fibrous wrought iron breaks with tensile strains of 15 to 18 tons per square inch; the mean is 16·5 tons, and as the mean strength for a constant dead load is 25·7 tons, as shown by Table 1, we have the ratio $16 \cdot 5 \div 25 \cdot 7 = \cdot 64$ or $\frac{2}{3}$ nearly.

Soft steel was found by Wöhler to give under similar conditions of entirely relieved strain, from 22·5 to 25 tons per square inch; the mean is 23·7 tons, and as by Table 1 the mean tensile strength of steel for dead loads is 47·8 tons, we have the ratio $23 \cdot 7 \div 47 \cdot 8 = \cdot 5$, which, being less than the ratio for wrought iron, seems to indicate less perfect elasticity, and must be incorrect. We will therefore assume that with totally relieved strains the breaking weight of steel is $\frac{2}{3}$ of the statical breaking weight, or the same ratio as for wrought iron.

(910.) For cast iron we have the experiments of Mr. Hodgkinson and Captain James, the leading results of which are given in Table 140. In James' experiments the beams were deflected by cams revolving from 4 to 7 times per minute; one

of these was a step cam which gently deflected the bar and then suddenly relieved the pressure at each revolution; the other was a common excentric with a notched edge intended to give some vibration to the motion, but this effect would be very slight, and essentially the strains may be regarded as statical ones. The deflections produced were such as would have been given by certain fractions of the breaking weight.

With the step cam, three 3-inch bars bore without apparent injury, each 10,000 deflections with $\frac{1}{3}$rd the breaking weight, one broke with 25,486, and one with 51,538 deflections; another was not broken with 100,000 deflections. This last bar had evidently been strained very nearly up to the breaking point, two similar bars having broken with a smaller number of deflections, and yet its strength was not impaired, as was proved by breaking it afterwards by a dead load. It broke with 2831 lbs. in the centre, the strength of a similar new bar of the same iron was 2834 lbs. Similarly the three bars which bore 10,000 deflections without failing, were subsequently broken with dead loads; the mean breaking weight was 3050 lbs., new and unstrained bars, as we have seen, breaking with 2834 lbs.

(911.) We find from this that with an intermittent load off-and-on without shock and in one direction only, the breaking weight is $\frac{1}{3}$rd of the Statical or dead load, for although four out of the six bars experimented upon were not broken, it is probable that they would all have failed with a greater number of deflections. It should be observed that the highest number of changes in these experiments was very much less than would occur in practice:—for instance, with pump-rods making 25 strokes per minute for 10 hours per day, we evidently have $25 \times 60 \times 10 = 15000$ changes per day. In the course of the years which such rods would be expected to last without breaking, the changes would amount to many millions, and the strength should be adapted to these practical conditions.

Other experiments were made with a load equal to $\frac{1}{2}$ the statical breaking weight variously applied, namely, by the step cam, the rough cam, and a traversing load passing to and fro over the beam, but so slowly as not to produce any shock as a rapidly rolling load would do (831). With one exception the

TABLE 140.—Of the RESISTANCE of BEAMS to FATIGUE, from the EXPERIMENTS of Mr. HODGKINSON and Captain JAMES, R.E.

Sizes of Bars.			Fraction of the		Changes of Load.	Effect.
Depth. Inches.	Breadth Inches.	Length. Feet.	Ultimate Deflection.	Breaking Weight.		
			Cast-iron Bars, subjected to Blows: Hodgkinson.			
3	3	13½	⅓	·488	1,085	Broke.
"	"	"	"	"	4,000	Not broken, 2.
"	"	"	½	·645	3,026	Broke.
"	"	"	"	"	127	Broke.
2	2	9	⅓	·456	4,000	Not broken, 2.
"	"	"	½	·627	3,965	Broke.
"	"	"	"	"	1,282	Broke.
"	"	"	"	"	29	Broke.
"	"	"	⅔	·780	474	Broke.
"	"	"	"	"	127	Broke.
1	1	4½	⅓	·434	4,000	Not broken.
"	"	"	½	·606	4,000	Not broken.
"	"	"	$\frac{7}{12}$	·684	3,700	Broke.
			Cast-iron Bars, Deflected by Rough Cam: James.			
3	3	13½	..	⅓	10,000	Not broken, 3.
"	"	"	..	½	30,000	Not broken.
"	"	"	..	"	28,602	Broke.
			Cast-iron Bars, Deflected by Step Cam: James.			
3	3	13½	..	⅓	10,000	Not broken, 3.
"	"	"	..	"	25,486	Broke.
"	"	"	..	"	51,538	Broke.
"	"	"	..	"	100,000	Not broken.
"	"	"	..	½	900	Broke.
"	"	"	..	"	617	Broke.
"	"	"	..	"	490	Broke.
			Cast-iron Bars, Traversing Load: James.			
3	3	13½	..	½	31,380	Broke.
"	"	"	..	"	8,416	Broke.
"	"	"	..	"	96,000	Not broken.
2	2	13½	..	"	12,918	Broke.
"	"	"	..	"	7,250	Broke.
"	"	"	..	"	7,196	Broke.
(1)	(2)	(3)	(4)	(5)	(6)	

TABLE 140.—Of the RESISTANCE of BEAMS to FATIGUE, from the EXPERIMENTS of Mr. HODGKINSON and Captain JAMES, R.E.—*continued.*

Sizes of Bars.			Fraction of the		Changes of Load.	Effect.
Depth. Inches.	Breadth. Inches.	Length. Feet.	Ultimate Deflection.	Breaking Weight.		
Plate-iron Girder, Intermittent Load: Fairbairn.						
16	4	20	..	$\frac{1}{4}$	596,790	Not broken.
”	”	”	..	$\frac{2}{7}$	403,210	Not broken.
”	”	”	..	$\frac{2}{5}$	5,175	Broke.
Same Beam repaired.						
16	4	20	..	$\frac{1}{4}$	3,150,000	Not broken.
”	”	”	..	$\frac{1}{3}$	313,000	Broke.
(1)	(2)	(3)	(4)	(5)	(6)	

whole of these bars were broken with from 490 to 51,538 changes.

(912.) "*Differential Strains.*"—With partially relieved or Differential strains, the destructive effect, or tendency to break, according to Wöhler's experiments, is proportional to the difference of the maximum and minimum strains. It is stated that a bar of steel with tensile strains varying continuously between the extremes of 40 and 20 tons per square inch, will bear those strains for a certain time, the difference being $40 - 20 =$ tons; but if the load of 40 tons be removed altogether each time, then, although the maximum strain is the same as before, the difference, or *destructive effect*, is $40 - 0 = 40$, and the bar will break.

It was also found that a bar of Steel loaded alternately with 35 and $12\frac{1}{2}$ tons, the difference being $23\frac{1}{2}$ tons, would be strained to about the same extent as by alternating loads of 40 and 20 tons, where the difference is 20 tons.

The effect of Intermittent and Differential Strains may be represented by the Rule:—

(913.) $$W_D = \left\{ W - w) \times R_v \right\} + w.$$

In which W = the maximum, and w the minimum alternating loads; W_D = the equivalent dead load: R_0 = the Ratio of the effect of an Intermittent load, that of the same load acting as a dead weight being $1 \cdot 0$ as given in (909) to (911): for Cast iron $R_0 = 3$, for Wrought iron and Steel $= \frac{3}{2}$. The value of R_0 for other materials is given by the "Ratios" in Table 141: thus for wrought Copper and Brass, Slate, &c., it is = 2, and for Cast Metals generally = 3.

We may now apply Rule (913) to Wöhler's experiments in (912): with 35 and $12\frac{1}{2}$ tons on the Steel bar $W_D = \left\{(35 - 12 \cdot 5) \times \frac{3}{2}\right\} + 12 \cdot 5 = 46 \cdot 25$ tons per square inch, as the equivalent dead load: with 40 and 20 tons, we obtain $W_D = \left\{(40 - 20) \times \frac{3}{2}\right\} + 20 = 50$ tons dead load, being nearly the same as the other, as found by Wöhler. The mean of the two is $(46 \cdot 25 + 50) \div 2 = 48 \cdot 12$ tons per square inch, which is almost precisely the mean tensile strength of Steel, which may be taken at 48 tons: see Table 1. Wöhler found, as we have seen, that if the same maximum load of 40 tons be wholly relieved each time, the bar would break, which it would be very likely to do, for we have then $W_D = \left\{(40 - 0) \times \frac{3}{2}\right\} + 0 = 60$ tons per square inch, equivalent dead load.

(914.) The mean Statical Breaking weight of Steel being 48 tons per square inch tensile strain, then with Factor 3 we have $48 \div 3 = 16$ tons working dead load, which with an intermittent strain becomes $16 \times \frac{2}{3} = 10 \cdot 7$ tons off-and-on. With differential strains; we may find the effect of different loads by Rule (913), observing that it must never exceed W_D, the dead load which the material should bear, or in our case 16 tons per square inch: then

With $10 \cdot 7$ and $0 \cdot 0$ tons, the difference $= 10 \cdot 7$ tons, and

$$W_D = \left\{(10 \cdot 7 - 0 \cdot 0) \times \tfrac{3}{2}\right\} + 0 \cdot 0 = 16 \text{ tons.}$$

With 12·7 and 6·0 tons, the difference = 6·7 tons, and

$$W_D = \Big\{ 12 \cdot 7 - 6 \cdot 0) \times \tfrac{3}{2} \Big\} + 6 \cdot 0 = 16 \text{ tons.}$$

With 14·7 and 12·0 tons, the difference = 2·7 tons, and

$$W_D = \Big\{ 14 \cdot 7 - 12 \cdot 0) \times \tfrac{3}{2} \Big\} + 12 \cdot 0 = 16 \text{ tons.}$$

With 15·7 and 15·0 tons, the difference = 0·7 tons, and

$$W_D = \Big\{ 15 \cdot 7 - 15 \cdot 0) \times \tfrac{3}{2} \Big\} + 15 \cdot 0 = 16 \text{ tons.}$$

With wrought iron the mean Statical Breaking weight = 25·7 tons per square inch Tensile strain by Table 1: with Factor 3 we have 25·7 ÷ 3 = 8·6 tons, which with an Intermittent load becomes $8 \cdot 6 \times \frac{2}{3} = 5 \cdot 7$ tons. With Rule (913) W_D being restricted to 8·6 tons, we obtain:—

With 5·7 and 0·0 tons, the difference = 5·7 tons, and

$$W_D = \Big\{ 5 \cdot 7 - 0 \cdot 0) \times \tfrac{3}{2} \Big\} + 0 \cdot 0 = 8 \cdot 6 \text{ tons.}$$

With 6·7 and 2·9 tons, the difference = 3·8 tons, and

$$W_D = \Big\{ 6 \cdot 7 - 2 \cdot 9) \times \tfrac{3}{2} \Big\} + 2 \cdot 9 = 8 \cdot 6 \text{ tons.}$$

With 7·7 and 5·9 tons, the difference = 1·8 tons, and

$$W_D = \Big\{ 7 \cdot 7 - 5 \cdot 9) \times \tfrac{3}{2} \Big\} + 5 \cdot 9 = 8 \cdot 6 \text{ tons.}$$

With 8·5 and 8·3 tons, the difference = 0·2 tons, and

$$W_D = \Big\{ 8 \cdot 5 - 8 \cdot 3) \times \tfrac{3}{2} \Big\} + 8 \cdot 3 = 8 \cdot 6 \text{ tons.}$$

With Cast iron, the mean Statical *Transverse* strength = 2063 lbs. (335); with Factor 3 we have 2063 ÷ 3 = 688 lbs. working dead load, which with an intermittent strain becomes $688 \times \frac{1}{3} = 229$ lbs. off-and-on. By Rule (913), W_D being restricted to 688 lbs., we obtain

With 229 and 0·0 lbs., the difference = 229 lbs., and

$$W_D = \Big\{ 229 - 0) \times 3 \Big\} + 0 = 688 \text{ lbs.}$$

With 458 and 353 lbs., the difference = 105 lbs., and

$$W_D = \left\{458 - 353) \times 3\right\} + 353 = 688 \text{ lbs.}$$

With 600 and 556 lbs., the difference = 44 lbs., and

$$W_D = \left\{600 - 556) \times 3\right\} + 556 = 688 \text{ lbs.}$$

With 660 and 646 lbs., the difference = 14 lbs., and

$$W_D = \left\{660 - 646) \times 3\right\} + 646 = 688 \text{ lbs.}$$

With 685 and 683·5 lbs., the difference = 1·5 lbs., and

$$W_D = \left\{685 - 683{\cdot}5) \times 3\right\} + 683{\cdot}5 = 688 \text{ lbs.}$$

Cases of differential Strain are very numerous in practice: thus, in a Railway bridge, the weight of the structure itself is the minimum, and that weight plus the weight of the train is the maximum. Again, with long rods to deep-well pumps the maximum strain is the pressure on the bucket due to the head of water added to the weight of the rods, &c.; the minimum strain being the latter alone, &c.

(915.) "*Alternating Strains.*"—When a strain is alternately tensile and compressive, as, for instance, with the piston-rod of a steam-engine, or again, when the transverse strain on a beam acts in both directions, up-and-down, resolving itself eventually into alternating tensile and crushing strains, the destructive action or tendency to break is very severe. Indeed, instinct teaches us that the easiest mode of breaking anything is to bend it to-and-fro repeatedly, a very moderate strain thus exerted sufficing to effect the purpose.

From Wöhler's experiments it appears that the destructive effect of alternate strains in opposite directions is expressed by the *sum* of those strains:—thus 5 tons tensile, alternating with 5 tons compressive strain is, in its tendency to break the material, equivalent to 10 tons acting intermittently or off-and-on in one direction only.

With wrought iron we found in (914) that the safe intermittent tensile strain was 5·7 tons per square inch; the com-

pressive strain would be the same, or $5 \cdot 7$ tons also, because the tensile and crushing strengths of wrought iron are equal to one another (377). With equal strains, alternately tensile and compressive, we should have therefore $5 \cdot 7 \div 2 = 2 \cdot 85$ tons each way, which is $\frac{1}{8}$th of the statical breaking weight, and is equivalent to $2 \cdot 85 + 2 \cdot 85 = 5 \cdot 7$ tons off-and-on in one direction only.

(916.) But with many materials the resistance to these two strains is very unequal, as shown by Table 79, and this fact complicates the question considerably. For instance, with cast iron, the tensile breaking strain is $7 \cdot 14$ tons; Factor 3 gives $7 \cdot 14 \div 3 = 2 \cdot 28$ tons statical safe load, which for an intermittent strain is reduced to $2 \cdot 28 \times \frac{1}{3} = \cdot 76$ ton, and for an alternating strain, to $\cdot 76 \times \frac{1}{2} = \cdot 38$ ton, which is $\frac{1}{18}$ of the statical breaking weight.

But the crushing strength of cast iron is 43 tons, hence the statical safe load becomes $43 \div 3 = 14 \cdot 3$; the safe intermittent strain $14 \cdot 3 \times \frac{1}{3} = 4 \cdot 8$ tons; and the alternating or crushing and tensile strains $4 \cdot 8 \times \frac{1}{2} = 2 \cdot 4$ tons per square inch. We have just seen, however, that the alternating tensile strain does not exceed $\cdot 38$ ton or about $\frac{1}{6}$th of the same kind of compressive strain, and we find from this, that where, in any material, the power of resistance to these two strains is unequal, the case is governed, and the alternating strain limited by the strength of the weaker.

(917.) We have here supposed that the tensile and compressive *loads* are equal to one another, which is usually the case in practice; for instance, in a piston-rod or a double-acting pump-rod the two loads, or those during the up-stroke and the down-stroke, are practically equal. But in some exceptional cases they may be so unequal as to enable us to utilise the whole strength of the material, or that in both directions; for instance, with cast iron, a compressive strain of $2 \cdot 4$ tons per square inch might be arranged so as to alternate with a tensile strain of $\cdot 38$ ton. But in most cases the two loads are of necessity equal to one another, and in the case of cast iron, $\cdot 38$ ton would become the working tensile *and* compressive strain.

(918.) With the transverse and torsional strains the com-

plication which arises from unequal tensile and compressive strength in the material is eliminated, for although the former strains resolve themselves eventually into the tensile and compressive, the inequalities of strength adjust themselves to one another, and the transverse strength is the mean effect of the two combined.

An alternating transverse strain is very common in practice; the beam of an ordinary steam-engine, and the lever working a double-acting pump are familiar instances. Another case is that of an over-hung axle heavily loaded at the end, as in Fig. 204: if the axle were stationary it would simply be deflected from A, its normal position, to B. If while thus strained the axle could make half a revolution, it would evidently be carried round to the position C, but the weight W continuing to act, it is retained in the position B. Thus at every revolution, the shaft is *in effect* strained *both ways*, or up and down, namely, from B to C, and, as shown by Wöhler's experiments (915), the destructive effect is proportional, not to A, B, only, but to B, C.

The strain in this case is peculiar, 1st, although intermittent and alternate, or in both directions, it is effected entirely without shock: and, 2nd, the axle is strained not only in two directions, or up and down, but in *all directions* equally, which would probably be more destructive than an equivalent strain in two opposite directions only.

DYNAMIC FATIGUE.

(919.) The philosophy of dynamic fatigue may be easily explained: let Fig. 206 represent an unloaded beam A, deflected 8 inches or to the position B, by a strain or weight of 8 lbs., being 1 inch per lb. Now, as shown in (775), the mean strain is $(8 + 0) \div 2 = 4$ lbs., which acting through 8 inches gives as the *mechanical power* producing the deflection $4 \times 8 = 32$ inch-lbs., therefore a weight D of 1 lb. falling 32 inches from E to B would deflect the beam to B as before—neglecting inertia (781).

If the elasticity of the material were perfect, the bar would sustain any number of similar blows without injury or increase

of deflection. But it is shown in (751) that when a beam (of cast iron more particularly) is deflected by a transverse strain, it never returns to its primitive form on the relief of the strain, but takes a permanent set. Now, let us suppose that in our case, a permanent set of 1 inch occurs, so that when, unloaded, the beam returns, not to A, but to G; if then again loaded with the same weight of 8 lbs. quietly laid on, the bar would deflect to B as before, and the mean strain would also be $(0+8) \div 2 = 4$ lbs. as before, but as the bar now deflects with that load $8 - 1 = 7$ inches, we should have $4 \times 7 = 28$ inch-lbs. only, which would not absorb the whole force of the second blow by $32 - 28 = 4$ inch-lbs. The beam would therefore be deflected below the point B by about $0 \cdot 52$ inch, or to $8 \cdot 52$ inches below A: then the mean strain during the second blow would be $(0 + 8 \cdot 52) \div 2 = 4 \cdot 26$ lbs., which acting through $8 \cdot 52 - 1 = 7 \cdot 52$ inches will give $4 \cdot 26 \times 7 \cdot 52 = 32$ inch-lbs., as required.

We have seen in (752) that the permanent set is nearly proportional to the square of the strain; if, therefore, the deflection of 8 inches gave 1 inch set, $8 \cdot 52$ inches would give $1 \times 8 \cdot 52^2 \div 8^2 = 1 \cdot 134$ inch set. The third blow will therefore deflect the beam still lower than before, say to $8 \cdot 58$ inches, or the point due to $8 \cdot 58$ lbs. dead load, the *mean* strain will then be $(0 + 8 \cdot 58) \div 2 = 4 \cdot 29$ lbs., which acting through $8 \cdot 58 - 1 \cdot 134 = 7 \cdot 446$ inches, will give a resistance of $4 \cdot 29 \times 7 \cdot 446 =$ 32 inch-lbs. nearly, as required.

The permanent set due to $8 \cdot 58$ inches deflection will be $1 \times 8 \cdot 58^2 \div 8^2 = 1 \cdot 15$ inch: the fourth blow will therefore deflect the bar to say $8 \cdot 6$ inches, the mean strain being then $(0 + 8 \cdot 6) \div 2 = 4 \cdot 3$ lbs., which acting through $8 \cdot 6 - 1 \cdot 15 = 7 \cdot 45$ inches, will give a resistance of $4 \cdot 3 \times 7 \cdot 45 = 32$ inch-lbs., as required.

Thus with four successive and equal blows we have deflections and strains of $8 \cdot 0$, $8 \cdot 52$, $8 \cdot 58$, and $8 \cdot 60$ inches respectively, increasing with every blow, but in a rapidly diminishing ratio, namely, $\cdot 52$, $\cdot 06$, and $\cdot 02$ inch respectively. Evidently, although the first blow may have strained the bar to a small fraction only of the breaking weight, a succession of similar

blows would eventually break it: in fact, it becomes a question of number of blows as much as of amount of load.

The extreme slowness with which the deflection under successive blows increases, shows that the number of blows necessary to produce fracture may be very great, extending possibly to millions and occupying years (911). This will help to explain the well-known fact that parts of machinery (such as a pump-rod) often fail with a strain which is a small fraction only of the normal breaking weight, and one, moreover, which had been borne successfully for many years. Thus, the fact that a given dynamic strain has been sustained for a lengthened period, is no guarantee that it will be borne for ever.

(920.) Where a load literally falls upon a beam, as in Fig. 186, its effect must be calculated by the laws of Impact (789), as illustrated in (805). Mr. Hodgkinson made an extensive series of experiments on the power of impact with cast and wrought bars; many of his results are given by Tables 121, 122, 140, &c. The experiments in Table 140 are liable to be misunderstood; the bars were subjected not to given fractions of the breaking weights, which is the usual and most convenient course, but to strains producing given fractions of the ultimate deflection (696), which is quite a different thing with such an imperfectly elastic material as cast iron.

From the Diagram, Fig. 219, in which the deflections up to the breaking weights are shown graphically, we find that bars 1, 2, and 3 inches square are deflected to $\frac{1}{3}$rd of the ultimate deflection by ·434, ·456, and ·488 of the respective breaking weights, whereas, of course, with perfect elasticity we should have had $\frac{1}{3}$ or ·333 in all cases. We have thus obtained col. 5 in Table 140.

(921.) The Table shows that with 1-inch bars, 4000 blows, deflecting the bar to $\frac{1}{3}$ of ultimate deflection, due by col. 5 to ·434 of the breaking weight, failed to break the bar; another bar was not broken with 4000 blows, deflecting it to $\frac{1}{2}$ the ultimate deflection, due to ·606 of the breaking weight; another broke with 3700 blows, deflecting it to $\frac{7}{12}$ the ultimate deflection, due to ·684 of the breaking weight.

With 2-inch bars, two bore 4000 blows, deflecting them to $\frac{1}{3}$

the ultimate deflection, due to ·456 of the breaking weight:— three others were all broken when deflected to $\frac{1}{2}$ the ultimate deflection, due to ·627 of the breaking weight, with 29, 1282, and 3695 blows respectively. Two others with $\frac{2}{3}$ of the ultimate deflection, due to ·78 of the breaking statical weight, broke with 127 and 474 blows respectively.

With 3-inch bars deflected to $\frac{1}{3}$ the ultimate deflection, due to ·488 of the breaking weight, one broke with 1085 blows; two others were not broken with 4000 blows each. Two others, with $\frac{1}{2}$ the ultimate deflection, due to ·645 of the breaking weight, broke with 127 and 3026 blows respectively.

These bars appear to have been stronger in resisting impact than those experimented upon by Captain James (910), especially considering the nature of the strains; we should have expected that a given deflection produced by a *blow*, as in Mr. Hodgkinson's experiments, would have been more destructive than the same deflection produced gently by a cam, as in James' experiments. But, in the former, the number of impacts was not carried far enough to exhibit fully the effect of fatigue; one of the bars, however, did fail with $\frac{1}{3}$ of the ultimate deflection, and with 1085 blows; possibly the others would have failed also with a greater number of impacts, such as would occur in practice (911), (919).

(922.) We will therefore retain for cast iron the ratio of the breaking intermittent load at $\frac{1}{3}$ of the statical breaking weight, as found from Captain James' experiments (911); the ratio for Steel and Wrought iron being $\frac{2}{3}$, as in (909).

These probably represent the extremes, steel and wrought iron having the most perfect elasticity of all the materials used in the arts, and cast iron the least perfect. We have no experimental information for other materials, but supposing them to occupy an intermediate position, we may admit for them the ratio for an intermittent load to be $\frac{1}{2}$ the Statical, which will apply to Timber, wrought metals, such as Copper, Brass, &c., also to Slate, York-paving, and other kinds of Stone, &c.

For cast metals, such as Copper, Brass, Lead, &c., we may adopt the ratio $\frac{1}{3}$ as for cast iron.

(923.) "*Fatigue from Rolling Load.*"—The effect of a rolling

load in straining a beam is shown in (832), &c., to depend on the horizontal velocity; at *very* low speeds the effect is similar to the action of a cam, which quietly deflects the beam; but as the velocity rises the deflection increases until, at a certain velocity varying with the span of the beam and the elasticity of the material, the deflection becomes *double* that due to the same load acting statically, or as a dead load. We may therefore admit that a rolling load should not exceed $\frac{1}{2}$ the statical or dead load under otherwise similar conditions, and this ratio may be applied for all ordinary cases.

Thus, by Table 66, the Transverse strength of wrought iron is 4000 lbs. breaking-down dead load; with Factor 3 we have $4000 \div 3 = 1333$ lbs. safe dead working load. Therefore, $1333 \times \frac{2}{3} = 888$ lbs. intermittent dead working load, and $888 \times \frac{1}{2} = 444$ lbs. rolling or dynamic working load. This last is $\frac{1}{9}$th of 4000 lbs., the Statical Breaking-down load. Again, with Cast iron, the transverse strength for dead load is 2063 lbs.; with Factor 3 we have $2063 \div 3 = 688$ lbs. safe dead load, $688 \times \frac{1}{3} = 229$ lbs. intermittent load, and $229 \times \frac{1}{2} = 115$ lbs. rolling load, which is $\frac{1}{18}$th of the statical breaking weight. Again, with English Oak, the transverse strength for dead load is 509 lbs. breaking weight; with Factor 5 we obtain $509 \div 5 = 102$ lbs. safe dead load, then $102 \times \frac{1}{2} = 51$ lbs. intermittent load, and finally $51 \times \frac{1}{2} = 26$ lbs. rolling load, which is $\frac{1}{20}$th of the statical breaking weight.

(924.) In very many cases the strains on the different parts of machinery are not strictly rolling loads, but acting with a certain amount of shock they may be taken as similar in their action to rolling loads, this being in many cases the best approximation that can be made. Thus, with the rods of single-acting pumps, worked by a 3-throw crank, there is a certain amount of shock in passing the centres, and we may take it as doubling the strain in the same way as a rolling load would act on, say, the vertical rods of a suspension bridge. Then, taking the tensile strength of *welded* joints, as in our case, at 21 tons per square inch (see Table 1), we have with Factor 3, $21 \div 3 = 7$ tons safe dead load, $7 \times \frac{2}{3} = 4 \cdot 67$ tons intermittent dead load; and finally $4 \cdot 67 \times \frac{1}{2} = 2 \cdot 33$ tons,

or 5220 lbs. per square inch, dynamic or rolling load. This agrees with practice, as shown by col. 10 of Table 30, which shows that rods with 5070 lbs. stand their work, but others with 6600 lbs. fail repeatedly. Those parts of pump-rods which work through the glands or stuffing-boxes are commonly made of wrought copper, principally to avoid rust, to which iron rods would be liable in case of stoppage for a few days. Wrought copper breaks with a tensile strain of 15 tons per square inch; then with Factor 3 we have $15 \div 3 = 5$ tons safe dead load; $5 \times \frac{1}{2} = 2 \cdot 5$ tons intermittent load; and $2 \cdot 5 \times \frac{1}{2} = 1 \cdot 25$ tons, or 2800 lbs. per square inch dynamic load; and this, it should be observed, is the strain at the reduced section, or where the area is reduced by the key or screw-thread (210). Table 29 shows that in practice copper rods are much more heavily loaded than we have thus calculated, some having as much as 5670 lbs. per inch, &c.; but, as a matter of fact, these rods are frequently breaking, and in some cases duplicate or spare rods are kept on hand ready for that contingency.

(925.) But, wherever possible, the constant, or Factor of Safety, for any particular machinery should be obtained direct from cases working well in practice: the whole matter of the strains in machinery is so complicated and obscure that no other course is likely to be perfectly safe and satisfactory. It is only in those cases where no direct data are attainable that the theoretical methods we have explained and illustrated should be used.

(926.) When a dynamic strain acts in both directions, or is an alternated strain, we must apply the ratio for that circumstance, as explained in (915). Thus, taking the cases in (923), wrought iron becomes $444 \times \frac{1}{2} = 222$ lbs., or $\frac{1}{18}$th of the statical breaking weight; cast iron becomes $115 \times \frac{1}{2} = 57$ lbs., or $\frac{1}{36}$th of the statical breaking weight; and Oak is reduced to $26 \times \frac{1}{2} = 13$ lbs., which is $\frac{1}{46}$th of the statical breaking weight, &c.

(927.) "*Fatigue of Plate-iron Beams.*"—Mr. Fairbairn made some valuable experiments showing the effect of fatigue from oft-repeated strains on a riveted plate-iron beam, Fig. 133, the

load being completely relieved and laid on again about 8 times per minute by a crank-arrangement. To imitate as nearly as possible the strain to which Railway bridges are subjected by the passage of heavy trains, the apparatus was designed to lower the load quickly, and to produce a considerable amount of vibration, as the large lever with its load was left suspended on the beam at each stroke. The beam was 16 inches deep, and 20 feet between supports; Table 140 gives a compendium of the experimental results.

By (909) with an intermittent dead load, the breaking weight of wrought iron is $\frac{2}{3}$rds of the breaking dead weight; and by (923) half that amount, or $\frac{1}{3}$rd where the load rolls over the beam at a certain velocity, or where it acts in a manner analogous to a rolling load, which is our case; accordingly Mr. Fairbairn found that with $\frac{1}{3}$rd of the statical breaking weight, the beam broke with 313,000 changes of load.

This beam bore first 596,790 changes of $\frac{1}{4}$ the statical breaking weight without apparent injury, or manifesting distress by increasing in deflection, which remained practically the same throughout, namely ·16 or ·17 inch. It was then loaded with $\frac{2}{7}$ or $\frac{1}{3\cdot5}$ of the breaking weight and bore 403,210 changes without distress, the deflection remaining constant throughout at ·22 or ·23 inch. The load was then increased to $\frac{2}{5}$ or $\frac{1}{2\cdot5}$ of the breaking statical weight, and the beam broke with 5175 changes.

The beam was then thoroughly repaired, and with $\frac{1}{4}$th the breaking weight bore without apparent injury 3,150,000 changes, the deflection remaining constant throughout at ·17 or ·18 inch, and the permanent set at ·01 inch. The load was then increased to $\frac{1}{3}$rd of the statical breaking weight, with which the beam broke after 313,000 changes, but without manifesting distress by increase of deflection, which remained constantly throughout at ·2 inch.

(928.) Mr. Fairbairn concludes from these experiments that with $\frac{1}{3}$rd of the statical breaking weight, Railway bridges would be decidedly unsafe, but that with $\frac{1}{4}$th of that weight, a wrought-iron bridge would be perfectly safe for a great number of years. Nevertheless, he allows in practice a larger margin

for safety, namely, $\frac{1}{5}$ or $\frac{1}{6}$ of the breaking weight; many of our leading Railway Engineers, such as R. Stephenson, J. Cubitt, P. W. Barlow, &c., adopt $\frac{1}{6}$th as the ratio in practice, as shown by (892) and Table 138.

We have shown in (840) that in Railway bridges, the velocity is never high enough to give anything approaching to the maximum effect, or to produce a deflection double of that due to the same load acting statically; moreover, the inertia of the bridge itself gives a considerable resistance. The combined effect of these two circumstances is, that the strain is very little greater than that due to a dead load, which is proved to be the fact by the experiments on the Ewell and other bridges (839). The strains on Railway bridges may therefore practically be regarded as dead loads: they are, however, intermittent, and for wrought iron should be $\frac{2}{3}$, and for cast iron $\frac{1}{3}$ of the equivalent constant dead loads. Then with Factor 3 as given by Table 137 we finally obtain for wrought-iron Girders the Ratio $\frac{2}{3} \div 3 = \cdot 222$ or $\frac{1}{4\cdot 5}$, hence the working Factor of Safety $= 4\cdot 5$: for Cast-iron girders we obtain $\frac{1}{3} \div 3 = \frac{1}{9}$ of the statical Breaking weight, the Factor being $= 9$. As we have seen (928), the leading Railway Engineers, making no distinction between cast and wrought iron, have adopted the ratio $\frac{1}{6}$ or Factor 6, which is intermediate between $4\cdot 5$ and 9.

(929.) Collecting these results and applying them to the three great Strains, namely, the Tensile, Crushing, and Transverse strains, we obtain for various Materials, the series of equivalent strains for varying conditions of loading given by Table 141, combining which with the Ratios or Factors of Safety in Table 137, we may find the proper load under all ordinary conditions. As the matter is essentially and necessarily a complicated one, we may give examples which will help to make it more clear.

Say, we have a single-acting pump, 18 inches diameter, 150 feet head of water, &c.: the strain being Intermittent and Dynamic, col. 3 of Table 141 gives $8\cdot 6$ tons per square inch Breaking weight, and Table 137 gives the Factor of Safety $= 3$: hence we obtain $8\cdot 6 \div 3 = 2\cdot 87$ tons Working load. Then 18 inches diameter $= 254$ square inches area, and the pressure due to the water $= 150 \div 2\cdot 3 = 65$ lbs. per square

inch, giving $254 \times 65 \div 2240 = 7 \cdot 4$ tons, requiring $7 \cdot 4 \div 2 \cdot 87 = 2 \cdot 6$ square inches area at the *key-way*, equivalent by (210) to $2 \cdot 6 \times 2 = 5 \cdot 2$ square inches of the body of the rod, or $2\frac{5}{8}$ inches diameter where it works through the gland of the pump. With a deep-well pump, there would be long rods to the surface with welds here and there: by Table 1 the strength of *welded* joints = 47266 lbs., or 21 tons per square inch Breaking weight, which by the "Ratio" in col. 3 of Table 141 is reduced to $21 \times \frac{1}{3} = 7$ tons Dynamic strain, and with Factor 3, to $7 \div 3 = 2 \cdot 33$ tons Working strain. Hence we obtain $7 \cdot 4 \div 2 \cdot 33 = 3 \cdot 2$ square inches, or 2 inches diameter: Table 28 gives practically the same result: the Working load for 2-inch rods being 15,708 lbs., or 7 tons, being nearly 7·4 tons, the strain in our case.

If this same pump had been double-acting, the strain being alternately Tensile and compressive, col. 5 of Table 141 gives 3·2 tons per square inch Breaking weight, or $3 \cdot 2 \div 3 = 1 \cdot 07$ ton working load, requiring $7 \cdot 4 \div 1 \cdot 07 = 7$ square inches area, or 3 inches diameter of the body of the rod. Here the key-way question is eliminated, being covered by the large diameter due with an alternating strain (210).

(930.) Again: say that we have a short pillar of English Oak subjected to an intermittent load of 10 tons acting without shock: then, col. 2 of Table 141 gives 1·85 tons per square inch Breaking weight, Table 137 gives the Factor = 5, hence we obtain $1 \cdot 85 \div 5 = \cdot 37$ ton per square inch working load, and require $10 \div \cdot 37 = 27$ square inches area, say $5\frac{1}{4}$ inches square. If this Oaken rod had worked a Double-acting pump, the strain being both Alternating and Dynamic, col. 5 of Table 141 gives 0·46 ton Breaking weight, or $\cdot 46 \div 5 = \cdot 092$ ton working load, which is $\frac{1}{8} \div 5 = \frac{1}{40}$th of 3·7 tons, the *Crushing* dead load by col. 1: we then require $10 \div \cdot 092 = 109$ square inches area, or $10\frac{1}{2}$ inches square, &c.

Again: say we have a rocking-beam working a double-acting pump, 12 inches diameter, 100 feet head of water $= 100 \div 2 \cdot 3 = 44$ lbs. per square inch: the area of 12 = 113 square inches, hence $113 \times 44 \div 2240 = 2 \cdot 2$ tons, which being a Dynamic and Alternating load is by the "Ratio" in col. 5 of Table 141, equivalent to $2 \cdot 2 \times 6 = 13 \cdot 2$ tons dead load: wrought-iron.

TABLE 141.—Of the STRENGTH of MATERIALS with DIFFERENT KINDS of STRAIN: being the Ultimate or Breaking Loads.

Materials.		Dead Load.	Intermittent Load Off-and-on continuously.			
			One Direction.		Both Directions.	
			Without Shock.	Dynamic or Rolling.	Without Shock.	Dynamic or Rolling.
Wrought Iron and Steel	Ratios	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$
Wrought Copper and Brass, Slate, Timber, &c.	"	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$
Cast Metals: Iron, Copper, Brass, Lead, &c.	"	1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$
		Tensile Strains: Tons per Square Inch.				
Steel: ordinary, bar		48	32	16	16	8
Wrought Iron: rolled bar ..		25·7	17·1	8·6	6·3*	3·2*
" " " (limit of Elasticity)		12·8	8·5	4·3	4·3	2·2
Cast Iron		7·1	2·4	1·2	1·2	0·6
Copper, bolts		16	8	4	4	2·0
" cast		9	3	1·5	1·5	0·75
Gun-metal, cast..		14	4·7	2·3	2·3	1·16
Brass, yellow, cast		8	2·67	1·33	1·33	0·67
Lead, drawn pipe		1·12	0·374	0·187	0·187	0·094
Ash		7·4	3·7	1·85	1·00*	0·50*
Oak, English		5·5	2·75	1·37	0·92*	0·46*
Larch		4·2	2·1	1·05	0·50*	0·25*
		Crushing Strains: Tons per Square Inch.				
Steel, in pillars		52	34·7	17·3	16*	8*
Wrought Iron, in pillars		19	12·6	6·3	6·3	3·15
" " " (limit of Elasticity)		12·8	8·5	4·3	4·3	2·15
Cast Iron		43	14·3	7·16	1·2*	0·60*
Gun-metal, cast		15	5	2·5	2·3*	1·16*
Ash		4	2	1	1	0·50
Oak, English		3·7	1·85	0·92	0·92	0·46
Larch		2	1	0·5	0·5	0·25
		(1)	(2)	(3)	(4)	(5)

NOTE.—In alternating strains, the loads marked * are limited by the strength in the opposite direction.

TABLE 141.—Of the STRENGTH OF MATERIALS with DIFFERENT KINDS of STRAIN, &c.—*continued.*

Materials.	Dead Load.	Intermittent Load Off-and-on continuously. One Direction. Without Shock.	One Direction. Dynamic, or Rolling.	Both Directions. Without Shock.	Both Directions. Dynamic, or Rolling.
	Transverse Strains in Lbs., on Bars 1 Inch Square, 1 Foot Long.				
Steel, ordinary bar	6720	4480	2240	2240	1120
" (limit of Elasticity)	5600	3733	1867	1867	933
" (working load) ..	3360	2240	1120	1120	560
Wrought Iron, plain bars ..	4000	2667	1333	1333	667
" " (limit of Elasticity)	2000	1333	667	667	333
" plain bars (working load) ..	1330	888	444	444	222
" T and I bars ..	3200	2132	1066	1066	533
" " (limit of Elasticity)	1500	1000	500	500	250
" T and I bars (working load) ..	1120	747	374	374	187
Cast Iron	2063	688	344	344	172
Gun-metal, cast	1830	610	305	305	153
Slate, Bangor, split	421	210	105	105	53
York Paving	73	36	18	18	9
Ash	680	340	170	170	85
Oak, English	509	255	128	128	64
Larch	380	190	95	95	48
	(1)	(2)	(3)	(4)	(5)

Say that the beam is a cantilever 5 feet long, equal by (431) to a beam $5 \times 4 = 20$ feet long, supported at both ends: then with $M_T = 4000$ lbs., or $1 \cdot 8$ ton, and assuming the thickness or $B = 1\frac{1}{2}$ inch, we may find the depth D by Rule (325) or $D = \sqrt{(13 \cdot 2 \times 20) \div (1 \cdot 8 \times 1\frac{1}{2})} = 9\frac{7}{8}$ inches deep. In this case the working load $= \frac{1}{6} \div 3 = \frac{1}{18}$ of the dead load. Taking the value of $M_T = 667 \div 2240 = \cdot 3$ ton, from col. 5 of Table 141, the Rule (324) gives $W = 9\frac{7}{8}^2 \times 1\frac{1}{2} \times \cdot 3 \div 20 =$ $2 \cdot 2$ tons breaking dead load, as before.

APPENDIX.

EFFECT OF SIZE OF CASTING ON THE STRENGTH OF CAST IRON.

(931.) The transverse strength of cast iron has usually been determined by experiments on bars 1 inch square, and it was supposed that the data thus obtained were applicable without correction to bars of all sizes and to girders of all forms of section.

More recent observations have shown that these conclusions were not correct, and that, 1st, bars of large sizes are specifically weaker than small ones: 2nd, that in bars of rectangular section the strength is governed by the thickness or least dimension rather than by the greater.

(932.) "*Effect of Thickness on Transverse Strength.*"—All the properties of cast iron seem to be more or less affected by the size or rather by the least thickness of the casting; so far as we have experimental knowledge, that is to say between 1 inch and 3 inches square, the tensile, crushing, and transverse strengths, also the Modulus of Elasticity (738) are reduced as the size of the casting is increased.

Table 142 gives reduced results of experiments by Mr. Hodgkinson and Captain James, R.E.; the latter are more numerous and more consistent among themselves than those of Mr. Hodgkinson, which are anomalous, giving the same specific strength for 2-inch as for 3-inch bars; they give, however, one important fact, that in a rectangular bar of unequal dimensions, namely $3 \times 1\frac{1}{2}$, the transverse strength is practically the same as that of a bar $1\frac{1}{2}$ inch square, as shown by the Table, where the value of M_T, or the specific transverse strength, is nearly an arithmetical mean between the strength of 1-inch and 2-inch square bars. This experiment seems to show that the larger dimension of rectangular bars, has no sensible influence on the

TABLE 142.—SHOWING the EFFECT of SIZE of CASTING on the TRANSVERSE STRENGTH of CAST IRON.

No. of Ex-periments.	Kind of Iron.	Sizes of Bars.			Mean Breaking Weight.	Value of M_T.			Authority.
		d.	b.	L.		Observed.	Mean.	Ratios.	
		inches.	inches.	feet.	lbs.				
3	No. 3 Clyde	1	1	2¼	1136	2556	..	..	James, R.E.
3	”	1	1	4½	567	2552	2554	1·0000	”
7	”	2	2	4½	3586	2017	..	..	”
3	”	2	2	9	1842	2072	..	..	”
3	”	2	2	13½	1066	1798	1962	·7682	”
8	”	3	3	6¾	6732	1683	..	..	”
3	”	3	3	13½	3217	1608	1646	·6445	”
3	No. 2 Blaenavon	1	1	4½	556	2558	..	1·0000	”
3	”	2	2	9	1507	1695	..	·6626	”
6	”	3	3	13½	2834	1418	..	·5543	”
3	Llynvi	1	1	4½	483	2174	..	1·0000	”
3	”	2	2	9	1400	1575	..	·7245	”
3	”	3	3	13½	2868	1434	..	·6596	”
4	No. 2 Blaenavon	1	1	4½	447	2012	..	1·0000	Hodgkinson.
4	”	1½	3	13½	819	1638	..	·8141	”
6	”	2	2	9	1274	1433	..	·7122	”
4	”	3	3	13½	2865	1432	..	·7122	”
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	

strength; nearly the same result was found to prevail with the Modulus of Elasticity (744), bars $6 \times 1\frac{1}{2}$ and $3\frac{1}{2} \times 1\frac{1}{2}$ being nearly alike, and also about the same as a bar $1\frac{1}{2}$ inch square, occupying in the Diagram, Fig. 214, a position intermediate between 1-inch, and 2-inch square bars.

(933.) Captain James' experiments in Table 142 were 51 in number, they were made on three different kinds of iron, and the mean combined result of the whole series is that bars 1, 2, and 3 inches square, have transverse strengths in the ratios 1·0, ·7184, and ·6195. Mr. Hodgkinson's ratios are 1·0, ·7122, and ·7122 respectively; and for $1\frac{1}{2} \times 3$ inch bar, ·8141.

The combined results of the experiments are represented approximately by the Rule:—

(934.) $$z = 1 \div \sqrt[2\cdot3]{t}.$$

In which t is the least dimension of a rectangular bar, and z the ratio of the transverse strength, that of 1-inch square bar being = 1·0. Thus, for 3 inches thick, the logarithm of 3, or ·477121 ÷ 2·3 = ·2074, the natural number due to which is 1·612; then 1 ÷ 1·612 = ·62 = z, or the ratio of the strength, that with 1 inch thick being 1·0: for thicknesses of:—

$\frac{1}{2}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3

inches, the rule gives as the ratios of transverse strengths

1·35	1·0	·907	·838	·783	·739	·671	·62

respectively, experiment giving

..	1·0	..	·8141	..	·7184	..	·6195

respectively. In col. 7 of Table 18 this rule is applied to extreme thicknesses, far beyond the limits of experiment, and the results may be more or less inaccurate; but the practical Engineer has often to deal per-force with extreme cases whether he has reliable data or not, and it is better under such circumstances to have a Rule based on inadequate experiments than to have no rule at all (86).

The results of experiment are represented graphically by

Fig. 212, the line F being found by the Rule, and the means of Mr. Hodgkinson and Captain James' experiments are added for comparison.

It is shown in (942) that Stirling's toughened cast iron, which is a mixture of cast and wrought iron, is affected by the thickness very similarly to ordinary cast iron: thicknesses of 1·0, $1\frac{1}{2}$, and 2 inches giving specific transverse strengths of 1·0, ·824, and ·784 respectively.

(935.) "*Effect of Thickness on the Tensile and Crushing Strength.*"—We have no direct experiments on this subject, but Captain James made some instructive experiments on bars $\frac{3}{4}$ inch square planed out of the centre of 3-inch and 2-inch bars of No. 3 Clyde iron, which were compared with other $\frac{3}{4}$-inch bars, cast of that size from the same iron, and from these we may perhaps obtain by analysis the tensile and crushing strains with approximate accuracy.

The $\frac{3}{4}$ bars from the 3-inch ones gave anomalous results, we will therefore take those from the centre of the 2-inch ones: the length being $4\frac{1}{2}$ feet, the transverse breaking weight by experiment was 134 lbs. for the $\frac{3}{4}$-inch planed bars, and 193 lbs. for the cast bars, the ratio being $134 \div 193 = \cdot 694$ to 1·0.

The crushing strength in the planed-out bars was by experiment = 60233 lbs. per square inch, whereas Mr. Hodgkinson's experiments on ordinary No. 3 Clyde iron gave 106,039 lbs.; the Ratio for the Crushing strengths is therefore $60233 \div 106039 = \cdot 568$, or 56·8 per cent.

The Tensile strength of the iron in the centre of the 2-inch bars was not observed, but we may calculate it by the Rule (499) from the known values of C and W.

$$T = \frac{134 \times 4{\cdot}5 \times 4{\cdot}5}{\left\{\cdot 75 - \sqrt{134 \times 4{\cdot}5 \times 4{\cdot}5 \div (60233 \times \cdot 75}\right\}^2 \times \cdot 75}$$

= 14200 lbs. per square inch. Mr. Hodgkinson's direct experiments on ordinary No. 3 Clyde iron gave 23,468 lbs. per square inch; hence the ratio is $14200 \div 23468 = \cdot 605$, or 60·5 per cent.

(936.) Thus for the three strains, Transverse, Tensile, and Crushing, the ratios of strengths are ·694, ·605, and ·568 respectively in $\frac{3}{4}$ bars out of the centre of 2-inch ones, ordinary iron in bars cast about 1 inch square being 1·0 in all cases.

Obviously, this does not give the strengths for the whole area of the 2-inch bars which we desire to find, but those of the centre only, but if we admit that the outside of the 2-inch bar has the ordinary strength, and the central part that found for the $\frac{3}{4}$-inch planed bars, the mean of the two gives for the transverse strength $(1 + ·694) \div 2 = ·847$; for the tensile $(1 + ·605) \div 2 = ·803$; and for the crushing $(1 + ·568) \div 2 = ·784$. These ratios differ little among themselves, the extreme difference being $·847 \div ·784 = 1·08$, or 8 per cent.: on the whole it will be preferable to take the Ratios given by the Rule (934) and col. 7 of Table 18 as correct for *all* the strains.

(937.) "*Effect of Thickness on Girders.*"—An important question arising out of this inquiry is, how far these Ratios will apply to large girders of ordinary sections, that is to say, if we had three girders of the same form of section, but 1, 2, and 3 inches thick all over respectively, whether the strength would be governed by the respective transverse strengths of rectangular bars 1, 2, and 3 inches square respectively, namely, 1·0, ·72, and ·62.

Mr. Hodgkinson gives an experiment on a large girder, Fig. 187, 27 feet 5 inches between bearings, which broke with 76·6 tons in the centre: the general thickness was about 2 inches, and the question is how far the strength was governed by that thickness. By the mode of calculation explained in (350) and taking M_T, or the mean transverse strength of cast iron, at ·9 ton, being the multiplier derived from experiments on 1-inch bars, we have: top flange $2·33^2 \times 5·1 = 27$; vertical web $(28·43^2 - 2·33^2) \times 2·08 = 1670$; and bottom flange $(30·5^2 - 28·43^2) \times 12·1 = 1476$. The total reduced value of $d^2 \times b$ thus becomes $27 + 1670 + 1476 = 3173$; then $3173 \times ·9 \div 27·4 = 104·2$ tons breaking weight in the centre;—but the experimental breaking weight was 76·6 tons only, and we

have the ratio $76 \cdot 6 \div 104 \cdot 2 = \cdot 73$, being very nearly that for 2-inch square bars by Captain James' experiments (933), which was ·72 nearly.

In Mr. Owen's experiments, Table 68, on the contrary, the thickness of the bottom flange was $1\frac{3}{4}$ inch, for which by Rule (934) the strength should be ·783, but the experimental strength was $38 \cdot 3 \div 39 \cdot 8 = \cdot 96$, or 96 per cent. of that due by calculation on the basis of data from 1-inch bars. In this girder, however, the thickness of top flange and vertical web was 1 inch only, and the strength was possibly modified thereby, still we should have expected that the thickness of the bottom flange would have been more influential than it appears to have been.

Stirling's Toughened Cast Iron.

(938.) Many years ago Mr. Stirling introduced his Patent process for increasing the strength of ordinary cast iron by mixing with it given proportions of wrought-iron scrap: it was stated that with

10	20	30	40

per cent. of wrought iron, the transverse strength was increased

$22\frac{1}{2}$	$31\frac{1}{2}$	60	33

per cent., the effect being a maximum with 30 per cent., that is to say, with a mixture of 100 cast iron to 30 wrought iron.

Table 143 gives the result of experiments, and shows by col. 5 that the effect of Stirling's process on the strength in resisting the three principal kinds of strain is very unequal, the Tensile strength being increased 74 per cent.; the Transverse, 60 per cent., and the Crushing, 30 per cent. only. The transverse strength is of course dependent on the tensile and compressive strengths, and calculation will show that an increase of 60 per cent. in the transverse strength is almost exactly that due to an increase of 74 per cent. in the tensile, and 30 per cent. in the crushing strengths.

(939.) Thus the mean value of T for British Cast-iron (4) is $7 \cdot 142$ tons, and of C, 43 tons per square inch (132): then by

TABLE 143.—Of Experiments on Stirling's Toughened Cast Iron.

Kind of Strain.	Stirling's Iron.			Mean Ordinary Cast Iron, O.	Ratio, $\frac{S}{O}$.	No. of Experiments.	Authorities.
	Maximum.	Minimum.	Mean, S.				
Tensile, T, in tons ..	14·32	10·47	12·46	7·142	1·74	5	Owen, Grissell, &c.
Crushing, C, in tons	71·14	53·18	55·7	43·0	1·30	12	Hodgkinson.
Transverse, M_T, in lbs.	3906	2898	3300	2063	1·60	17	Owen, Grissell, &c.
	(1)	(2)	(3)	(4)	(5)	(6)	

Rule (496) we obtain the Specific Transverse strength, or M_T, which becomes $W = \left(\frac{\sqrt{43}}{\sqrt{43} + \sqrt{7 \cdot 142}}\right)^2 \times 7 \cdot 142 \div 4 \cdot 5 =$ $\cdot 8016$ ton, or 1795 lbs.

By the application of Stirling's process to this iron T is increased 74 per cent., and becomes $7 \cdot 142 \times 1 \cdot 74 = 12 \cdot 4$ tons, while C is increased 30 per cent., and becomes $43 \times 1 \cdot 30 = 56$ tons per square inch. Then, M_T by rule (496) gives $W = \left(\frac{\sqrt{56}}{\sqrt{56} + \sqrt{12 \cdot 4}}\right)^2 \times 12 \cdot 4 \div 4 \cdot 5 = 1 \cdot 274$ ton, or 2854 lbs.: hence we obtain $2854 \div 1795 = 1 \cdot 59$, or 59 per cent. increase in transverse strength, being very nearly 60 per cent., as given by experiment.

We should have obtained the same result with the identical iron used by Mr. Stirling, namely the Calder, which being a weak iron is specially adapted for his process (941). By Table 31 Mr. Hodgkinson's experiments on that iron give T = 6·13, and C = 33·92 tons per square inch, with which Rule (496) gives $W = \left(\frac{\sqrt{33 \cdot 92}}{\sqrt{33 \cdot 92} + \sqrt{6 \cdot 13}}\right)^2 \times 6 \cdot 13 \div 4 \cdot 5 = \cdot 6708$ ton. With this iron treated by Stirling's process T becomes $6 \cdot 13 \times 1 \cdot 74 = 10 \cdot 67$ tons, and $C = 33 \cdot 92 \times 1 \cdot 30 = 44 \cdot 1$ tons per square inch: then $W = \left(\frac{\sqrt{44 \cdot 1}}{\sqrt{44 \cdot 1} + \sqrt{10 \cdot 67}}\right)^2 \times 10 \cdot 67 \div$ $4 \cdot 5 = 1 \cdot 066$ ton is the value of M_T, giving $1 \cdot 066 \div \cdot 6708 =$

1·59, or an increase in the transverse strength of 59 per cent., as before.

(940.) Another important question connected with this subject is to determine the effect of Stirling's process on Cast iron in girders of ordinary sections, Mr. Berkley's experiments (897) having shown that with ordinary iron, the strength of girders is not simply proportional to the transverse strength of small test-bars cast from the same metal.

Table 68 gives the result of experiments by Mr. Owen, H.M. Inspector of Metals, on a large girder of Mr. Hodgkinson's form, Fig. 79; there were 13 experiments on common cast iron of different kinds, and 11 with Stirling's iron, or cast iron mixed with different proportions of wrought-iron scrap, varying from 17 to 33 per cent. of the cast iron, the mean of the whole being 22 per cent.

The mean breaking weight with common cast iron was 38·3 tons, and with Stirling's iron 52·3 tons, the ratio is 1 to $52 \cdot 3 \div 38 \cdot 3 = 1 \cdot 366$, showing an increase of 36·6 per cent. only, whereas the small test-bars gave, as we have seen in (938), an increase of 60 per cent.

(941.) But, analysis of the details of these experiments will show that the effect of Stirling's process varies very much with the strength of the particular cast iron to which it is applied, weak iron being very greatly improved in strength, while very strong irons are scarcely affected at all. Thus, with the Calder, which is a very weak iron, experiments 2 and 4 give 33 and 34 tons respectively, the mean being 33·5 tons :—by experiments 14, 18, 21, and 24, this same iron mixed with 25 per cent. of wrought-iron scrap gave 48, 52, 52½, and 60½ tons respectively, the mean being 53·25 tons, and we have $53 \cdot 25 \div 33 \cdot 5 = 1 \cdot 59$, or an increase in strength of 59 per cent., agreeing very nearly with that given for rectangular bars by Table 143, which was 60 per cent., and agreeing exactly with the calculations in (939).

But in experiments 12 and 13 we have a strong mixture of irons which gave 47 and 47¼ tons respectively, the mean being 47·1 tons, whereas Calder iron gave 33·5 tons only. Now this strong iron mixed with 20 per cent. of wrought-iron scrap, gave

in experiment 15 a breaking weight of 48·5 tons, and we have $48 \cdot 5 \div 47 \cdot 1 = 1 \cdot 03$, an increase in strength of 3 per cent. only.

In accordance with these facts, Mr. Stirling states that the special object he had in view was to raise the weaker irons to the strength of the strongest; generally speaking, he finds that Scotch iron requires a larger proportion of wrought iron than Staffordshire, and Welsh least of all. For No. 1 Scotch he recommends from 18 to 21 per cent.; No. 2, from 27 to 36 per cent.; No. 3 he does not recommend at all.

But it should be observed that the section of girder adopted by Mr. Owen, with bottom and top flanges having areas as 6 to 1, was specially adapted to give maximum results with ordinary cast iron whose crushing and tensile strengths were in that same ratio. But with Stirling's iron, the ratios of those strengths is $55 \cdot 7 \div 12 \cdot 46 = 4 \cdot 47$ to 1·0, and the flanges should have had that ratio in order to obtain a maximum effect.

(942.) Experiments have shown that the strength of Stirling's iron is affected by the size or thickness of the casting, as we found to be the case with ordinary cast iron. Thus Calder iron with 42 per cent. of wrought iron and in bars 1 inch, $1\frac{1}{2}$ inch, and 2 inches square, gave as the value of M_T, or specific transverse strength, 3514, 2895, and 2754 lbs. respectively; the ratios being 1·0, ·824, and ·784 respectively. The rule in (934) gives for these same sizes with ordinary cast iron, the ratios 1·0, ·838, and ·739; from which it appears that the effect of size or thickness of casting is practically the same for Stirling's as for ordinary cast iron.

ON THE STRENGTH OF WHEEL-TEETH.

(943.) The tooth of a wheel may be regarded as a simple cantilever, and where the strain upon it is known, as for example with the gearing of a crane, it appears to be a very simple matter to calculate the strength and to adapt it to the strain. But while this may be done satisfactorily for a *dead* load such as that on the large 1st motion wheel of a crane, it will be found not to apply to the other wheels of the train, for although

the motion is a very slow one, the strength seems to be governed by the laws of Impact, or of forces in motion, which differ entirely from those dominating a statical force or dead load.

Examples of the proper method of calculating the strength of the teeth in a train of crane-wheels are given in (594) and (598), therefore need not be repeated here; but we will consider the strength of wheels carrying the power of Steam-engines or other motors,—an important matter which is fully considered in the Author's Treatise on 'Mill-Gearing.'

For iron-and-iron toothed wheels we have the Rules:—

(944.) $$H_N = \sqrt{D \times R} \times p^2 \times w \times \cdot 043.$$

(945.) $$H_I = \sqrt{D \times R} \times p^2 \times w \times \cdot 0645.$$

For Mortise Wheels the Rules become:—

(946.) $$H_N = \sqrt{D \times R} \times p^2 \times w \times \cdot 05.$$

(947.) $$H_I = \sqrt{D \times R} \times p^2 \times w \times \cdot 075.$$

In which H_N = Nominal Horse-power; H_I = net Indicated Horse-power; D = diameter of the wheel at pitch line in feet; p = pitch in inches; w = width in inches; R = revolutions per minute; M_N and M_I = Multipliers for nominal and Indicated Horse-power respectively. The relations of the Nominal and Indicated powers are explained and illustrated in (572).

Thus, an iron toothed wheel 6 feet diameter, 5 inches wide, 2 inches pitch, 24 Revolutions, gives $H_N = \sqrt{6 \times 24} \times 2^2 \times 5 \times \cdot 043 = 10 \cdot 32$ Nominal Horse-power. Again; a spur mortise wheel 4 feet diameter, $2\frac{1}{2}$ inches pitch, 7 inches wide, 30 Revolutions, gives by Rule (947), $H_I = \sqrt{4 \times 30} \times 2\frac{1}{2}^2 \times 7 \times \cdot 075 = 36$ Net indicated Horse-power, or by Rule (946), $H_N = \sqrt{D \times R} \times 2\frac{1}{2}^2 \times 7 \times \cdot 05 = 24$ Nominal Horse-power, &c.

The width of wheels on the face is to a great extent arbitrary; a good proportion is given by the Rule:—

(948.) $$w = p^2 \div \sqrt{p} \times 1 \cdot 8.$$

In which $p =$ pitch, and $w =$ width in inches: col. 2 of Table 144 has been calculated by that rule: now, multiplying p^2 by the width thus found we obtain the *ratios* of power in col. 3, which shows how rapidly the power rises with the pitch, being in fact proportional to the $3\frac{1}{2}$ power of the pitch, or $p^{3 \cdot 5}$, and col. 4 has been calculated by *that* rule. Thus a wheel of any diameter and revolutions which with 1-inch pitch gives 1 Horse-power, would with 4-inch pitch, &c., give 128 Horse-power, &c.

TABLE **144.**—Of the RATIO of the POWER of TOOTHED WHEELS.

Pitch.		Width.		Ratios of Power.	
				Product.	As $p^{3 \cdot 5}$.
1^2	$\times$	$1\frac{3}{4}$	=	1·75	1·00
$1\frac{1}{4}^2$	$\times$	$2\frac{1}{2}$	=	3·90	2·18
$1\frac{1}{2}^2$	$\times$	$3\frac{1}{4}$	=	7·31	4·13
$1\frac{3}{4}^2$	$\times$	4	=	12·25	7·09
2^2	$\times$	$5\frac{1}{4}$	=	21·00	11·31
$2\frac{1}{4}^2$	$\times$	6	=	30·37	17·9
$2\frac{1}{2}^2$	$\times$	7	=	43·75	24·7
$2\frac{3}{4}^2$	$\times$	$8\frac{1}{4}$	=	62·38	34·5
3^2	$\times$	$9\frac{1}{4}$	=	83·25	46·8
$3\frac{1}{4}^2$	$\times$	$10\frac{1}{2}$	=	110·9	61·9
$3\frac{1}{2}^2$	$\times$	$11\frac{3}{4}$	=	143·5	80·2
$3\frac{3}{4}^2$	$\times$	13	=	224·8	102·1
4^2	$\times$	$14\frac{1}{2}$	=	232·0	128·0
(1)		(2)		(3)	(4)

STRENGTH OF CRANK-PINS, ETC.

(949.) "*Crank-pins.*"—The strain on the crank-pin of a Steam-engine may be found with sufficient accuracy from the area of the piston, pressure of steam, &c. Then, regarding the pin as a cantilever, its strength will be directly proportional to d^3 and inversely as the length, but inasmuch as the length is usually proportional to the diameter, the strength is reduced to d^2 simply, and we have the empirical Rule:—

(950.) $$S_c = d^2 \times 1000.$$

TABLE 145.—Of the STRENGTH and PROPORTIONS of CRANK-PINS to STEAM-ENGINES. Cases in Practice.

Nom. Horse-power.	Diameter of Cylinder.	Assumed Pressure in Lbs.		Diameter of Crank-pin.		Maker.
		Per Sq. In.	Total.	Actual.	Calculated.	
			Marine Engines.			
280	88	20	121,640	12	11	Fairbairn.
120	57	20	51,040	8	$7\frac{1}{4}$	Maudslay.
110	$55\frac{1}{2}$	20	48,384	$7\frac{3}{4}$	7	"
100	$52\frac{1}{2}$	20	43,300	$7\frac{3}{8}$	$6\frac{5}{8}$	"
90	50	20	39,270	7	$6\frac{1}{4}$	"
80	48	20	36,200	$6\frac{1}{2}$	6	"
70	46	20	33,240	6	$5\frac{3}{4}$	"
60	43	20	29,040	$5\frac{1}{2}$	$5\frac{1}{2}$	"
50	40	20	25,140	5	5	"
40	$36\frac{1}{2}$	20	20,920	$4\frac{1}{2}$	$4\frac{1}{2}$	"
30	32	20	16,080	4	4	"
25	$29\frac{1}{2}$	20	13,670	$3\frac{3}{4}$	$3\frac{3}{4}$	"
20	27	20	11,452	$3\frac{1}{2}$	$3\frac{1}{2}$	"
15	24	20	9,048	3	3	"
10	20	20	6,280	$2\frac{1}{2}$	$2\frac{1}{2}$	"
			Low-pressure Condensing Engines.			
80	$45\frac{1}{2}$	20	32,520	6	$5\frac{3}{4}$	Hick.
60	42	20	27,720	$5\frac{3}{8}$	$5\frac{1}{4}$	"
50	38	20	22,680	$4\frac{5}{8}$	$4\frac{3}{4}$	"
45	$34\frac{3}{4}$	20	18,968	$4\frac{1}{4}$	$4\frac{3}{8}$	"
40	33	20	77,160	$4\frac{1}{4}$	$4\frac{1}{8}$	"
30	30	20	14,140	$3\frac{5}{8}$	$3\frac{3}{4}$	"
25	28	20	12,320	$3\frac{3}{8}$	$3\frac{1}{2}$	"
			High-pressure Engines.			
24	19	45	12,757	$3\frac{1}{2}$	$3\frac{1}{2}$	Easton and Amos.
*10	$11\frac{1}{2}$	45	6,411	$2\frac{1}{2}$	$2\frac{1}{2}$	"
*8	$10\frac{1}{2}$	45	5,344	2	$2\frac{5}{16}$	"
*6	$9\frac{1}{2}$	45	4,564	2	$2\frac{1}{8}$	"
3	7	45	1,732	$1\frac{1}{4}$	$1\frac{5}{16}$	"
(1)	(2)	(3)	(4)	(5)	(6)	

* Unequal-ended Beams, increasing the strain on Pin.

In which S_C = the strain on the pin in lbs.; d = diameter of pin in inches. Table 145 gives the sizes of many crank-pins in practice; col. 6 has been calculated by the rule and agrees fairly with the actual sizes in col. 5.

There are other considerations besides strength which should be regarded in fixing the sizes of crank-pins: the proportions should be such as not only to give adequate strength, but also large area of surface, &c., to avoid heating with high velocities, and abrasion with low ones: see the Author's Treatise on 'Mill-Gearing.'

"*Cross-heads.*"—In the old form of Marine Engines, the piston carried a long cross-head with two side-rods, &c., as in Fig. 200; this arrangement is still used for some Marine and other Engines. To find the strength of such cross-heads we have the empirical Rule:—

$$(951.) \qquad d^2 \times t = w \times l \times \cdot 00018.$$

In which d = the depth, and t = the thickness, both at or near the centre, in inches; l = the length between the centres of side-rods in inches, and w = strain on the piston-rod in lbs. Table 146 gives the particulars of long cross-heads in practice: col. 9 has been calculated by the rule, &c.

"*Beam-gudgeons.*"—The main gudgeon in the centre of a Steam-engine Beam is in effect a cylindrical beam supported at each end by the bearings, and loaded in the centre by the strain due to the pressure on the piston, &c. The diameter at the centre for gudgeons of wrought iron will be given by the Empirical Rule:—

$$(952.) \qquad d = \sqrt[3]{\mathsf{H}_N \times L \times \cdot 3}.$$

In which H_N = the Nominal Horse-power of the Engine, L = length, or distance between centres of bearings in inches, and d = the diameter of the gudgeon at the centre, in inches. Col. 6 of Table 147 has been calculated by this rule, and agrees well with the actual sizes in practice given by col. 7.

"*Steam-engine Entablatures.*"—Fig. 41 shows a common arrangement of Beam-engines, in which an Entablature A is built into side-walls, and supported by two columns. The

TABLE **146.**—Of the STRENGTH and PROPORTIONS of LONG CROSS-HEADS for MARINE and other STEAM-ENGINES. From Cases in Practice.

Nominal. Horse-power.	Diameter of Cylinder.	Pressure in Lbs.		Length, Centres of Side-rods, in Inches.	Depth at Centre in Inches. d.	Thickness. Inches. t.	$d^2 \times t$.		Maker, &c.
		Per Square Inch.	Total.				Actual.	Calculated by Rule.	
120	57	20	51,040	83	14	$4\frac{1}{8}$	808	763	Maudslay, Marine.
90	50	20	39,270	72	$12\frac{1}{2}$	$3\frac{1}{2}$	546	509	ditto ditto
60	43	20	29,040	63	$10\frac{1}{4}$	3	315	329	ditto ditto
30	32	20	16,080	48	$7\frac{1}{2}$	$2\frac{1}{4}$	127	139	ditto ditto
10	20	20	6,280	33	5	$1\frac{1}{2}$	37	37	ditto ditto
12	13	45	5,985	$48\frac{1}{2}$	6	$1\frac{5}{8}$	58	52	Easton and Amos.
6	10	45	3,529	$44\frac{1}{2}$	5	$1\frac{1}{4}$	31	29	ditto ditto
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	

depth of the Entablature will be given by the Empirical Rule:—

(953.) $$D = \sqrt{H_N} \times 2.$$

In which H_N = the Nominal Horse-power of the Engine, and D = the depth of the Entablature in inches: thus for 40-Horse we have $\sqrt{40} = 6 \cdot 3$; hence $D = 6 \cdot 3 \times 2 = 12 \cdot 6$ inches, &c.; col. 2 of Table 43 has been calculated by this rule.

TABLE **147.**—Of the SIZES of BEAM-GUDGEONS for STEAM-ENGINES. Cases in Practice.

Nominal Horse-power.	Length, Centres. Inches.		M_G.		Diameter. By Rule.	Diameter. Actual.	Makers.
Double-cylinder Engines.							
100 ×	69 =	6900 ×	·3 =	2070 ∛ =	$12\frac{3}{4}$	$12\frac{1}{2}$	Easton and Amos.
60 ×	52 =	3120 ×	·3 =	936 ∛ =	$9\frac{3}{4}$	8	"
42 ×	42 =	1764 ×	·3 =	529 ∛ =	$8\frac{1}{16}$	$7\frac{1}{4}$	"
30 ×	36 =	1080 ×	·3 =	324 ∛ =	$6\frac{7}{8}$	$6\frac{3}{4}$	"
22 ×	36 =	832 ×	·3 =	250 ∛ =	$6\frac{5}{16}$	6	"
20 ×	30 =	600 ×	·3 =	180 ∛ =	$5\frac{5}{8}$	$5\frac{1}{2}$	"
16 ×	30 =	480 ×	·3 =	144 ∛ =	$5\frac{1}{4}$	5	"
12 ×	30 =	360 ×	·3 =	108 ∛ =	$4\frac{3}{4}$	$4\frac{3}{4}$	"
Low-pressure Condensing Engines.							
80 ×	54 =	4320 ×	·3 =	1296 ∛ =	$10\frac{7}{8}$	11	Rothwell and Hick.
70 ×	49 =	3430 ×	·3 =	1029 ∛ =	$10\frac{1}{8}$	10	"
65 ×	48 =	3120 ×	·3 =	936 ∛ =	$9\frac{3}{4}$	$9\frac{1}{2}$	"
45 ×	44 =	1980 ×	·3 =	594 ∛ =	$8\frac{3}{8}$	8	"
30 ×	40 =	1200 ×	·3 =	360 ∛ =	$7\frac{1}{8}$	7	"
(1)	(2)	(3)	(4)	(5)	(6)	(7)	

"*Steam-engine Beams.*"—The beam of a Steam-engine is subjected to a transverse strain, and the strength might possibly be calculated by the ordinary Rules, but in most cases the proportions may be determined more easily and satisfactorily by Empirical Rules. The ratio of the depth at the centre to the

length is, to some extent, arbitrary; but a good proportion will be given by the Rule

(954.) $$d = l \times \cdot 15.$$

In which l = the length of the beam between centres in inches and d = the depth at the centre in inches; thus, in the 42-Horse Engine in Table 148, $l = 15$ feet, or 180 inches: hence $d = 180 \times \cdot 15 = 27$ inches, as in col. 5, which happens to be precisely the depth in practice, as given in col. 3.

TABLE **148.**—Of the PROPORTIONS of CAST-IRON BEAMS to STEAM-ENGINES. Cases in Practice.

Nominal-Horse-power.	Length.		Depth at the Centre in Inches.			Thickness of the Web, in Inches.		Breadth of Flanges.	Stroke.	
			Actual.	By Rule (956).	By Rule (954).	Actual.	By Rule (955).			
	ft.	in.						inches.	ft.	in.
100	25	0	45	44·85	45	$2\frac{1}{8}$	2·11	..	8	0
60	18	3	33	32·25	32·85	2	1·90	$4\frac{1}{2}$	6	0
42	15	0	27	29·18	27	$1\frac{1}{2}$	1·75	$4\frac{1}{2}$	5	0
30	13	6	23	24·87	24·3	$1\frac{3}{8}$	1·60	$4\frac{1}{4}$	4	6
22	12	6	21	19·9	22·5	$1\frac{1}{2}$	1·34	$4\frac{1}{2}$	4	3
12	9	0	$16\frac{1}{2}$	16·11	16·2	1	0·95	$3\frac{7}{8}$	2	10
(1)	(2)		(3)	(4)	(5)	(6)	(7)	(8)	(9)	

Having found the depth, the thickness may be determined by the Rule

(955.) $$t = \frac{H_N \times L \times 5}{\sqrt[3]{L} \times d^2}.$$

In which H_N = the Nominal or Reputed power of the Engine, L = Length of the beam in feet, d = depth at the centre in inches, and t = the thickness of the main central web in inches. Thus, with the 100-Horse beam, in which **L** = 25, $d = 45$, we have $t = \dfrac{100 \times 25 \times 5}{\sqrt[3]{25} \times 45^2} = 2 \cdot 11$ inches, as in col. 7, agreeing with the actual thickness, which was $2\frac{1}{8}$ inches by col. 6.

2 L

When the thickness is given, the depth may be found by the Rule :—

(956.) $$d = \sqrt{\left(\frac{H_N \times L \times 5}{\sqrt[3]{L} \times t}\right)}.$$

Thus, with the 60-Horse Beam, L being $18\frac{1}{4}$ feet and $t = 2$ inches, we obtain $d = \sqrt{\left(\frac{60 \times 18 \cdot 25 \times 5}{\sqrt[3]{18 \cdot 25} \times 2}\right)} = 32 \cdot 25$ inches, as in col. 4; the actual depth was 33 inches by col. 3.

These Rules assume that the length of the beam is simply proportional to the stroke of the piston, and that the speed of piston is directly proportional to $\sqrt{}$ of the stroke (therefore of the length of Beam). The strain with a given Horse-power will, therefore, be inversely proportional to $\sqrt[3]{}$ of the length of the beam, and is taken to be so in the Rule (956). Thus, if the stroke of the piston is in the ratio 1, 2, 3, the length of the beam will be in the *ratio* 1, 2, 3, the velocity of piston in the ratio $\sqrt[3]{1} = 1 \cdot 0$; $\sqrt[3]{2} = 1 \cdot 26$; and $\sqrt[3]{3} = 1 \cdot 44$: the strain per Horse-power being inversely proportional to the velocity of the piston, becomes $1 \div 1 = 1 \cdot 0$; $1 \div 1 \cdot 26 = \cdot 8$; and $1 \div 1 \cdot 44 = \cdot 7$ respectively.

(957.) "*Variableness of Materials.*" — Experiments have shown that there is great variableness in the strength of all materials, even when apparently of the same kind and quality. The *mean* strength, as found by numerous experiments, is usually taken as a Standard for calculation, and it becomes a matter of considerable practical importance that the Engineer should know within what limits the strength may probably vary, and particularly that the probable minimum should be known. Table 149 gives a collected statement of the variableness of many materials under the three great strains, namely, the Transverse, Tensile, and Crushing strains, reduced to percentages for convenience of reference. For example, the Transverse strength of cast-iron bars 1 inch square (931) may be 27 per cent. above the average, or 100 − 79 = 21 per cent below it from 221 experiments. Col. 4 gives the *range* of probable variations, and shows that there are great differences, Larch being very variable, and Canadian oak very equable, &c.

TABLE 149.—Of the VARIABLENESS in the STRENGTH of MATERIALS in resisting different STRAINS.

Material.	Max.	Mean.	Min.	Extreme Variation, Max./Min.	No. of Experiments.
Ratios of Transverse Strength.					
Cast iron	127	100	79	1·61	221
" Stirling's toughened	118	100	88	1·34	17
Slate, split	129	100	84	1·54	9
Brick	121	100	75	1·61	9
Ash	120	100	72	1·67	14
Beech	121	100	89	1·36	4
Birch	119	100	75	1·58	3
Cedar	133	100	80	1·66	4
Chestnut	125	100	80	1·56	3
Deal	110	100	83	1·33	5
Elm	132	100	82	1·61	6
Fir, Riga	141	100	71	2·00	58
Larch	172	100	59	2·91	21
Mahogany, Honduras, &c. ..	108	100	85	1·27	4
Oak, English	189	100	72	2·63	38
" Dantzic	147	100	63	2·34	29
" Canadian	106	100	97	1·09	6
" African	109	100	90	1·21	8
Pine, Pitch	158	100	63	2·51	35
" Red	132	100	84	1·57	7
" Yellow	113	100	85	1·33	5
" White	155	100	87	1·78	10
Teak	123	100	88	1·40	7
Ratios of Cohesive Strength.					
Cast iron	147	100	79	1·84	23
" Stirling's toughened	116	100	87	1·34	5
Wrought iron, Rolled Bar ..	120	100	77	1·55	188
" welded joints ..	121	100	68	1·77	18
Steel, tilted bar	155	100	68	2·27	66
" welded joints	115	100	85	1·36	2
Boiler-plate, wrought-iron ..	129	100	67	1·93	327
" steel	119	100	79	1·51	80
Crane-chain, 1½ to ¾ inch ..	114	100	86	1·31	125
" 1 inch diameter ..	119	100	82	1·45	61
	(1)	(2)	(3)	(4)	(5)

TABLE 149.—Of the VARIABLENESS in the STRENGTH of MATERIALS in resisting different STRAINS—*continued.*

Material.	Max.	Mean.	Min.	Extreme Variation, $\frac{\text{Max.}}{\text{Min.}}$	No. of Experiments.
Ratios of Crushing Strength.					
Cast iron	156	100	61	2·57	139
" Stirling's toughened	120	100	89	1·34	12
Granite	149	100	58	2·56	17
Red Sandstone	180	100	50	3·61	4
Slate, Valencia	111	100	90	1·23	4
Cragleith	127	100	80	1·58	5
York Paving	136	100	78	1·73	4
Marble	116	100	81	1·43	7
Teak	105	100	93	1·12	12
Willow	136	100	64	2·12	2
Larch	127	100	73	1·74	2
Oak—cohesion	160	100	72	2·23	9
Teak "	102	100	97	1·06	3
Ash "	108	100	73	1·47	8
	(1)	(2)	(3)	(4)	(5)

(958.) "*Agreement of Rules with Experiment.*"—There are very great difficulties in preparing a Practical Work on the Strength of Materials, as compared, or rather contrasted, with a purely Theoretical one. Theorists follow fundamental Laws, which are as fixed and inflexible as the laws of gravitation, and the accuracy of their conclusions can be mathematically demonstrated. But when confronted with the results of experiment many of these correct laws are found to lead to manifestly erroneous conclusions, and have to be relinquished in favour of Empirical Rules, for which nothing can be said, except that they are correct as proved by Experiment.

(959.) Many of the Rules in this work are of a more or less Empirical character; in most cases the Theoretical laws were first taken as a basis, but had to be laboriously modified tentatively, by the teachings of experience. In almost all cases,

TABLE 150.—Of the AGREEMENT of RULES with EXPERIMENTS.

Subject of Table.	No. of Table.	Reference Paragraph.	Difference, or Error per Cent.			No. of Experiments.
			Greatest + Error.	Greatest − Error.	Mean.	
Pillars, Cast-iron, Cylindrical, solid and hollow ..	38	161	19·8	22·1	+0·532	40
,, Wrought-iron, solid, cylindrical	44	201	13·8	15·3	+0·293	15
,, ,, cylindrical, thin iron	52	224	30·4	33·0	−0·461	36
,, ,, solid, Rectangular	53	240	22·4	34·8	0·000	21
,, ,, Rectangular, thin iron	55	264	28·5	23·0	−2·25	27
,, Timber, Square and Rectangular	57	291	15·9	15·5	−0·427	11
Beams, plate-iron, Tubular	77	410	21·8	24·9	−1·56	16
Connection of Transverse strain with T and C	79	508	30·0	15·0	+0·276	22
Impact on Loaded Beams	125	806	24·4	30·3	−4·5	8
	(1)	(2)	(3)	(4)	(5)	(6)

their correctness is shown at every point by comparison with experiment. It will be interesting, however, to collect a few of the most important of these in the form of a table, which is done in Table 150.

In some cases the agreement of the Rules with Experiment is remarkable, partaking evidently more or less of an accidental character: some of these are given in paragraphs 21, 139, 375, 685, 777, 878, &c.

(960.) "*Real and Apparent Factor of Safety.*"—Essentially, the "Factor of Safety" is, or should be, the Ratio of the Breaking weight to the Safe Load, the latter being 1·0, and its special object is to give a margin of strength to cover unknown and unexpected contingencies. But this simple purpose becomes complicated by the fact that the breaking weight varies with the *character* of the strain: thus for a Cast-iron beam, it is shown by Table 141, that the breaking weight for constant dead load being 1·0, it is reduced to $\frac{1}{3}$ for an *Intermittent* dead load, or one acting in one direction only, or off-and-on continuously without shock; and to $\frac{1}{6}$ for a similar *Alternating* load, or one acting in both directions, up-and-down continuously. Say that the constant dead load = 18 tons Breaking weight: then the Intermittent Breaking weight becomes $18 \times \frac{1}{3} = 6$ tons, and the Alternating Breaking weight, $18 \times \frac{1}{6} = 3$ tons.

Now, if we take 3 for the Factor of Safety, and apply it to these three loads, we obtain 6, 2, and 1 ton respectively, as the Safe loads, which are equal to $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{18}$ of the constant dead load, the *Apparent* Factor of Safety becoming 3, 9, and 18 for the three cases, while the *real* Factor is the same in all, namely 3.

Illustrations of the application of these principles to cases in practice are given in (923), &c.: thus with Wrought-iron beams, the *Dynamic* rolling load is $\frac{1}{3}$ of the equivalent dead constant load, and with Factor 3 becomes $\frac{1}{3} \div 3 \times \frac{1}{9}$th of that dead load. Similarly, with Cast Iron, the Dynamic rolling load is $\frac{1}{6}$th of the equivalent dead constant load, and with the same Factor, 3, becomes $\frac{1}{6} \div 3 = \frac{1}{18}$th of that dead load: the apparent Factor = 18, although the real Factor is 3, as before. Again, with Timber, the Dynamic rolling load = $\frac{1}{4}$, which with Factor 5

becomes $\frac{1}{4} \div 5 = \frac{1}{20}$th of the dead constant Breaking weight, the apparent Factor being 20, while the real Factor is 5 only, &c.

In many cases it is convenient to use the apparent rather than the real Factor of Safety, and this course has been adopted frequently throughout this work: thus for Railway Bridges, the Factor used by most Engineers is 6, which is in fact the apparent Factor: being an intermittent strain with wrought iron, the Real Factor is $6 \times \frac{2}{3} = 4$. Thus, say we have a Bridge whose calculated breaking weight = 600 tons, then with Factor 4, we obtain $600 \div 4 = 150$ tons dead Safe load, or $150 \times \frac{2}{3} = 100$ tons intermittent Safe load: evidently we should have obtained the same result more easily by using the apparent Factor 6, which gives $600 \div 6 = 100$ tons, as before.

(961.) "*Strength of Flat Cover to Boiler.*"—A circular Boiler in America, 48 inches diameter, was provided with a plain, flat cover of cast iron $1\frac{7}{8}$ inch thick, and was subjected to a steam pressure of 160 to 170 lbs. per square inch, which it bore for about 6 months and then burst:—breaking at the edge all round. By Rule (367) we obtain $p = 1\frac{7}{8}^2 \times 148390 \div 48^2$ or $3 \cdot 5156 \times 148390 \div 2304 = 227$ lbs. per square inch: but applying the correction for *thickness* of metal as given in (934) and taking the value of z for say 2 inches thick at $\cdot 74$, we obtain $227 \times \cdot 74 = 168$ lbs. bursting pressure per square inch, or practically the pressure with which the cover actually burst, so that it is surprising that it did not fail before.

(962.) "*Low Resilience of Slate, &c.*"—Slate and York paving are frequently used for flooring in cases where they are supported at the ends only and act as beams: as there will always be a probability of a *blow* from the load falling on the floor, and those materials are excessively weak in resisting Impact, that fact should be borne in mind and extra strength provided to guard against failure. Table 67 shows by col. 6, that a Cast-iron plate of a given thickness, &c., will bear a safe falling load $6 \cdot 78 \div \cdot 2 = 34$ times the safe falling load for a similar plate of Slate; and $6 \cdot 78 \div \cdot 06 = 113$ times!! the safe falling load for York paving.

(963.) "*Graphic Ratios of Strength, &c.*"—Figures give, of course, very precise information as to the Specific Strengths of

Materials, but fail to give a clear idea of their comparative Ratios. Plate 27 gives a graphic representation of the Strength and Elasticity under various strains, which, appealing to the eye, will convey a more definite impression.

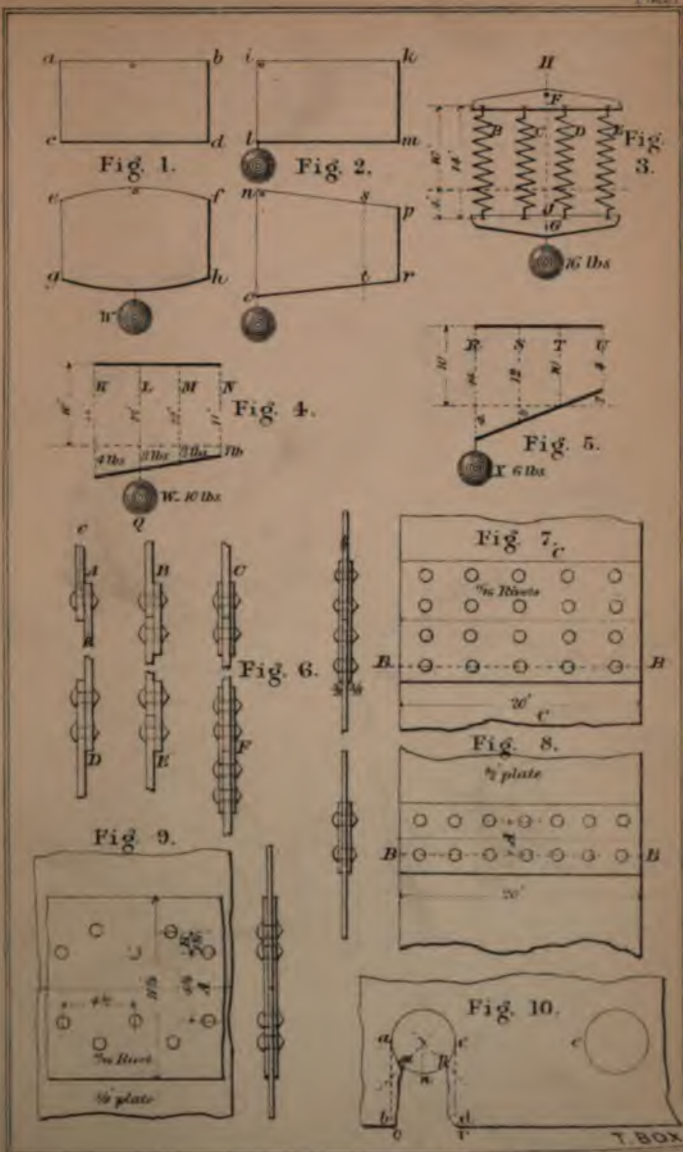
(964.) "*Weight of Materials.*"—For convenience of reference, and in order to make this work as complete and self-contained as possible, the Weight of many Materials used in the Arts is given by Table 151, which has been taken from the Author's 'Treatise on Heat.'

TABLE **151.**—Of the SPECIFIC GRAVITY and WEIGHT of MATERIALS Water at 62° being 1·000.

Material.	Specific Gravity.	Weight of a Cubic Foot in Pounds.	Weight of a Cubic Inch in Pounds.	No. of Cubic Feet in One Ton.
Mercury	13·596	847·3	·4903	2·644
Lead	11·352	707·5	·4094	3·166
Copper, sheet	8·785	547·5	·3168	4·091
Gun-metal, cast	8·670	540·3	·3127	4·145
Copper, cast	8·607	536·4	·3104	4·176
Brass, cast	8·393	523·1	·3027	4·282
Wrought Iron	7·788	485·3	·2809	4·615
Tin, cast	7·291	454·4	·2630	4·930
Zinc, sheet	7·190	448·1	·2593	4·999
Cast Iron, British, mean	7·087	441·6	·2556	5·07
Zinc, cast	6·861	427·6	·2474	5·24
Slate	2·835	176·7	·1022	12·68
Glass	2·760	172·0	·0995	13·02
Granite, Cornish	2·662	165·9	·0960	13·50
Sandstone, Yorkshire	2·506	156·2	·0904	14·34
Brick, London Stock	1·841	114·7	·0664	19·52
Sand, River	1·546	96·35	·0558	23·25
Coal, British, mean	1·313	81·83	·0474	27·37
Water, distilled	1·000	62·321	·03606	35·95
Ice, at 32°	·93	57·96	·03354	38·65
Alcohol	·813	50·67	·02932	44·21
Oil, Olive	·9153	57·04	·03301	39·27
Oak, seasoned	·777	48·42	·02802	46·26
Elm, "	·588	36·65	·0212	61·13
Mahogany, Honduras, seasoned	·560	34·9	·02 2	64·18
Pine, Yellow, seasoned	·483	30·1	·01742	74·41
Coke, Gas, in measure	·353	22·0	·01273	161·8
Cork	·24	14·96	·00866	149·7

INDEX.

Plate I



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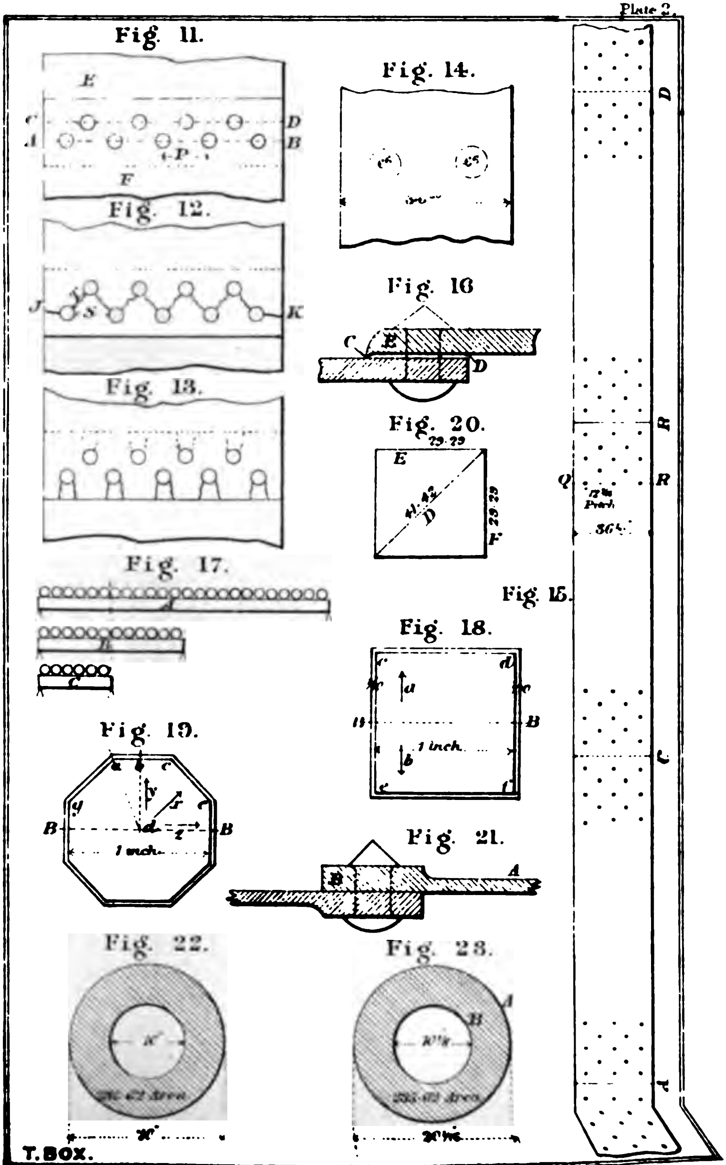

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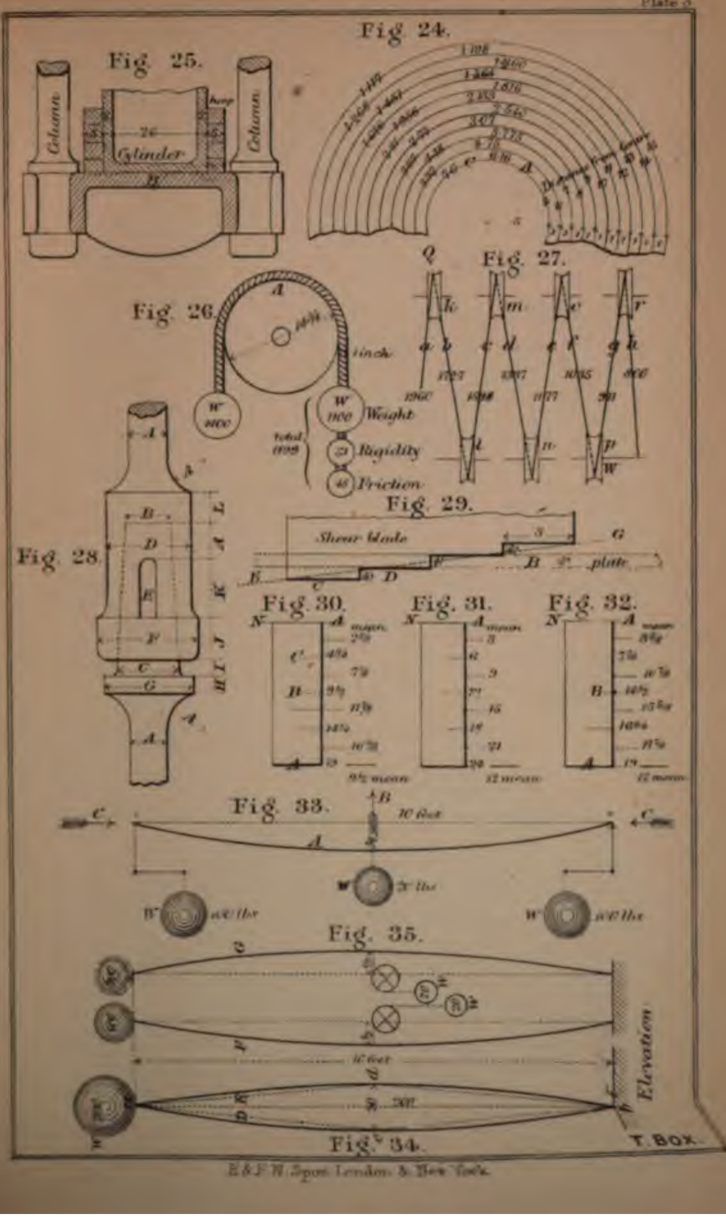

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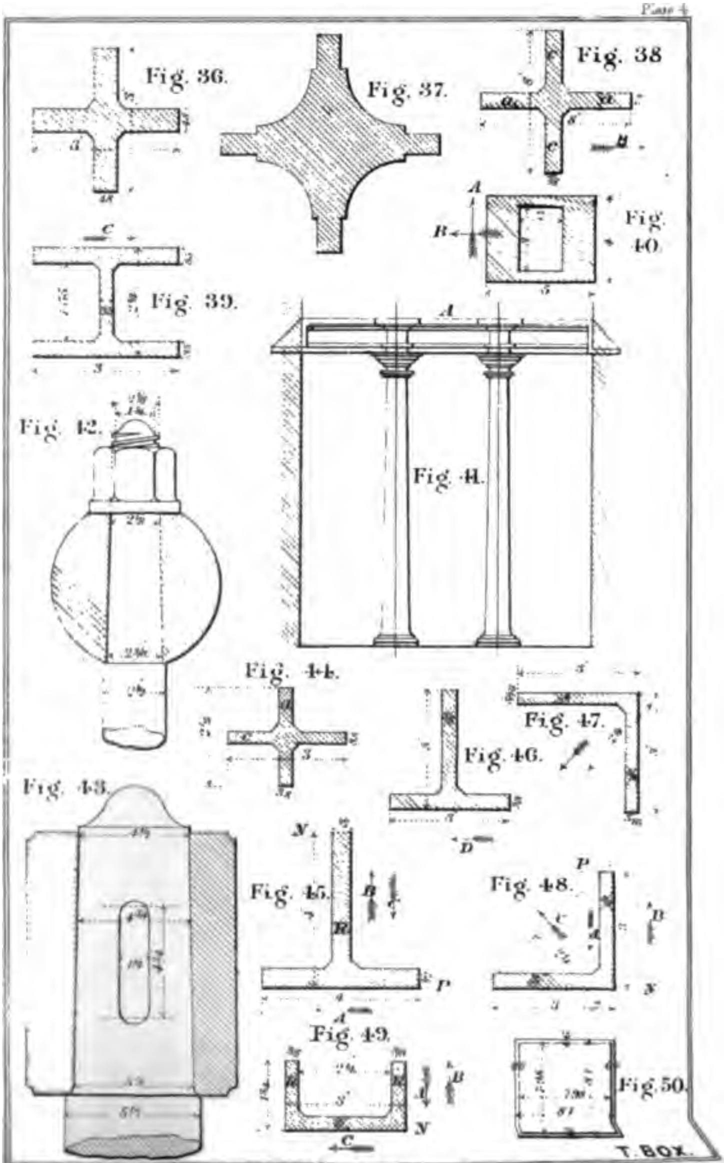

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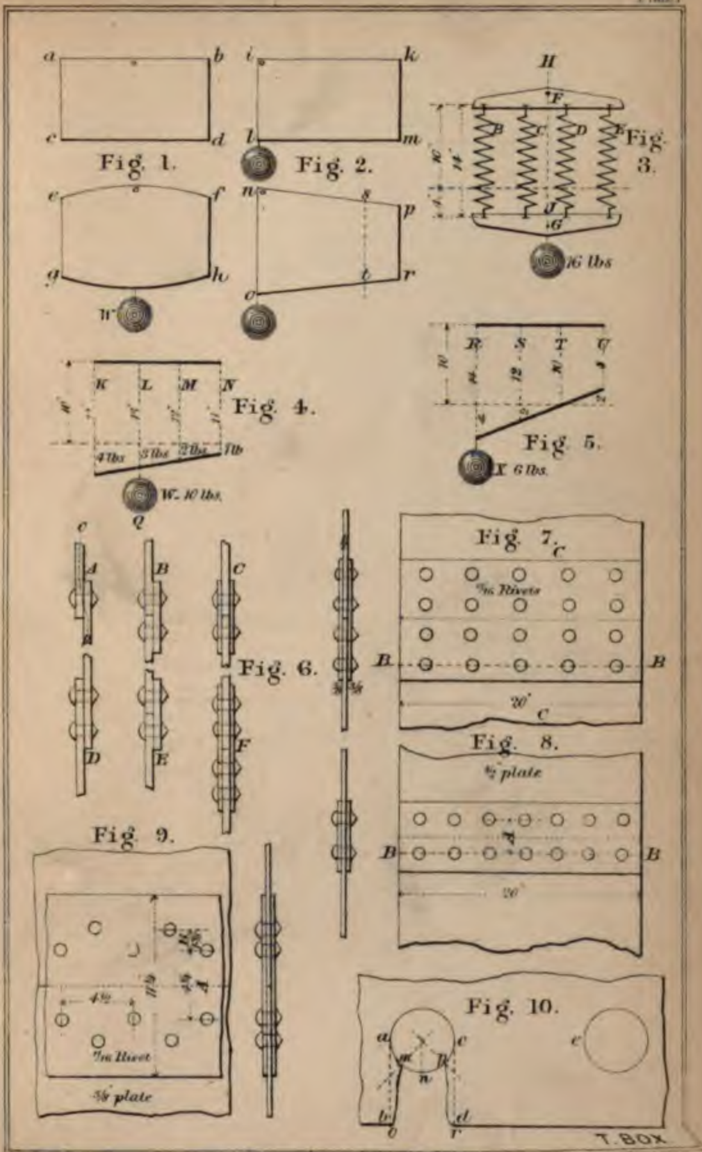

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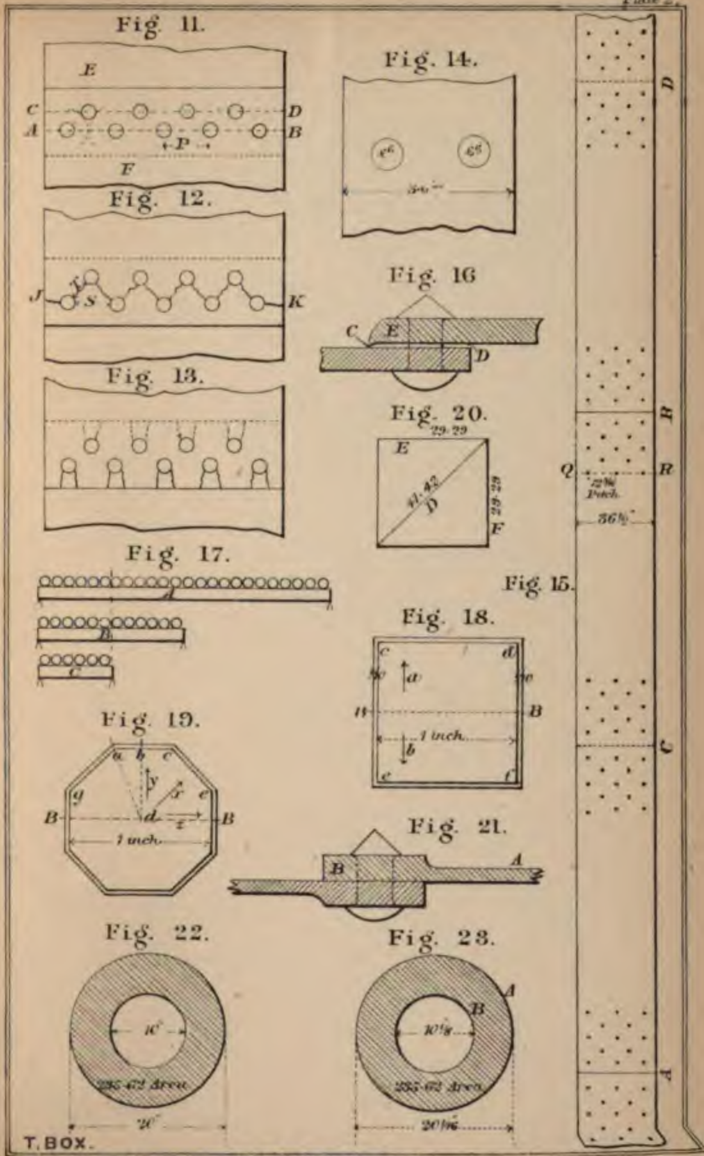

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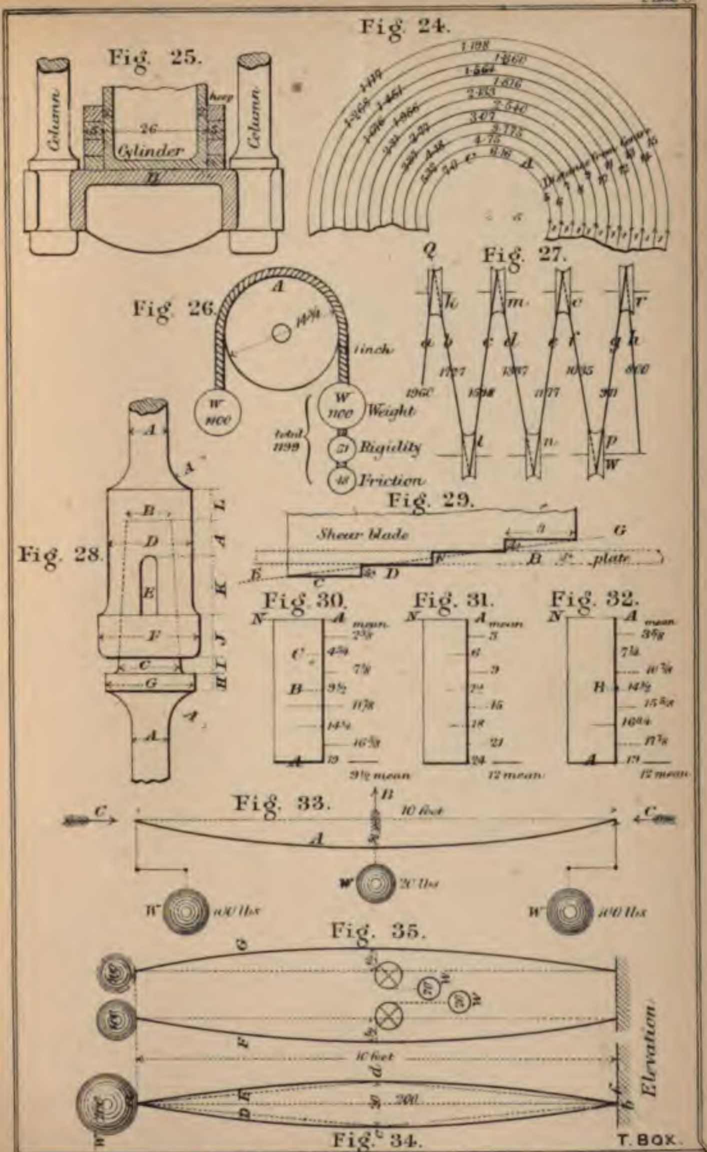

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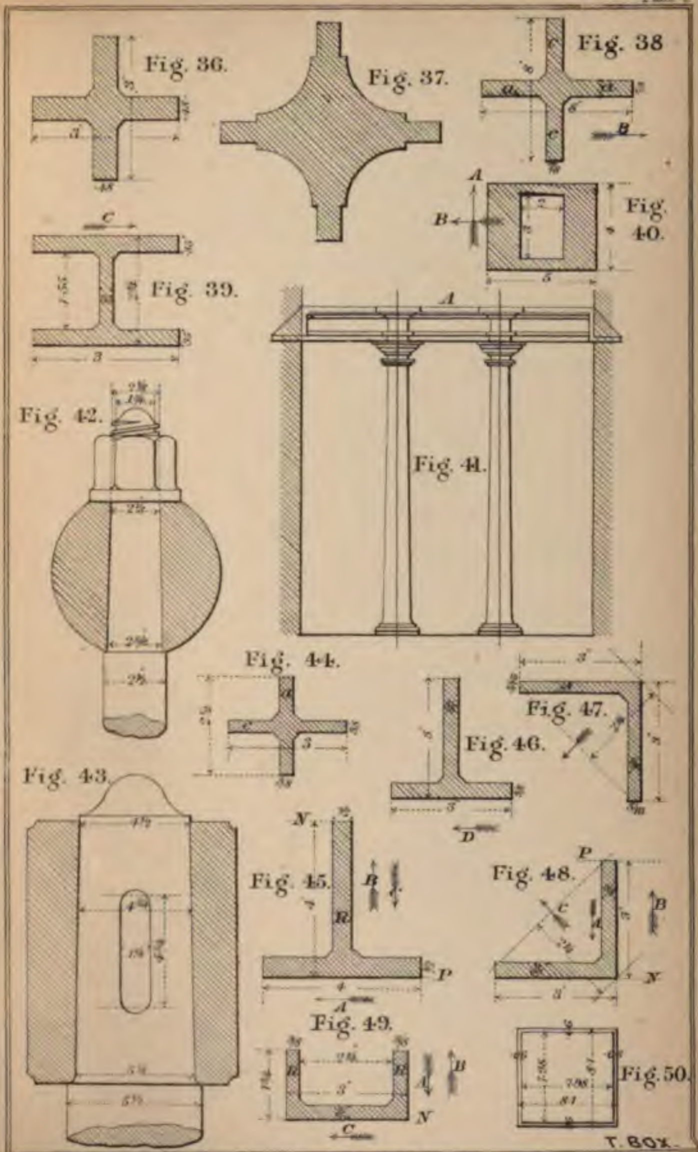

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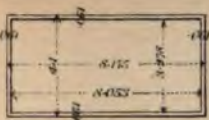
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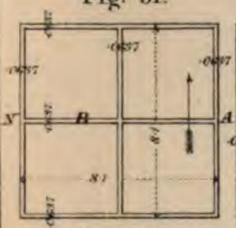
Fig. 52.


Fig. 53.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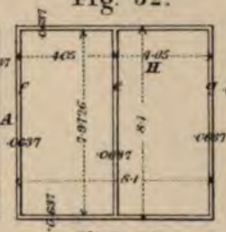
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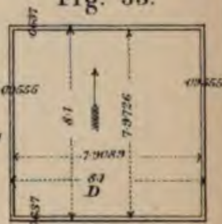


Fig. 56.


Fig. 57.

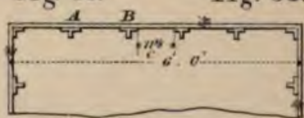
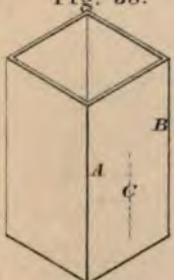


Fig. 62.

Fig. 58.



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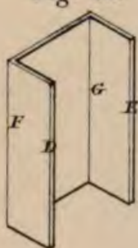

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Fig. 61.

Fig. 63.

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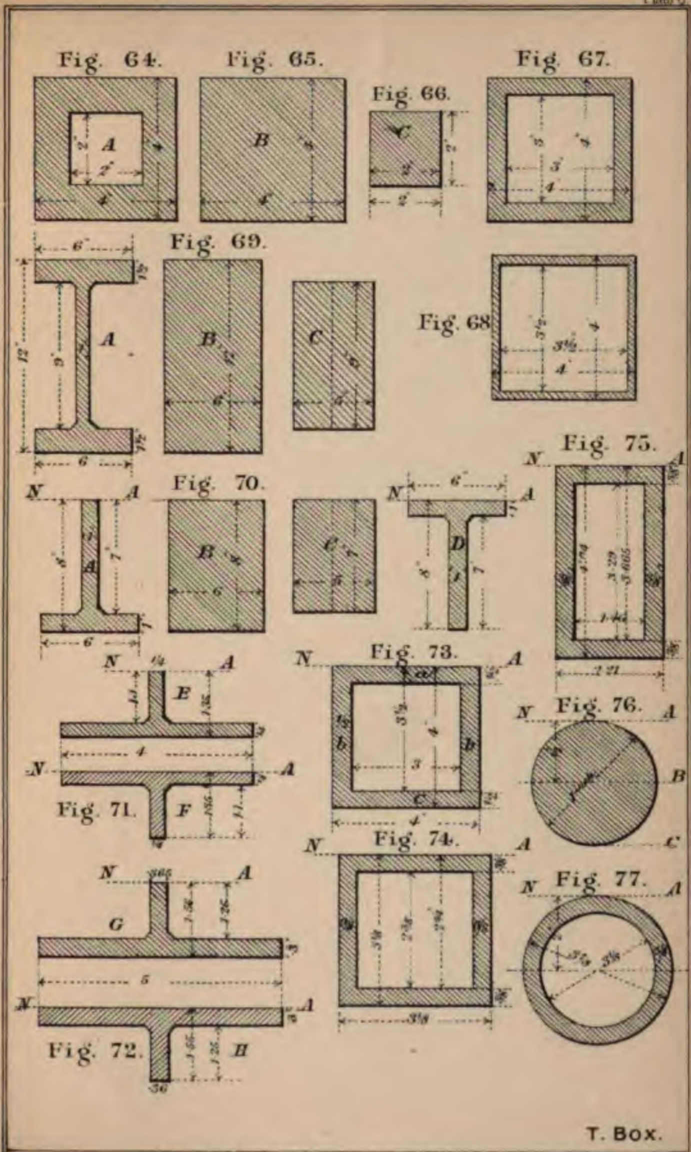

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Fig. 78.

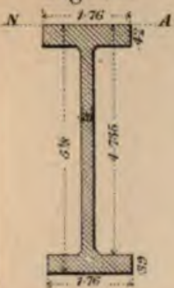
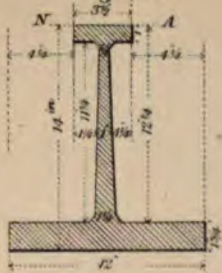


Fig. 79.



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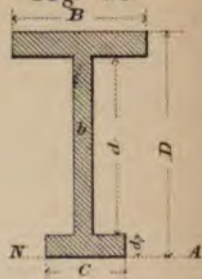

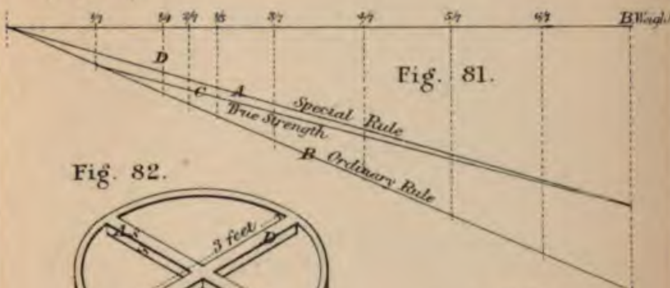

Fig. 81.

Fig. 82.

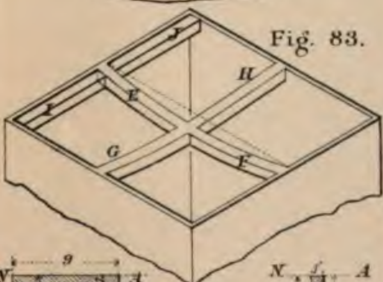

Fig. 83.

Section.

Fig. 84.

Plan.

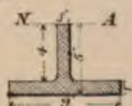

Fig. 86.

Fig. 85.

T. Box.

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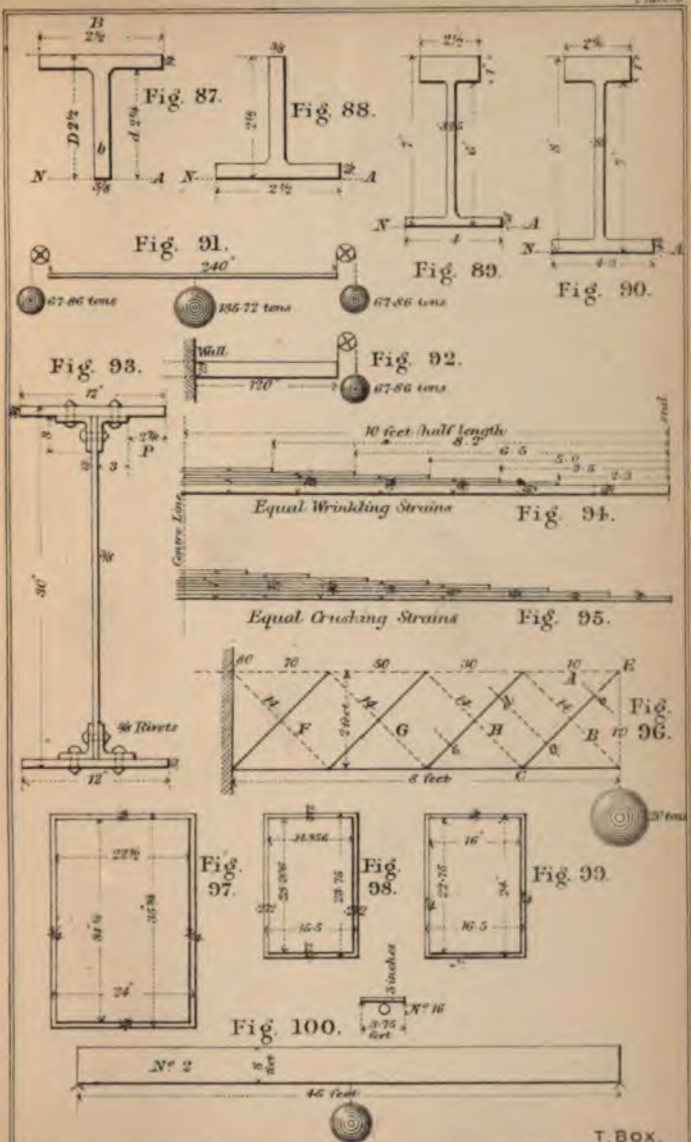
Plate 8



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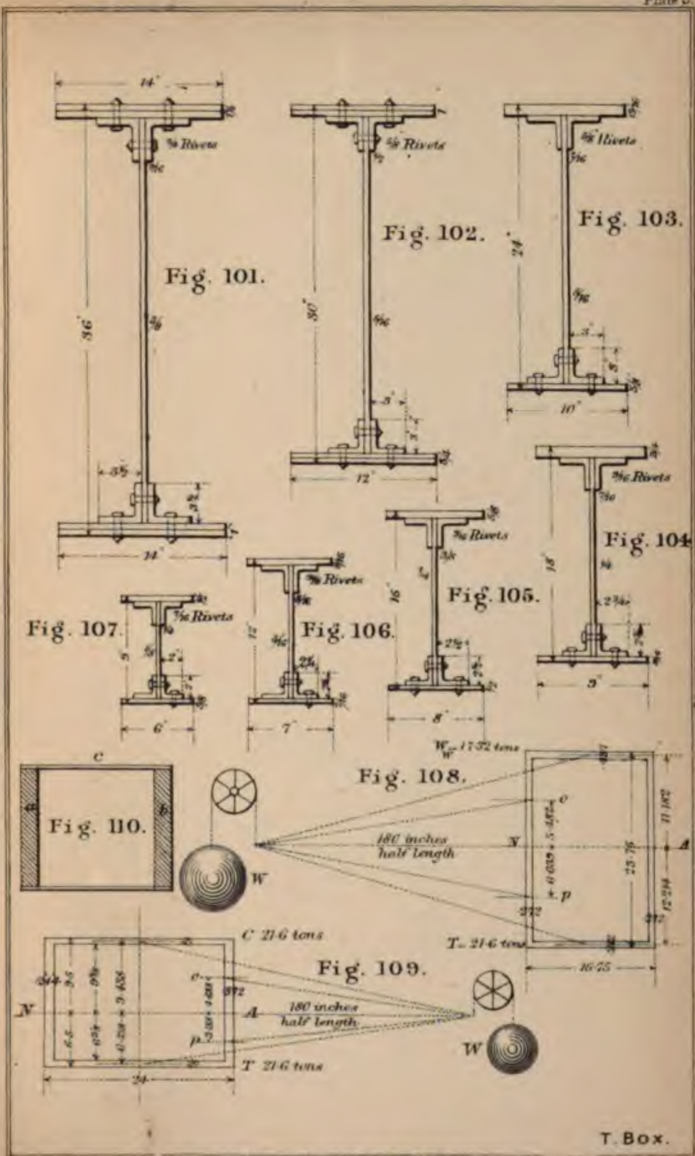

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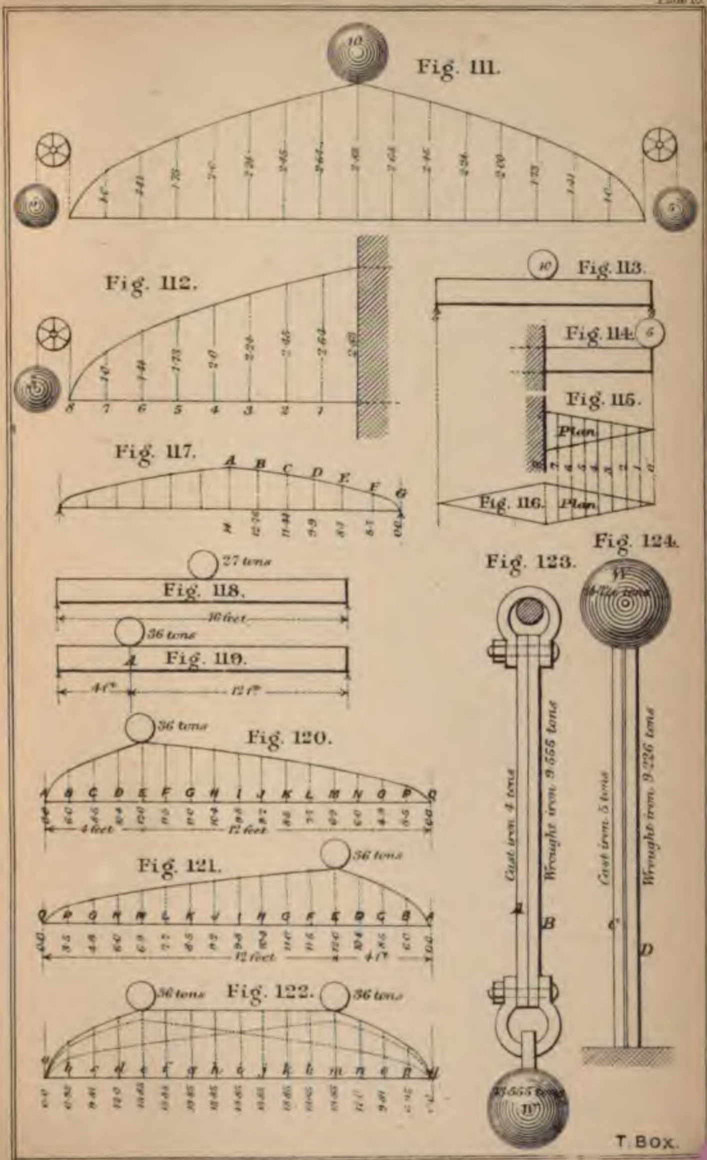
Fig. 111.
Fig. 112.
Fig. 113.
Fig. 114.
Fig. 115.
Plan
Fig. 116.
Plan
Fig. 117.
27 tons
Fig. 118.
16 feet
36 tons
Fig. 119.
36 tons
Fig. 120.
4 feet
12 feet
Fig. 121.
36 tons
12 feet
Fig. 122.
36 tons
36 tons
Fig. 123.
Cast iron 4 tons
Wrought iron 9.555 tons
Fig. 124.
Cast iron 5 tons
Wrought iron 9.226 tons
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Plate II.

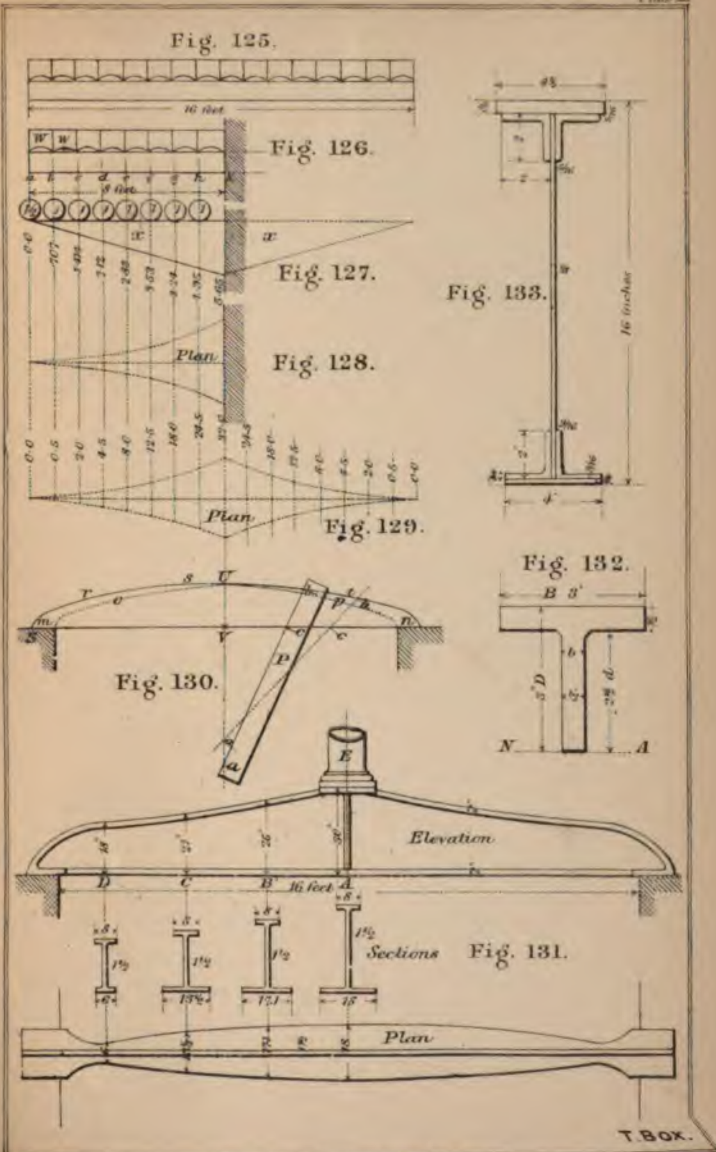

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Plate 12.

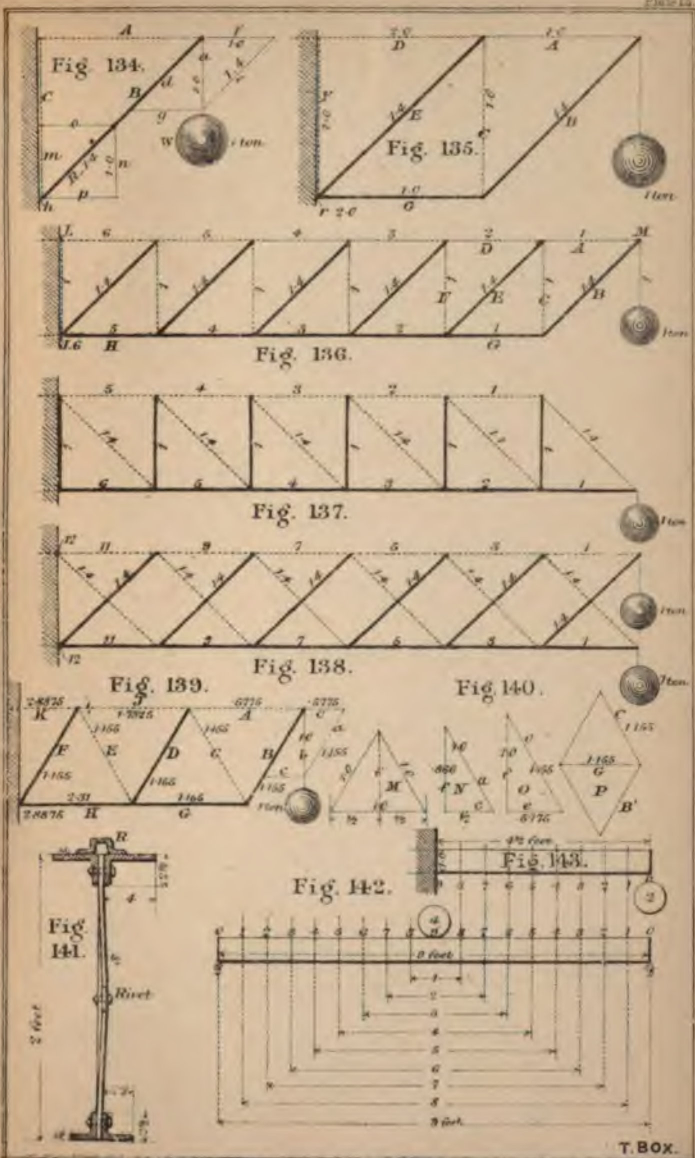

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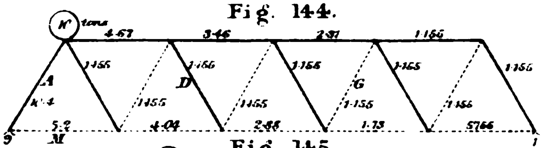
Plate 13

Fig. 144.



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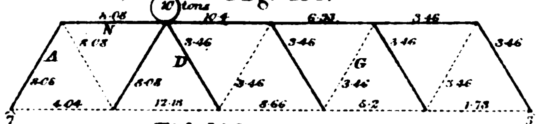

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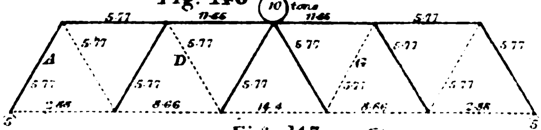

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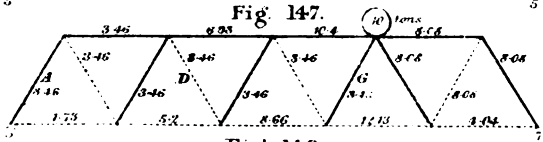

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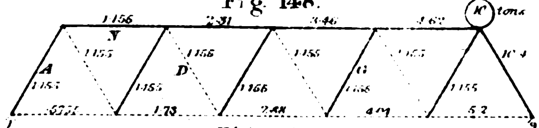

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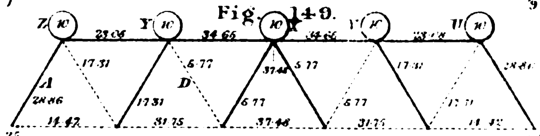

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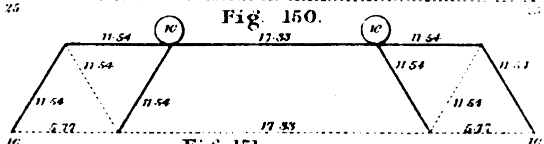
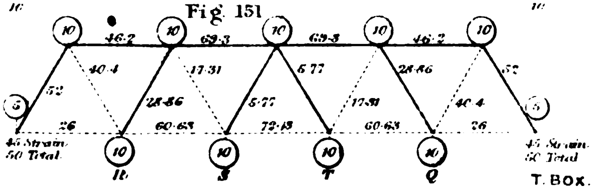


Fig. 151



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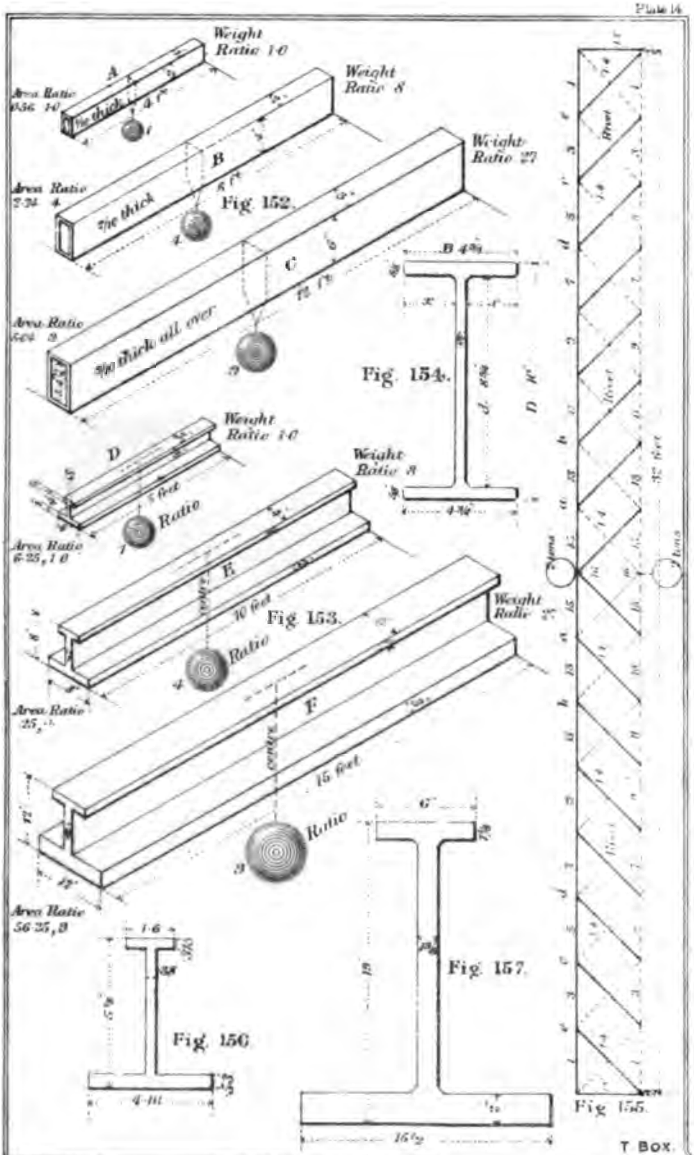

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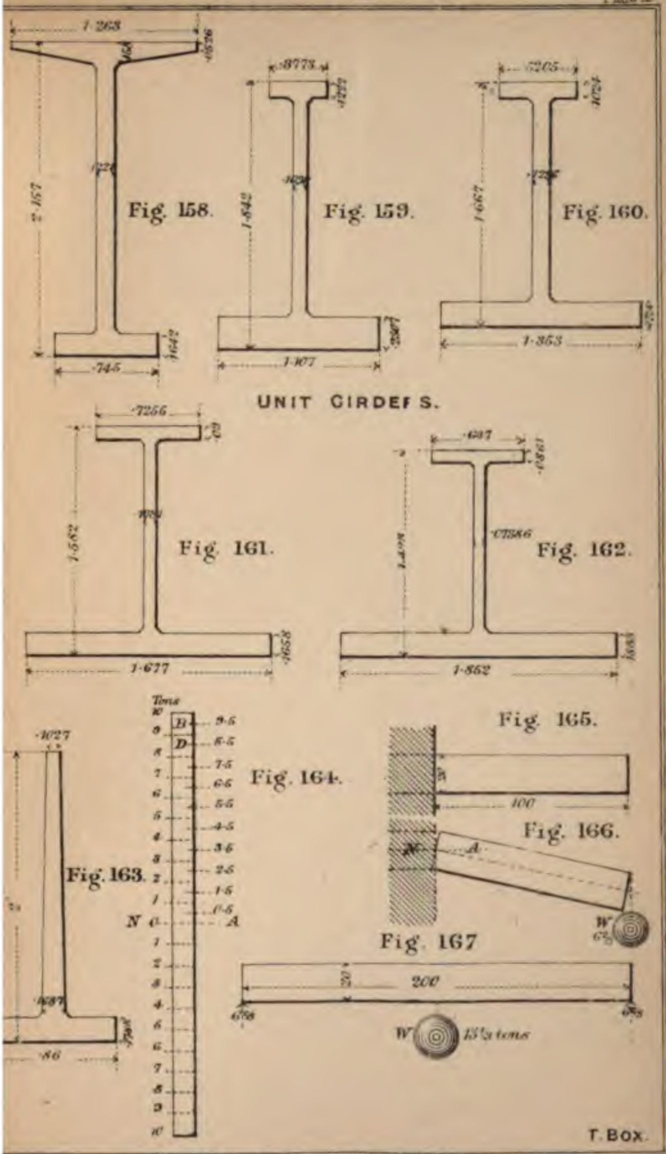

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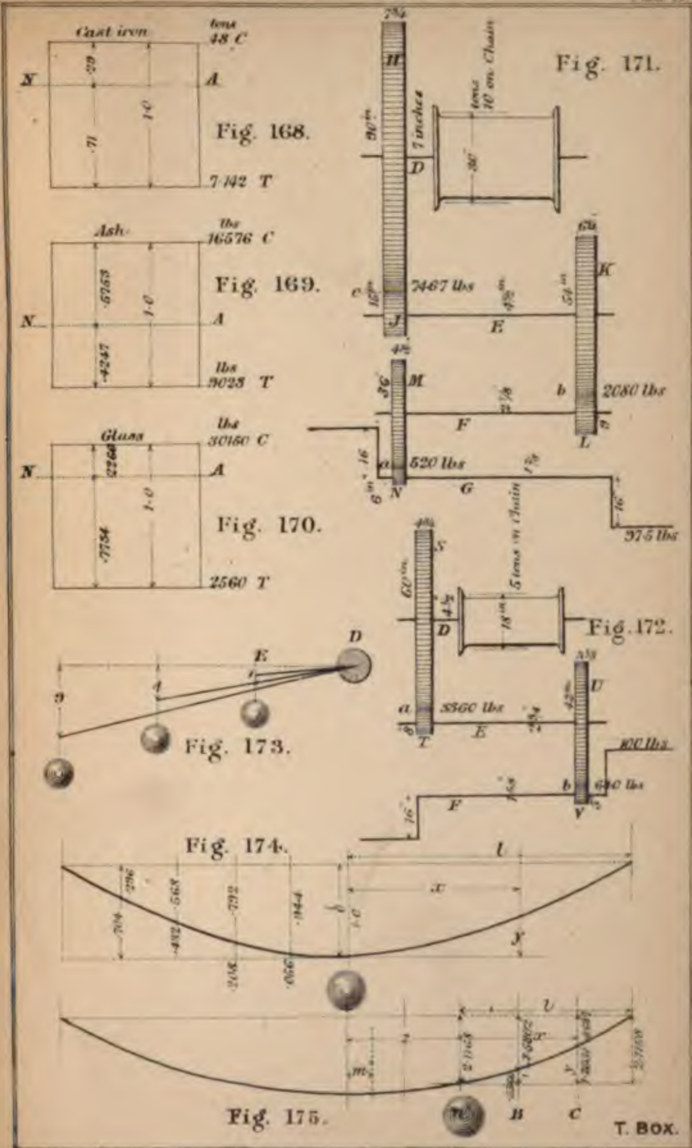

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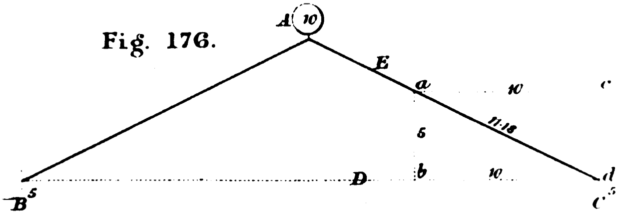
Fig. 176.



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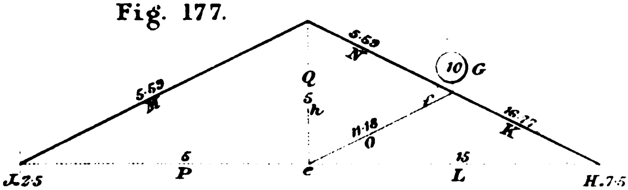

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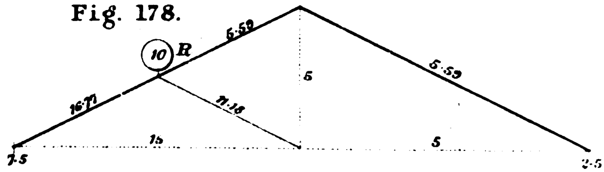
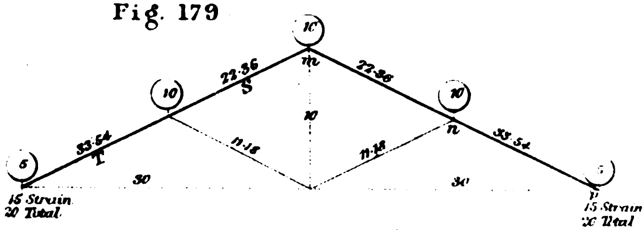


Fig. 179



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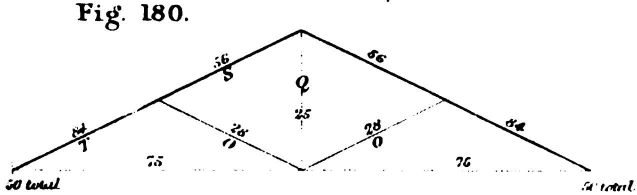


T. BOX.

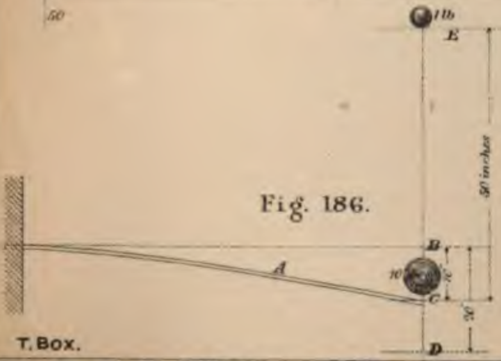
Fig. 181.

Fig. 182.

Fig. 183.

Fig. 184.

Fig. 185.



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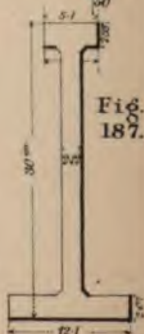

Fig. 187.

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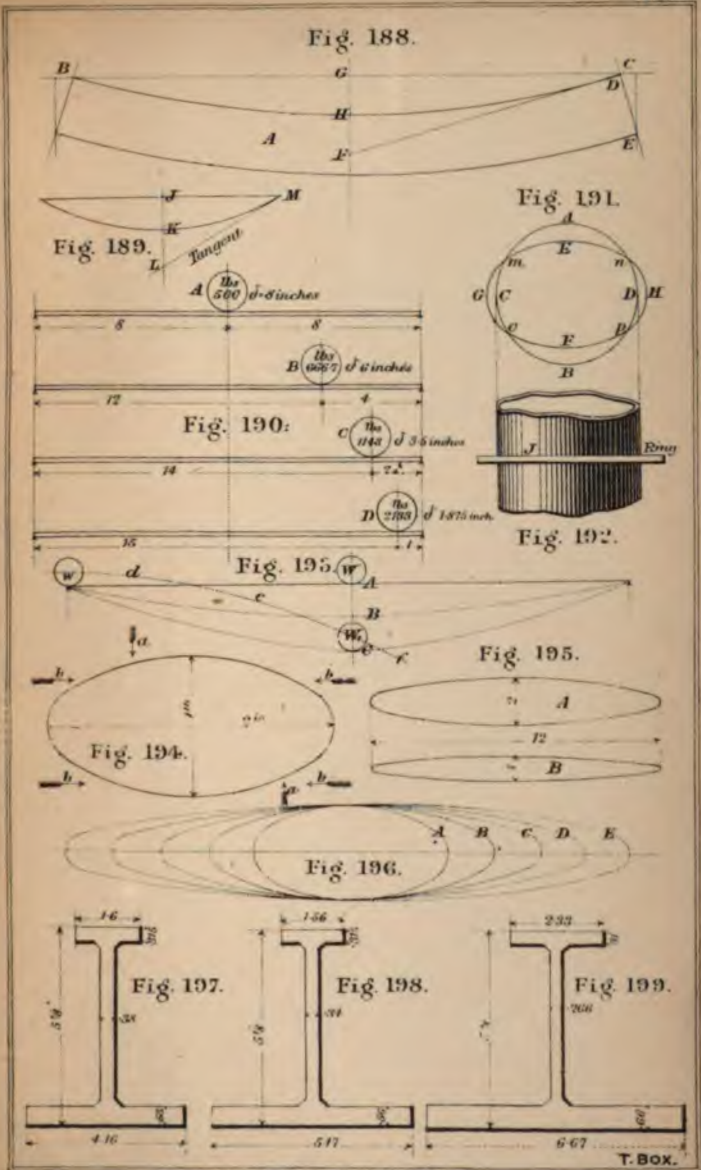

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Plate 2[illegible]

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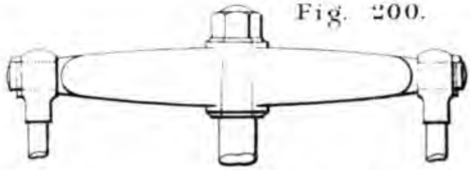

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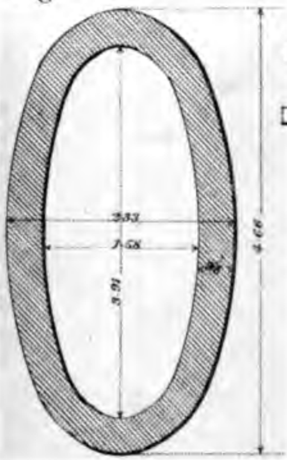

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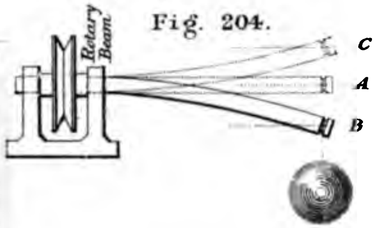

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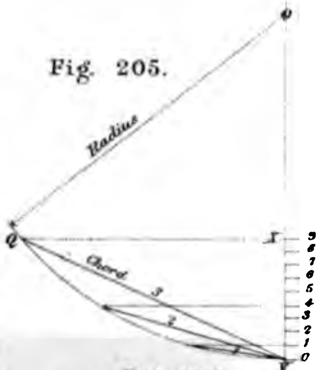

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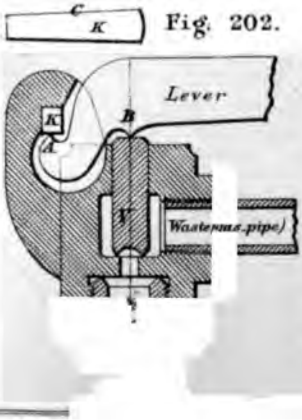

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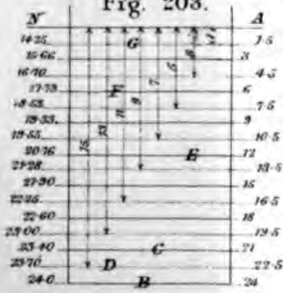

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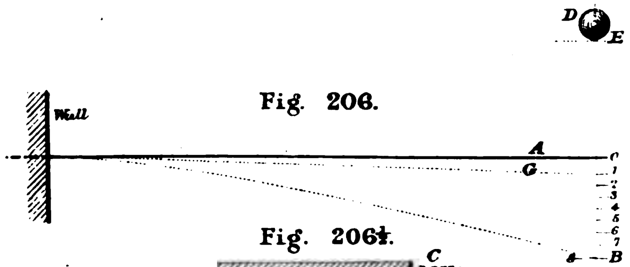

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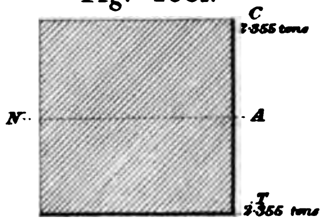

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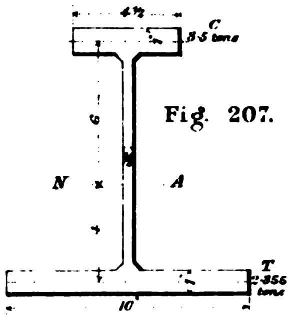

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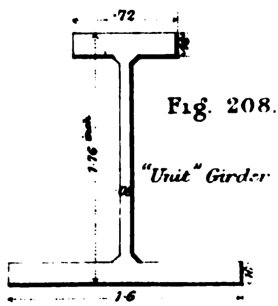

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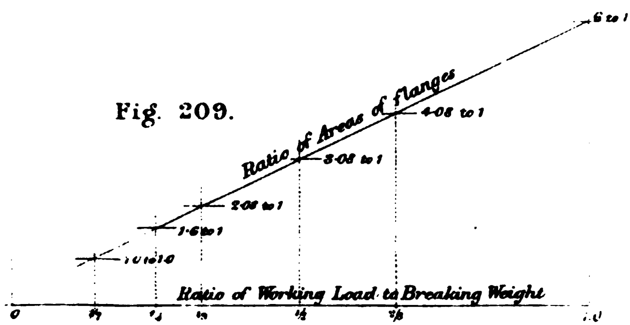

Fig. 209.

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